

2025. 12. 8(-) ~ 12. 9(=) Stop ~ 12. 26(=) Ku 考生姓名: _____

一、填充題 (每格 5 分, 共 70 分, 請寫在答案卷上, 答案須化為最簡分數或最簡根式)

1. 請以牛頓法取初始值 $a_1 = 0$, 求方程式 $x^3 - 3x + 1 = 0$ 在 $0 < x < 1$ 範圍的實根之第三個近似值 $a_3 =$ 25/72

(A) $y - f(a_n) = f'(a_n)(x - a_n)$ 過 $(a_{n+1}, 0)$ $f(x) = x^3 - 3x + 1$ $a_2 = 0 - \frac{1}{-3} = \frac{1}{3}$

$\Rightarrow a_{n+1} = a_n - \frac{f(a_n)}{f'(a_n)}$ $f'(x) = 3x^2 - 3$ $a_3 = \frac{1}{3} - \frac{\frac{1}{27} - 1 + 1}{-\frac{8}{3}} = \frac{25}{72}$

2. 已知 $i = \sqrt{-1}$, 若 $\frac{1}{i^{2025}} - \frac{2}{i^{2024}} + \frac{3}{i^{2023}} - \frac{4}{i^{2022}} + \dots - \frac{2024}{i^2} + \frac{2025}{i} = a + bi$, 其中 a, b 為實數, 則 $a - b =$ (B)

$\frac{1}{i^{4k+1}} = -i$ $a - b = \begin{pmatrix} -2+4 \\ -6+8 \\ \vdots \\ -2022+2024 \end{pmatrix} - \begin{pmatrix} -1+3 \\ -5+7 \\ \vdots \\ -2021+2023 \end{pmatrix} = 2025$

$\frac{1}{i^{4k}} = 1$

$\frac{1}{i^{4k-1}} = i$

$\frac{1}{i^{4k-2}} = -1$

3. 創創自稱射擊之命中率為 0.6。某日創創表演射擊, 直到第 n 次時才首次命中靶面並停止, 設隨機變數 X 的取值表示創創首次命中靶面之射擊次數, 現在以顯著水準 $\alpha = 0.01$, 對命中率 0.6 進行假設檢定, 求「不拒絕」創創自稱射擊命中率为 0.6 的最大 n 值为 (C)。

$H_0: p = 0.6$ vs $H_1: p \neq 0.6$ $\sum_{k=1}^n 0.4^{k-1} \cdot 0.6 < |-0.01|$ $0.4^5 = 0.01024 > 0.01 \Rightarrow 5$

$X \sim \text{Geo}(p=0.6)$ $\Rightarrow |-0.4^n| < |-0.01| \Rightarrow 0.4^n > 0.01$ $0.4^6 = 0.004096 < 0.01$

4. 已知區域 R 滿足 $\begin{cases} x^2 + (y-2)^2 \leq 4 \\ y \geq 2 \end{cases}$, 將區域 R 繞 x 軸旋轉所得的旋轉體體積為 (D) (請以符號 π 表示答案)。

$\frac{32}{3}\pi + 8\pi^2$ $y = 2 + \sqrt{4-x^2}$ $2 \int_0^2 \pi \left((2 + \sqrt{4-x^2})^2 - 2^2 \right) dx$

$= 2\pi \left(\int_0^2 (4\sqrt{4-x^2} + 4 - x^2) dx \right) = 2\pi \left(4 \cdot \frac{\pi \cdot 4}{4} + 8 - \frac{8}{3} \right) = \frac{32}{3}\pi + 8\pi^2$

5. 守守將 10 元存入一家銀行推出的高頻複利定存方案, 該方案的年利率為 50%, 並且每年分成 n 次計息。當計息次數 n 趨近無限大時, 守守的這筆投資在一年後將趨近於某個金額 S 。已知 $e \approx 2.71828$, 請求出 S 的整數部分為 (E) (請用正整數表示)。

$10 \left(1 + \frac{1}{n} \right)^n$ $10 \left(1 + \frac{1}{2n} \right)^{2n} \rightarrow 10 \cdot e^{\frac{1}{2}} \div 16$

$\frac{1}{1} \overline{) 2.71}$
 $\frac{2}{2} \overline{) 171}$

6. 將橢圓 $\Gamma: \frac{x^2}{4} + \frac{y^2}{2} = 1$ 在坐標平面上，以原點 O 為中心逆時針旋轉 30° 得到橢圓 Γ' ，試求 Γ' 上任意一點 $P(x, y)$ 到直

