

$$P(x) = x^4 - 1 = (x+1)(x-1)(x+i)(x-i), f(x) = (x-d_1)(x-d_2)(x-d_3)(x-d_4)(x-d_5).$$

$$\Rightarrow P(d_1) \cdot P(d_2) \cdot P(d_3) \cdot P(d_4) \cdot P(d_5)$$

$$[(d_1-1)(d_2-1)(d_3-1)(d_4+1)(d_5-1)] [(d_1+1)(d_2+1)(d_3+1)(d_4+1)(d_5+1)] [(d_1+i)(d_2-i)(d_3-i)(d_4+i)(d_5-i)] [(d_1-i)(d_2-i)(d_3-i)(d_4-i)(d_5-i)]$$

$$= 0 \times \text{circled empty} = 0.$$

2.

$$\begin{vmatrix} 13 & -9 \\ 9 & 13 \end{vmatrix} = 250.$$

$$\Rightarrow \frac{1000\pi}{250} = 4\pi.$$

3.

$$y = xe^x \text{ 和 } y = \ln x \text{ 對稱於 } y=x.$$

算 $y=x$ 到 $y=\ln x$ 最短距離 x_2 即可.

$$\Rightarrow \frac{x - \ln x}{\sqrt{2}} \Rightarrow (x - \ln x)' = (1 - \frac{1}{x}) \Rightarrow \text{當 } x=1 \text{ 有最大值}$$

$$\Rightarrow \frac{1 - \ln \frac{1}{2}}{\sqrt{2}} \cdot x_2 = \sqrt{2}(1 + \ln 2).$$

$$4. \sum_{n=1}^k \frac{n}{C(n+2)} = \sum_{n=1}^k \frac{6}{(n+1)(n+2)} \Rightarrow \lim_{k \rightarrow \infty} \sum_{n=1}^k \frac{6}{(n+1)(n+2)} = 6(\frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \frac{1}{4} + \dots) = 6 \times \frac{1}{2} = 3.$$

5.

$$\text{設 } x^2 - x + a = 0 \text{ 之根 } \alpha_1, \alpha_2, \alpha_1 + \alpha_2 = 1.$$

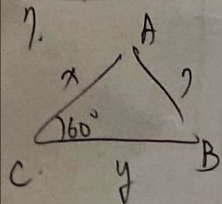
$$x^2 - x + b = 0 \text{ 之根 } \beta_1, \beta_2, \beta_1 + \beta_2 = 1.$$

$$\Rightarrow \alpha_1, \beta_1, \beta_2, \alpha_2 \text{ 互異 } \Rightarrow \alpha_1 = \frac{1}{4}, \alpha_2 = \frac{1}{4} + 3d \Rightarrow d = \frac{1}{6}.$$

$$6. \text{設 } y-2 = m(x-4) \Rightarrow y = mx - 4m + 2.$$

$$\Rightarrow \begin{cases} 4x^2 - y^2 = 9 \\ y = mx - 4m + 2 \end{cases} \Rightarrow 4x^2 - (mx - 4m + 2)^2 = 9 \Rightarrow \frac{8m^2 - 4m}{m^2 - 4} = 9 \Rightarrow m = 8 \Rightarrow 8x - y - 30 = 0.$$

7.



$$\Rightarrow x^2 + y^2 - xy = 49.$$

$$\Rightarrow \textcircled{1} x=1 \Rightarrow y \notin \mathbb{N} \quad \textcircled{5} x=5 \Rightarrow y=8.$$

$$\textcircled{2} x=2 \Rightarrow y \notin \mathbb{N} \quad \textcircled{6} x=6 \Rightarrow y \notin \mathbb{N}$$

$$\textcircled{3} x=3 \Rightarrow y=8 \quad \textcircled{7} x=7 \Rightarrow y=7.$$

$$\textcircled{4} x=4 \Rightarrow y \notin \mathbb{N} \quad \textcircled{8} x=8 \Rightarrow y=3, 5.$$

$$\textcircled{9} x=9 \Rightarrow y \text{ 無實數解}$$

$\Rightarrow 5 \text{ 個}$

8.

