填充：

1. 每 105 個數有 $35+21+15-7-5-3+1=57$ 個與 3 或 5 或 7 不互質，有 $105-67=48$ 個互質，再計算 $2026=48\times 42+10$，且前 10 個為 $1,2,4,8,11,13,16,17,19,\boxed{22}$，則 $a_{2026}=105\times 42+22=4432$

2. $xa+yb+z=\log\left(2^{3x+2y+z}\times 3^{y}\times 5^{-2x+z}\right)=\log\left(2^{0}\times 3^{1}\times 5^{1}\right)$，解 $\left(x,y,z\right)=\left(\dfrac{-3}{5},1,\dfrac{-1}{5}\right)$

3. $P\left(\text{最後黑}\cap\text{紅比白後}\right)=\dfrac{5}{12}\times\dfrac{4}{7}=\dfrac{5}{21}$

4. 令 $g\left(x\right)=xf\left(x\right)-x^{2}-1$，則 $\deg\left(g\right)=11$ 且 $g\left(k\right)=0$，$k=1,2,3,\cdots,11$，可設

$g\left(x\right)=a\left(x-1\right)\left(x-2\right)\cdots\left(x-11\right)=xf\left(x\right)-x^{2}-1$，比較常數可知 $a=\dfrac{1}{11!}$。則

$g\left(13\right)=\dfrac{12!}{11!}=13f\left(13\right)-170\Rightarrow f\left(13\right)=\dfrac{182}{13}=14$

5. 設 $B\left(0,0\right)$，$C\left(5,0\right)$，$A\left(5,5k\right)$，$\overrightarrow{BA}=\left(1,k\right)\Rightarrow\overrightarrow{BD}=\left(4+3k,4k-3\right)\Rightarrow\overrightarrow{CD}=\left(3k-1,4k-3\right)$，

$\overrightarrow{CA}=\left(0,k\right)$，解 $\dfrac{(3k-1,4k-3)\bullet(0,k)}{\sqrt{\left(3k-1\right)^{2}+\left(4k-3\right)^{2}}\cdot k}=\dfrac{5}{13}\Rightarrow k=\dfrac{31}{33}$ 或 $\dfrac{41}{63}$ (平方增根不合) $\Rightarrow k=\dfrac{31}{33}$

6. $\left|z^{3}-3z-2\right|=\left|\left(z+1\right)^{2}\left(z-2\right)\right|$，令 $z=x+yi$ 且 $x^{2}+y^{2}=1$，可將原式改寫為 $\sqrt{\left(2+2x\right)^{2}\left(5-4x\right)}$ 且

$2\left(2+2x\right)+\left(5-4x\right)=9$，則 $\dfrac{\left(2+2x\right)+\left(2+2x\right)+\left(5-4x\right)}{3}\geq\sqrt[3]{\left(2+2x\right)^{2}\left(5-4x\right)}$，當

$2+2x=5-4x\Rightarrow x=\dfrac{1}{2}$ 時，$3\geq\sqrt[3]{\left(2+2x\right)^{2}\left(5-4x\right)}\Rightarrow 3\sqrt{3}\geq\sqrt{\left(2+2x\right)^{2}\left(5-4x\right)}$。

7. 令 $P\left(\cos\theta,\sin\theta\right)$，$\overrightarrow{PA}=\left(2-\cos\theta,-\sin\theta\right)$，$\overrightarrow{PB}=\left(\sin\theta,2-\cos\theta\right)$，

$B\left(\sin\theta+\cos\theta,2+\sin\theta-\cos\theta\right)$，其中 $\sin\theta+\cos\theta=\sqrt{2}\cos\left(\theta-45^{\circ}\right)$，

$\sin\theta-\cos\theta=\sqrt{2}\sin\left(\theta-45^{\circ}\right)$，則 $B$ 在 $x^{2}+\left(y-2\right)^{2}=2$ 上

8. 【解一】當 $\dfrac{x^{3}}{2}=x=\dfrac{2}{x}=\dfrac{4}{x^{3}}$，即 $x=\sqrt{2}$，以算幾不等式