$\frac{1+\sqrt{2}}{2}$

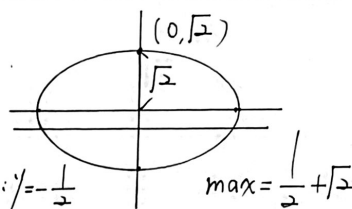
線 $L: x - \sqrt{3}y = 1$ 的距離最大值為 (F)

$L': x' - \sqrt{3}y' = 1 \Leftrightarrow L$

$x' + iy' \xrightarrow[\theta=30^\circ]{-\theta} x + iy$

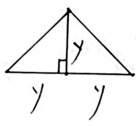
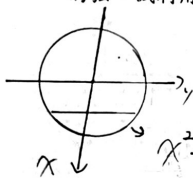
$x' + iy' = (x + iy) \left(\frac{\sqrt{3}}{2} + \frac{1}{2}i \right)$

$\frac{1}{2}(\sqrt{3}x - y) - \sqrt{3} \left(\frac{1}{2}(x + \sqrt{3}y) \right) = 1 \Rightarrow y = -\frac{1}{2}$



$\max x = \frac{1}{2} + \sqrt{2}$

7. 一個戶外露營活動需要搭建一個特殊形狀的立體帳篷。將帳篷的底座平放在 xy 平面上，為圓形區域 $x^2 + y^2 \leq 9$ 。已知帳篷內部每個垂直於 x 軸的截面，都是一個等腰直角三角形，且該三角形的斜邊恰好是底座圓上與 x 軸垂直的弦。試利用切片法，求出帳篷的內部體積為 (G)。



$\Delta = y^2 = 9 - x^2$

$2 \int_0^3 (9 - x^2) dx = 2 \left(27 - \frac{27}{3} \right) = 36$

2个骰子

1	2	3	4	5	6
1	2	3	4	5	6
2	3	4	5	6	7
3	4	5	6	7	8
4	5	6	7	8	9
5	6	7	8	9	10
6	7	8	9	10	11
7	8	9	10	11	12
8	9	10	11	12	13
9	10	11	12	13	14
10	11	12	13	14	15
11	12	13	14	15	16
12	13	14	15	16	17
13	14	15	16	17	18
14	15	16	17	18	19
15	16	17	18	19	20
16	17	18	19	20	21
17	18	19	20	21	22
18	19	20	21	22	23
19	20	21	22	23	24
20	21	22	23	24	25
21	22	23	24	25	26
22	23	24	25	26	27
23	24	25	26	27	28
24	25	26	27	28	29
25	26	27	28	29	30
26	27	28	29	30	31
27	28	29	30	31	32
28	29	30	31	32	33
29	30	31	32	33	34
30	31	32	33	34	35
31	32	33	34	35	36
32	33	34	35	36	37
33	34	35	36	37	38
34	35	36	37	38	39
35	36	37	38	39	40
36	37	38	39	40	41
37	38	39	40	41	42
38	39	40	41	42	43
39	40	41	42	43	44
40	41	42	43	44	45
41	42	43	44	45	46
42	43	44	45	46	47
43	44	45	46	47	48
44	45	46	47	48	49
45	46	47	48	49	50
46	47	48	49	50	51
47	48	49	50	51	52
48	49	50	51	52	53
49	50	51	52	53	54
50	51	52	53	54	55
51	52	53	54	55	56
52	53	54	55	56	57
53	54	55	56	57	58
54	55	56	57	58	59
55	56	57	58	59	60
56	57	58	59	60	61
57	58	59	60	61	62
58	59	60	61	62	63
59	60	61	62	63	64
60	61	62	63	64	65
61	62	63	64	65	66
62	63	64	65	66	67
63	64	65	66	67	68
64	65	66	67	68	69
65	66	67	68	69	70
66	67	68	69	70	71
67	68	69	70	71	72
68	69	70	71	72	73
69	70	71	72	73	74
70	71	72	73	74	75
71	72	73	74	75	76
72	73	74	75	76	77
73	74	75	76	77	78
74	75	76	77	78	79
75	76	77	78	79	80
76	77	78	79	80	81
77	78	79	80	81	82
78	79	80	81	82	83
79	80	81	82	83	84
80	81	82	83	84	85
81	82	83	84	85	86
82	83	84	85	86	87
83	84	85	86	87	88
84	85	86	87	88	89
85	86	87	88	89	90
86	87	88	89	90	91
87	88	89	90	91	92
88	89	90	91	92	93
89	90	91	92	93	94
90	91	92	93	94	95
91	92	93	94	95	96
92	93	94	95	96	97
93	94	95	96	97	98
94	95	96	97	98	99
95	96	97	98	99	100