$$z_1 = a_1 + b_1 i, \quad a_1^2 + b_1^2 = 2$$

$$z_2 = a_2 + b_2 i, \quad a_2^2 + b_2^2 = 5$$

$$z_3 = a_3 + b_3 i, \quad a_3^2 + b_3^2 = 4$$

$$a_1 + a_2 + a_3 = 0$$

$$b_1 + b_2 + b_3 = 0$$

$$\bar{z}_1 \cdot z_2 = (a_1 a_2 + b_1 b_2) + i \quad \bigcirc$$

$$\Rightarrow (a_1^2 + b_1^2) + (a_2^2 + b_2^2)$$

$$= (a_1 + a_2)^2 + (b_1 + b_2)^2 - 2(a_1 a_2 + b_1 b_2)$$

$$= a_3^2 + b_3^2 - 2(a_1 a_2 + b_1 b_2)$$

$$= 4 - 2(a_1 a_2 + b_1 b_2) = 7 \Rightarrow a_1 a_2 + b_1 b_2 = -\frac{3}{2}$$

9.

$$\begin{array}{cccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ & & & & & \text{反} & \text{正} & \text{正} \end{array}$$

$$\Rightarrow \frac{6}{16-3} = \frac{6}{13}$$

10.

$$\cos 30 = \frac{1}{3} \Rightarrow \cos 30 = -\frac{23}{25} \Rightarrow \cos(B(60-30)) = -\frac{23}{25}$$

$$\Rightarrow \overline{AB}^2 = \left(\frac{9}{2}\sqrt{3}\right)^2 + \left(\frac{9}{2}\sqrt{3}\right)^2 - 2\left(\frac{9}{2}\sqrt{3}\right) \cdot \left(-\frac{23}{25}\right)$$

$$\Rightarrow \overline{AB} = 15$$

11. (1)

$$L: y = 3a^2 x - 2a^3$$

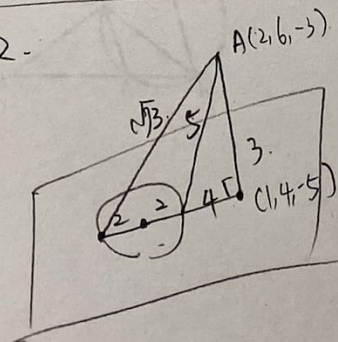
$$\Rightarrow Q(0, -2a^3) \Rightarrow \overline{PQ} \cdot \overline{QS} = 1 \cdot 2$$

$$S(2a, -8a^3)$$

11. (2)

$$\int_{-2a}^a x^3 - 3a^2 x + 2a^3 dx = \frac{21}{4} a^4$$

12.



13.

$$(1, 2, -2) \times (3, 1, -2) = (-2, -4, -5)$$

$$\Rightarrow -2x - 4y - 5z = -3 \Rightarrow h = \frac{1\sqrt{5}}{5}$$

$$\Rightarrow \left(\frac{1\sqrt{5}}{15} \times \frac{1}{\sqrt{3}} \times 2\right) \times \frac{\sqrt{3}}{4} = \frac{4\sqrt{3}}{135}$$

14.

$$\cos^2(A-B) + \cos^2(B-C) + \cos^2(C-A)$$

$$= 3 - \sin^2(A-B) - \sin^2(B-C) - \sin^2(C-A)$$

$$= 3 - \frac{1}{2}(3 - \cos(2A-2B) - \cos(2B-2C) - \cos(2C-2A))$$

$$= 3 - \frac{1}{2}(3 - \cos A \cos B - \sin A \sin B - \cos B \cos C - \sin B \sin C - \cos C \cos A - \sin C \sin A)$$

$$= 3 - \frac{3}{2} + \frac{1}{2}(\cos A \cos B + \sin A \sin B + \cos B \cos C + \sin B \sin C + \cos C \cos A + \sin C \sin A)$$

$$= \frac{3}{2} + \frac{1}{4}((\sin A + \sin B + \sin C)^2 + (\cos A + \cos B + \cos C)^2 - 3)$$

$$= \frac{3}{2} - \frac{3}{4} = \frac{3}{4}$$

15.

$$p = \frac{1}{4} + \frac{1}{4}p + \frac{1}{4}p^2 + \frac{1}{4}p^3 \Rightarrow p = \sqrt{2} - 1$$