

2025.7.22(二) ~ 7.25(五) Ru

第一部分：填充題 (每題 5 分，共 70 分)

1. 在某場比賽中，五位評審各給出一個介於 1 到 10 之間的整數分數，並計算這五位評審所給出分數的算術平均數、中位數和眾數。已知五個分數的算術平均數為 7、中位數為 8 且眾數為 9，請找出所有可能的分數組合。

$$\boxed{1} \quad 35-26=9 \quad \rightarrow \quad \boxed{2} \quad \begin{array}{c} 2 \\ 7 \\ 3 \\ 6 \\ 4 \\ 5 \end{array} \quad \begin{array}{c} 8 \\ 9 \\ 9 \end{array}$$

A : {2, 7, 8, 9, 9} 或 {3, 6, 8, 9, 9} 或 {4, 5, 8, 9, 9} 不全

$$\boxed{4} \quad a_1 \quad a_4 \quad \begin{array}{c} a_1 \quad a_2 \quad a_3 \quad a_4 \quad a_5 \\ 1 \quad 2 \quad 3 \quad 9 \quad 1 \quad 4 \quad 9 \quad 8 \end{array}$$

$$\begin{array}{c} 2 \\ 3 \\ 4 \\ 5 \end{array} \quad \begin{array}{c} 8 \\ 7 \\ 6 \end{array} \quad \begin{array}{c} \text{不} \\ \text{全} \end{array} \quad \begin{array}{c} 1 \\ 3 \\ 4 \\ 9 \end{array} \quad \begin{array}{c} 1 \\ 9 \\ 16 \\ 8 \end{array}$$

$$2. \text{ 試求 } \lim_{n \rightarrow \infty} \left(\frac{n}{(n+4)^2} + \frac{n}{(n+8)^2} + \dots + \frac{n}{(n+4n)^2} \right) = ?$$

$$\boxed{2} \quad \frac{1}{n} \cdot \frac{1}{(1+4(\frac{1}{n}))^2} \Rightarrow \int_0^1 \frac{1}{(4x+1)^2} dx$$

$$A : \frac{1}{5} \quad \text{令 } u = 4x+1, du = 4dx \quad = \int_1^5 \frac{1}{4u^2} du = \frac{1}{4u} \Big|_1^5 = \frac{1}{5}$$

$$= 114$$

$$= 110 = \boxed{10} + \boxed{10}^2 \quad \text{ok}$$

3. 平面 E 通過 $(2, -2, -1)$ 及 $(4, -4, -3)$ 。若 $P(a, b, c)$ 在平面 E 上的投影點為 $Q(1, 4, 0)$ ，且 $\overline{PQ} = 6\sqrt{2}$ ，求 P

的坐標？

$$\boxed{3} \quad \begin{array}{c} 1 \\ -1 \\ -1 \\ -1 \\ 6 \\ 1 \\ -1 \\ 6 \\ 1 \end{array} \quad \begin{array}{c} 2t^2 = 2 \cdot 36 \\ \Rightarrow t = \pm 6 \end{array}$$

$$A : (7, 4, 6), (-5, 4, -6)$$

$$\vec{n} \parallel (1, 0, 1) \quad \text{令 } (t+1, 4, t) \Rightarrow P(7, 4, 6) \text{ or } (-5, 4, -6)$$

4. 已知 $A = \{a_1, a_2, a_3, a_4, a_5\}$ 、 $B = \{a_1^2, a_2^2, a_3^2, a_4^2, a_5^2\}$ ，其中 $a_i \in N$ ，($i = 1, 2, 3, 4, 5$)。設

$a_1 < a_2 < a_3 < a_4 < a_5$ 且 $A \cap B = \{a_1, a_4\}$ ，若 $a_1 + a_4 = 10$ ，且 $A \cup B$ 元素之和為 224，求集合 $A = ?$

$$A : \{1, 3, 4, 9, 10\}$$

5. 85 藍歡慶營業一周年，凡買飲料超過百元者，皆贈送一張刮刮樂彩券。右圖為其中一種形式，此彩券共有 9 格，內有四個 1、三個 2、二個 3，玩法是從中任刮 2 格兌獎，否則作廢。若所刮 2 格的數字相同，則可得到此數字 72 倍之等價折價券；若所刮 2 格的數字相異，則只能領紀念品一份。今小胖拿到一張刮刮樂彩券，試問小胖刮得折價券金額之期望值？

1	2	1
1	3	1
2	2	3

$$A : 30 \text{ 元} \quad \begin{array}{c} 1 \quad 1 \quad 1 \\ 2 \quad 2 \quad 2 \\ 3 \quad 3 \end{array} \quad n(S) = C_9^2 = 36 \quad \begin{array}{c} 72 \\ 36 \end{array} \quad \begin{array}{c} (1 \times C_2^4 + 2 \times C_2^3 + 3 \times C_2^2) = 30 \\ = 36 \end{array}$$