$f\left(x\right)\geq 12\sqrt[12]{\dfrac{x^{3}}{2}\cdot\dfrac{x^{3}}{2}\cdot x\cdot x\cdot x\cdot\dfrac{2}{x}\cdot\dfrac{2}{x}\cdot\dfrac{2}{x}\cdot\dfrac{2}{x}\cdot\dfrac{2}{x}\cdot\dfrac{2}{x}\cdot\dfrac{4}{x^{3}}}=12\sqrt{2}$

【解二】令 $f\left(x\right)=x^{3}+3x+12x^{-1}+4x^{-3}\Rightarrow f'\left(x\right)=3x^{2}+3-12x^{-2}-16x^{-4}$

$=x^{-4}\left(3x^{6}+3x^{4}-12x^{2}-16\right)=x^{-4}\left(3x^{4}-12\right)\left(x^{2}+1\right)$，當 $x=\sqrt{2}$ 時 $f\left(x\right)$ 有極小值 $12\sqrt{2}$

9. $r_{xy}=\dfrac{Sxy}{\sqrt{Sxx}\sqrt{Syy}}=\dfrac{1}{2}$，$\sigma_x=\sqrt{\dfrac{Sxx}{n}}=1$，$\sigma_y=\sqrt{\dfrac{Syy}{n}}=2$，則$\dfrac{Sxy}{n}=1$，$\dfrac{Szz}{n}=\dfrac{Sxx}{n}+2\dfrac{Sxy}{n}+\dfrac{Syy}{n}=7$，

$\dfrac{Sxz}{n}=\dfrac{Sxx}{n}+\dfrac{Sxy}{n}=2$，$r_{xz}=\dfrac{Sxz}{\sqrt{Sxx}\sqrt{Szz}}=\dfrac{2}{\sqrt{7}}$

10. $F(x)-F(0)-8x^3+33x^2-18x=f(x)\Rightarrow f(x)-24x^2+66x-18=f'(x)$

$\Rightarrow f(x)-f'(x)=24x^2-66x+18\Rightarrow f(x)=24x^2-18x$

11. 令期望次數為$E$，$E_1$表示投出 1 後仍需的期望次數，$E_{12}$表示連續投出 12 後仍需的期望次數，

$E_{123}$表示連續投出 123 後仍需的期望次數，$E_{1234}=0$，則$E=1+\dfrac{1}{4}E_1+\dfrac{3}{4}E\Rightarrow E-E_1=4$，

$E_1=1+\dfrac{1}{4}E_{12}+\dfrac{1}{4}E_1+\dfrac{2}{4}E\Rightarrow E_1-E_{12}=12$，$E_{12}=1+\dfrac{1}{4}E_{123}+\dfrac{1}{4}E_1+\dfrac{2}{4}E\Rightarrow E_{12}-E_{123}=48$，

$E_{123}=1+\dfrac{1}{4}E_{1234}+\dfrac{1}{4}E_1+\dfrac{2}{4}E\Rightarrow E_{123}-E_{1234}=192\Rightarrow E_{123}=192\Rightarrow E_{12}=240\Rightarrow E_1=252\Rightarrow E=256$

12. 先在紙面上取$B$對$\overline{CE}$的垂足$P$，延長$\overline{BP}$交$\overline{AD}$為$H$，即為折起後$B$在$\overline{AD}$上的投影點，且紙面上所有三角形均為$1:2:\sqrt{5}$。整理有$\overline{AH}=\dfrac{3}{2}$，$\overline{EP}=\dfrac{2\sqrt{5}}{5}$，$\overline{BP}=\dfrac{4\sqrt{5}}{5}$，$\overline{PH}=\dfrac{7\sqrt{5}}{10}$，折起後