$\left(\frac{9}{5}, \frac{3}{2}, \frac{6}{5} \right)$

8. 已知空間中三向量 $\vec{OA}, \vec{OB}, \vec{OC}$ 所張的平行六面體體積為 60。若點 D 滿足 $\vec{OD} = x\vec{OA} + y\vec{OB} + z\vec{OC}$ ，求 $(x, y, z) =$ (H)

$12 = \frac{1}{6} \left| \begin{vmatrix} \vec{OA} \\ \vec{OB} \\ \vec{OD} \end{vmatrix} \right| = \frac{1}{6} \cdot z \cdot 60$

$15 = \frac{1}{6} \left| \begin{vmatrix} \vec{OA} \\ \vec{OC} \\ \vec{OD} \end{vmatrix} \right| = \frac{1}{6} \cdot y \cdot 60$

$18 = \frac{1}{6} \left| \begin{vmatrix} \vec{OB} \\ \vec{OC} \\ \vec{OD} \end{vmatrix} \right| = \frac{1}{6} \cdot x \cdot 60$

$\Rightarrow \left(\frac{9}{5}, \frac{3}{2}, \frac{6}{5} \right)$

9. 設函數 $f: (0, \infty) \rightarrow \mathbb{R}$ ，且當 $x > 0$ 時，恆有 $\lim_{n \rightarrow \infty} \frac{1}{3n-1} \left(f\left(\frac{x^2}{n}\right) + f\left(\frac{2x^2}{n}\right) + \dots + f\left(\frac{nx^2}{n}\right) \right) = \frac{\sqrt[3]{7+x}}{x}$ ，求 $f(1) =$ (I)

$\sum_{k=1}^n \frac{1}{3n} f\left(x^2 \cdot \frac{k}{n}\right)$

$u = x^2 t \Rightarrow du = x^2 dt$
 $0 \leq t \leq 1 \Rightarrow 0 \leq u \leq x^2$

$\int_0^{x^2} f(u) du = 3x^2 \left(\frac{1}{3} \right)^{\frac{1}{3}} \Rightarrow f(1) = \frac{3}{2} \left(\frac{1}{3} \cdot \frac{1}{4} + 2 \right)$

$\Rightarrow f(1) = \frac{3}{2} \left(\frac{1}{3} \cdot \frac{1}{4} + 2 \right)$

$3x^2 \cdot \frac{1}{3} \int_0^1 f(x^2 t) dt = \frac{(x+7)^{\frac{1}{3}}}{x} \cdot 3x^2$

$\Rightarrow f(x^2) \cdot 2x^2 = \left(x \cdot \frac{1}{3} (x+7)^{\frac{1}{3}} + (x+7)^{\frac{1}{3}} \right) = \frac{25}{8}$

10. 擲一顆公正骰子 6 次，令出現的點數依序為隨機變數 $X_1, X_2, X_3, X_4, X_5, X_6$ ，試求 $X_1 - X_2 + X_3 - X_4 + X_5 - X_6 = 10$ 的

機率為 (J)

3个骰子

18	3	1
17	4	1+2
16	5	1+2+3
15	6	1+2+3+4
14	7	1+2+3+4+5
13	8	1+2+3+4+5+6
12	9	2+3+4+5+6+5
11	10	3+4+5+6+5+4