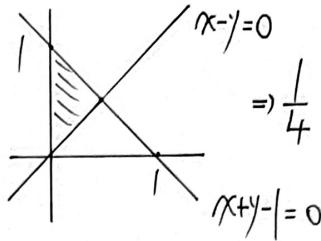
6. 求平面上滿足 $x^3y^2 - x^2y^2 - xy^4 + xy^3 \geq 0$ 且 $0 \leq x \leq y$ 的區域面積？

$$\Delta : \frac{1}{4}$$

$$xy^2(x^2 - x - y^2 + y) \geq 0$$

$$\Rightarrow xy^2(x-y)(x+y-1) \geq 0$$

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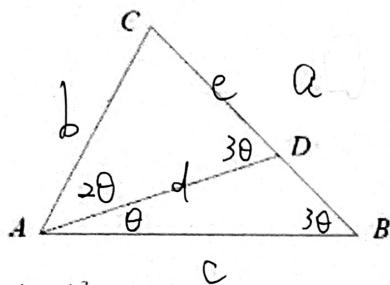
7. 如圖， $2\angle BAC = 3\angle ABC$ ，且 D 在 \overline{BC} 上，使得 $\angle DAC = 2\angle DAB$ 。假設 $\overline{BC} = a$, $\overline{AC} = b$ ，

$\overline{AB} = c$

$\overline{AD} = d$ 且 $\overline{CD} = e$ 。

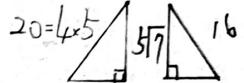
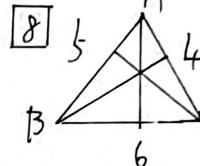
請用 a 、 b 和 c 表示 d 和 e 。求 $(d, e) = ?$

7



$$\frac{d}{b} = \frac{c}{a} \Rightarrow d = \frac{bc}{a}$$

$$\frac{e}{b} = \frac{c}{a} \Rightarrow e = \frac{b^2}{a}$$



$$15 = 3\sqrt{5}$$

$$9 = 3\sqrt{3}$$

$$\begin{aligned} \cos A &= \left(\frac{5}{2\sqrt{5} \cdot 4} \right)^2 \\ &= \frac{1}{64} \end{aligned}$$

$$\Delta : \left(\frac{bc}{a}, \frac{b^2}{a} \right)$$

8. H 為 $\triangle ABC$ 的垂心， $\cos B = \frac{3}{4}$ 、 $\cos C = \frac{9}{16}$ 。若 \overrightarrow{BH} 交 \overrightarrow{AC} 於 E ，且 \overrightarrow{CH} 交 \overrightarrow{AB} 於 F 。求 $\frac{\Delta AEF \text{ 面積}}{\Delta ABC \text{ 面積}} = ?$

$$\begin{aligned} A : \frac{1}{64} \quad \text{令 } Z_3 = x + iy \Rightarrow (x+iy)(-1 + \sqrt{3}i) &= (-x - \sqrt{3}y) + i(\sqrt{3}x - y) \Rightarrow y = \sqrt{3}x - 1 \Rightarrow y = -\frac{1}{3x+1} \\ \Rightarrow -(3x^2+1)x + \sqrt{3}x &= \sqrt{3}(x-1)/(3x+1) \Rightarrow x = \sqrt{3}(3x^2+1)/(3x^2+1)(3x+1) = \sqrt{3}/3x+1 \end{aligned}$$

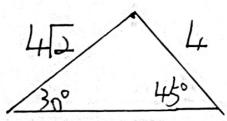
9. 設複數 $Z_1 = -1 + \sqrt{3}ai$, $Z_2 = \frac{\sqrt{3}(a-1)}{3a+1} + i$ ，其中 $a \in \mathbb{R}$ 且 $a \neq -\frac{1}{3}$ 。設複數 $Z_3 = \frac{Z_2}{Z_1}$ ，求 $\text{Arg}(Z_3) = ?$

$$Z_3 = \frac{2}{3a+1} e^{i(\frac{11\pi}{6})} \Rightarrow \text{Arg}(Z_3) = \frac{11}{6}\pi$$

$$a < -\frac{1}{3} : \frac{5}{6}\pi$$

10. A triangle with interior angles 30° and 45° , and the radius of the circumcircle is 4. What is the area of

the triangle?



$$\frac{1}{2} \cdot 4 \cdot 4\sqrt{2} \cdot \frac{\sqrt{6} + \sqrt{2}}{4} = 4\sqrt{3} + 4$$

$$\Delta : 4 + 4\sqrt{3}$$

11. George throws three unbiased dice and removes all of the dice that come up with a 5 or 6. Martha then

throws the dice that remain, if any. Determine the probability that exactly one of Martha's dice shows a 5

or 6.

$$A : \frac{98}{243}$$

Martha
丟
骰子

1

Martha
prob

$\frac{1}{3}$

George
prob

$\frac{2}{9}$

$$= \frac{18 + 48 + 32}{243}$$

$$P(A) = \frac{1}{3}, P(A') = \frac{2}{3}$$

$$2 \quad C_1^2 \left(\frac{2}{3} \right) \left(\frac{1}{3} \right) = \frac{4}{9} \quad C_1^3 \left(\frac{2}{3} \right)^2 \left(\frac{1}{3} \right) = \frac{4}{9} \quad = \frac{98}{243}$$

$$3 \quad C_1^3 \left(\frac{2}{3} \right)^2 \left(\frac{1}{3} \right) = \frac{4}{9} \quad C_0^3 \left(\frac{2}{3} \right)^3 \left(\frac{1}{3} \right)^0 = \frac{8}{27}$$