$\overline{BH}=\dfrac{\sqrt{5}}{10}\sqrt{8^2-7^2}=\dfrac{\sqrt{3}}{2}$，體積為$\dfrac{1}{3}\times\dfrac{(1+3)\times 4}{2}\times\dfrac{\sqrt{3}}{2}=\dfrac{4\sqrt{3}}{3}$

計算：

1. $\begin{cases}x-y=z-5\\-xy=z^2-3z+5\end{cases}$，則$x^2+y^2+xy=(x-y)^2+3xy=(z-5)^2-3(z^2-3z+5)=-2z^2-z+10$

$=-2\left(z+\dfrac{1}{4}\right)^2+\dfrac{81}{8}$且判別式$(z-5)^2-4(z^2-3z+5)\geq 0\Rightarrow -1\leq z\leq\dfrac{5}{3}$，則$z=\dfrac{-1}{4}$時有最大值$\dfrac{81}{8}$，

$z=\dfrac{5}{3}$時有最小值$\dfrac{25}{9}$。

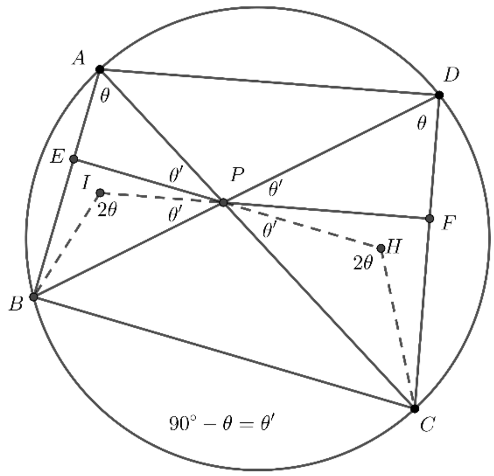
2. (1)令兩點為$A(a,a^2)$、$B(b,b^2)$在一、二象限且$b>0>a$，則$\overleftrightarrow{AB}:(a+b)x=y+ab$且

$\overrightarrow{OA}\bullet\overrightarrow{OB}=ab+a^2b^2=2\Rightarrow(ab-1)(ab+2)=0$，則$ab=-2$，必過點$P(0,2)$。

(2) 承上，則$OAB$面積$=\dfrac{1}{2}\left|\begin{vmatrix}a & a^2\\ b & b^2\end{vmatrix}\right|=b+(-a)$。又$F\left(0,\dfrac{1}{4}\right)$，則$COB$面積$=FOB$面積

$=\dfrac{1}{2}\left|\begin{vmatrix}b & b^2\\ 0 & \dfrac{1}{4}\end{vmatrix}\right|=\dfrac{b}{8}$，此時$OACB$面積為$\dfrac{9}{8}b+(-a)\geq 2\sqrt{\dfrac{-9ab}{8}}=3$

3. 根據圓心角與圓冪定理，$\theta' = 90° - \theta$
可由 $\Delta PAB \sim \Delta PDC$ 推得 $\Delta PEI \sim \Delta PFH$ ，
再由圓冪逆定理，可知 $E,F,H,I$ 四點共圓。




4. 由 $f(x) - f(x+5) \geq -3x - 9$ ①

及 $f(x+15) - f(x) \geq 9x + 72$ ②，

計算①+②得 $f(x+15) - f(x+5) \geq 6x + 63 \Rightarrow f(x+10) - f(x) \geq 6x + 33$ ③，

計算①+③得 $f(x+10) - f(x+5) \geq 3x + 24 \Rightarrow f(x+5) - f(x) \geq 3x + 9$ ④，

觀察①④可得 $f(x+5) - f(x) = 3x + 9$ ，再遞迴推出 $f(x+5k) - f(x) = 3kx + \dfrac{(15k+3)k}{2}$ 且

$2025 = 5 \times 405$，則 $f(x+2025) - f(x) = 1215x + 3039 \times 405$ 且 $f(0) = 1013$

$\Rightarrow f(2025) - f(0) = 3039 \times 405 \Rightarrow \dfrac{f(2025)}{2026} = \dfrac{1013 + 3039 \times 405}{2026} = 608$