(A+B+C)-(D+E+F)=10

1	3
2	4
3	5
4	6
5	7
6	8
7	9
8	10

$\frac{1}{2} \times 1 = \frac{1}{2} \times 2$
 $\frac{1}{3} \times 3 = \frac{1}{3} \times 6$
 $\frac{1}{4} \times 4 = \frac{1}{4} \times 12$
 $\frac{1}{5} \times 5 = \frac{1}{5} \times 20$
 $\frac{1}{6} \times 6 = \frac{1}{6} \times 30$
 $\frac{1}{7} \times 7 = \frac{1}{7} \times 42$
 $\frac{1}{8} \times 8 = \frac{1}{8} \times 56$
 $\frac{1}{9} \times 9 = \frac{1}{9} \times 72$
 $\frac{1}{10} \times 10 = \frac{1}{10} \times 90$
 $\frac{1}{11} \times 11 = \frac{1}{11} \times 110$
 $\frac{1}{12} \times 12 = \frac{1}{12} \times 132$
 $\frac{1}{13} \times 13 = \frac{1}{13} \times 156$
 $\frac{1}{14} \times 14 = \frac{1}{14} \times 182$
 $\frac{1}{15} \times 15 = \frac{1}{15} \times 210$
 $\frac{1}{16} \times 16 = \frac{1}{16} \times 240$
 $\frac{1}{17} \times 17 = \frac{1}{17} \times 272$
 $\frac{1}{18} \times 18 = \frac{1}{18} \times 306$
 $\frac{1}{19} \times 19 = \frac{1}{19} \times 342$
 $\frac{1}{20} \times 20 = \frac{1}{20} \times 380$
 $\frac{1}{21} \times 21 = \frac{1}{21} \times 420$
 $\frac{1}{22} \times 22 = \frac{1}{22} \times 462$
 $\frac{1}{23} \times 23 = \frac{1}{23} \times 506$
 $\frac{1}{24} \times 24 = \frac{1}{24} \times 552$
 $\frac{1}{25} \times 25 = \frac{1}{25} \times 600$
 $\frac{1}{26} \times 26 = \frac{1}{26} \times 650$
 $\frac{1}{27} \times 27 = \frac{1}{27} \times 702$
 $\frac{1}{28} \times 28 = \frac{1}{28} \times 756$
 $\frac{1}{29} \times 29 = \frac{1}{29} \times 812$
 $\frac{1}{30} \times 30 = \frac{1}{30} \times 870$
 $\frac{1}{31} \times 31 = \frac{1}{31} \times 930$
 $\frac{1}{32} \times 32 = \frac{1}{32} \times 992$
 $\frac{1}{33} \times 33 = \frac{1}{33} \times 1056$
 $\frac{1}{34} \times 34 = \frac{1}{34} \times 1122$
 $\frac{1}{35} \times 35 = \frac{1}{35} \times 1190$
 $\frac{1}{36} \times 36 = \frac{1}{36} \times 1260$
 $\frac{1}{37} \times 37 = \frac{1}{37} \times 1332$
 $\frac{1}{38} \times 38 = \frac{1}{38} \times 1406$
 $\frac{1}{39} \times 39 = \frac{1}{39} \times 1482$
 $\frac{1}{40} \times 40 = \frac{1}{40} \times 1560$
 $\frac{1}{41} \times 41 = \frac{1}{41} \times 1640$
 $\frac{1}{42} \times 42 = \frac{1}{42} \times 1722$
 $\frac{1}{43} \times 43 = \frac{1}{43} \times 1806$
 $\frac{1}{44} \times 44 = \frac{1}{44} \times 1892$
 $\frac{1}{45} \times 45 = \frac{1}{45} \times 1980$
 $\frac{1}{46} \times 46 = \frac{1}{46} \times 2070$
 $\frac{1}{47} \times 47 = \frac{1}{47} \times 2162$
 $\frac{1}{48} \times 48 = \frac{1}{48} \times 2256$
 $\frac{1}{49} \times 49 = \frac{1}{49} \times 2352$
 $\frac{1}{50} \times 50 = \frac{1}{50} \times 2450$
 $\frac{1}{51} \times 51 = \frac{1}{51} \times 2550$
 $\frac{1}{52} \times 52 = \frac{1}{52} \times 2652$
 $\frac{1}{53} \times 53 = \frac{1}{53} \times 2756$
 $\frac{1}{54} \times 54 = \frac{1}{54} \times 2862$
 $\frac{1}{55} \times 55 = \frac{1}{55} \times 2970$
