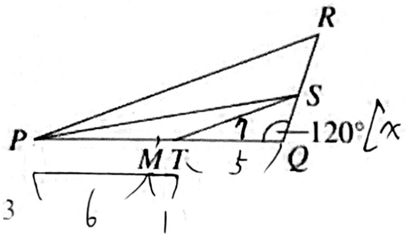
114南二中

12. In the diagram shown, M is the mid-point of PQ. The line PS bisects $\angle RPQ$ and intersects RQ at S. The line ST is parallel to PR and intersects PQ at T. The length of PQ is 12 and the length of MT is 1. The angle SQT is 120°. What is the length of SQ?



A : 3

$49 = 25 + x^2 + 5x$

$x^2 + 5x - 24 = 0$

$x + 8$ $\Rightarrow 3$
$x - 3$

13. The first two terms of an infinite geometric sequence, in order, are

$$3log_3 x,\ 2log_3 x, \text{ where } x > 0.$$

The first three terms of an arithmetic sequence, in order, are

$$log_3 x, log_3 \frac{x}{3}, log_3 \frac{x}{9}, \text{ where } x > 0.$$

Let S_6 be the sum of the first 6 terms of the arithmetic sequence. Given that S_6 is equal to one third of the sum of the infinite geometric sequence, find x.

A : 243 $\quad r = \frac{2}{3}$ $\quad S_6 = (2\log_3 x + 5(-1))\cdot 3 = \dfrac{\log_3 x}{1-\frac{2}{3}} \Rightarrow \log_3 x = 5 \Rightarrow x = 243$

$d = -1$

14. A geometric transformation T : $(x, y) \to (x', y')$ consists of three transformations in order :

- A rotation of θ radians $(0 \le \theta < 2\pi)$ about the origin, followed by
- An enlargement with scale factor 2, centred at the origin, followed by
- A translation of 1 unit right and 3 units down.

Given that the transformation T maps the point $\left(\frac{7}{5}, -\frac{1}{5}\right)$ to itself, find the angle θ of the rotation.

A : $\frac{\pi}{2}$ $\quad \frac{7}{5} - \frac{1}{5}i = \left(\frac{7}{5} - \frac{1}{5}i\right)\cdot 2(c + is) + (1 - 3i)$

$\Rightarrow \begin{array}{l} 7 = 2(7c+s)+5 \\ -1 = 2(-c+7s)-15 \end{array} \Rightarrow \begin{cases} 7c+s=1 \\ -c+7s=7 \end{cases} \Rightarrow \begin{array}{l} \sin\theta = 1 \\ \cos\theta = 0 \end{array} \Rightarrow \theta = \frac{\pi}{2}$

第二部分：計算證明題 (每題 10 分，共 30 分)

1. 已知 n 為任意正整數時，數列$\langle a_n\rangle$的前 n 項和S_n使得 $\frac{S_n}{S_n+n^2+n}$ 是一個定值，試證明：$\langle a_n\rangle$必不為等比數列。

① 令 $\frac{S_n}{S_n+n(n+1)}=k \Rightarrow S_n=\frac{k}{1-k}n(n+1) \Rightarrow a_n=\frac{k}{1-k}(2n) \Rightarrow \frac{a_2}{a_1}=\frac{2}{1}\neq\frac{3}{2}=\frac{a_3}{a_2}$

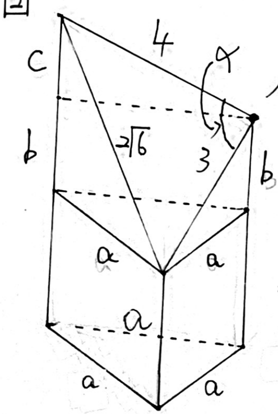
2. 今空間中位於平面 F 上的一正 ΔABC 在另一平面 E 上的投影為 $\Delta A'B'C'$，已知 $\Delta A'B'C'$ 之三邊長分別為 $\overline{A'B'}=4$、$\overline{A'C'}=3$、$\overline{B'C'}=2\sqrt{6}$，若 θ 表平面 E 與平面 F 之夾角，則 $\cos\theta=$？

$\pm\frac{\sqrt{69}}{15}$

3. 已知 a,b 為 $x^2+\frac{x^2}{(x+1)^2}=1$ 的兩個相異實根，試求 ab。

$1-\sqrt{2}$

②



$\sqrt{16-a^2}+\sqrt{9-a^2}=\sqrt{24-a^2}$

令 $t=9-a^2$

$\Rightarrow \sqrt{t+7}+\sqrt{t}=\sqrt{t+15}$

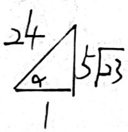
$\Rightarrow 2\sqrt{t(t+7)}=8-t$

$\Rightarrow 3t^2+44t-64=0$

$\begin{matrix}3 & -4\\ 1 & +16\end{matrix} \Rightarrow t=\frac{4}{3}=9-a^2$

$\Rightarrow a^2=\frac{23}{3}$

$\cos\varphi=\frac{1}{2\cdot3\cdot4}$



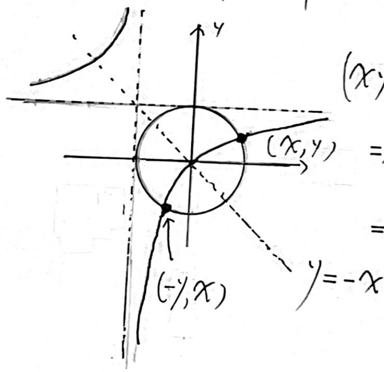
$|\cos\theta|=\frac{\Delta}{\Delta'}=\frac{\frac{\sqrt3}{4}\cdot\frac{23}{3}}{\frac12\cdot3\cdot4\cdot\frac{5\sqrt{23}}{24}}=\frac{\sqrt{69}}{15}$

$\Rightarrow \cos\theta=\pm\frac{\sqrt{69}}{15}$

③ 令 $y=\frac{x}{x+1}$

$\Rightarrow xy-x+y=0$

$\begin{cases}(x+1)(y-1)=-1\\ x^2+y^2=1\end{cases}$



$-ab=+xy$

$(xy)^2=(x-y)^2=1-2xy$

$\Rightarrow ab^2-2ab-1=0$

$\Rightarrow ab=1-\sqrt2$