 $\frac{1}{56} \times 56 = \frac{1}{56} \times 3080$
 $\frac{1}{57} \times 57 = \frac{1}{57} \times 3192$
 $\frac{1}{58} \times 58 = \frac{1}{58} \times 3306$
 $\frac{1}{59} \times 59 = \frac{1}{59} \times 3422$
 $\frac{1}{60} \times 60 = \frac{1}{60} \times 3540$
 $\frac{1}{61} \times 61 = \frac{1}{61} \times 3660$
 $\frac{1}{62} \times 62 = \frac{1}{62} \times 3782$
 $\frac{1}{63} \times 63 = \frac{1}{63} \times 3906$
 $\frac{1}{64} \times 64 = \frac{1}{64} \times 4032$
 $\frac{1}{65} \times 65 = \frac{1}{65} \times 4160$
 $\frac{1}{66} \times 66 = \frac{1}{66} \times 4290$
 $\frac{1}{67} \times 67 = \frac{1}{67} \times 4422$
 $\frac{1}{68} \times 68 = \frac{1}{68} \times 4556$
 $\frac{1}{69} \times 69 = \frac{1}{69} \times 4692$
 $\frac{1}{70} \times 70 = \frac{1}{70} \times 4830$
 $\frac{1}{71} \times 71 = \frac{1}{71} \times 4970$
 $\frac{1}{72} \times 72 = \frac{1}{72} \times 5112$
 $\frac{1}{73} \times 73 = \frac{1}{73} \times 5256$
 $\frac{1}{74} \times 74 = \frac{1}{74} \times 5402$
 $\frac{1}{75} \times 75 = \frac{1}{75} \times 5550$
 $\frac{1}{76} \times 76 = \frac{1}{76} \times 5700$
 $\frac{1}{77} \times 77 = \frac{1}{77} \times 5852$
 $\frac{1}{78} \times 78 = \frac{1}{78} \times 6006$
 $\frac{1}{79} \times 79 = \frac{1}{79} \times 6162$
 $\frac{1}{80} \times 80 = \frac{1}{80} \times 6320$
 $\frac{1}{81} \times 81 = \frac{1}{81} \times 6480$
 $\frac{1}{82} \times 82 = \frac{1}{82} \times 6642$
 $\frac{1}{83} \times 83 = \frac{1}{83} \times 6806$
 $\frac{1}{84} \times 84 = \frac{1}{84} \times 6972$
 $\frac{1}{85} \times 85 = \frac{1}{85} \times 7140$
 $\frac{1}{86} \times 86 = \frac{1}{86} \times 7310$
 $\frac{1}{87} \times 87 = \frac{1}{87} \times 7482$
 $\frac{1}{88} \times 88 = \frac{1}{88} \times 7656$
 $\frac{1}{89} \times 89 = \frac{1}{89} \times 7832$
 $\frac{1}{90} \times 90 = \frac{1}{90} \times 8010$
 $\frac{1}{91} \times 91 = \frac{1}{91} \times 8190$
 $\frac{1}{92} \times 92 = \frac{1}{92} \times 8372$
 $\frac{1}{93} \times 93 = \frac{1}{93} \times 8556$
 $\frac{1}{94} \times 94 = \frac{1}{94} \times 8742$
 $\frac{1}{95} \times 95 = \frac{1}{95} \times 8930$
 $\frac{1}{96} \times 96 = \frac{1}{96} \times 9120$
 $\frac{1}{97} \times 97 = \frac{1}{97} \times 9312$
 $\frac{1}{98} \times 98 = \frac{1}{98} \times 9506$
 $\frac{1}{99} \times 99 = \frac{1}{99} \times 9702$
 $\frac{1}{100} \times 100 = \frac{1}{100} \times 9900$

11. 設空間中三個向量如下： $\vec{a} = (2, 1, k)$ ， $\vec{b} = (1, 2, -2)$ ， $\vec{c} = (-2, 2, 1)$ 。若已知對於所有實數 t, s ， $|\vec{a} + t\vec{b} + s\vec{c}|$ 的最小值為 5，求所有滿足條件的實數 k 值之總和為 (K)。

$\begin{pmatrix} 2 & -2 & 1 \\ 2 & 1 & -2 \end{pmatrix} \Rightarrow \begin{pmatrix} 6 & 3 & 6 \end{pmatrix} \parallel \begin{pmatrix} 2 & 1 & 2 \end{pmatrix}$

$((t-2s+2)^2 + (2t+2s+1)^2 + (-2t+s+k)^2)(2^2 + 1^2 + 2^2) \geq (2(k+5))^2 = 25 \cdot 9$

$2(k+5) = 15 \text{ or } -15 \Rightarrow k = 5 \text{ or } -10 \Rightarrow -5$