112 年臺北市立陽明高級中學 第 1 次正式教師甄選 數學科 筆試試題

說明：本試卷測驗時間 120 分鐘，共 20 題，每題 5 分。 前 19 題為填空題，空白處供計算，僅需在各題 Answer 處填上答案，過程不計分 第 20 題為證明題，可用中文、英文作答。	評分	

一、填空題：

1. Find the difference between the sum of the squares of the first ten natural numbers and the square of the sum.

Answer：____________________

2. The sum

$$\frac{1}{2!}+\frac{2}{3!}+\frac{3}{4!}+\cdots+\frac{2022}{2023!}$$

Can be expressed as $a-\frac{1}{b!}$, where a and b are positive integers. What is $a-b$?

Answer：____________________

3. A data set consists of 6 (not distinct) positive integers : 1, 2, 4, 6, 7 and X. The average (arithmetic mean) of the 6 numbers equals a value in the data set. What is the sum of all positive values of X?

Answer：____________________

4. A perfect number is a number for which the sum of its proper divisors is exactly equal to the number. For example, the sum of the proper divisors of 28 would be $1+2+4+7+14=28$, which means that 28 is a perfect number.

A number n is called **deficient** if the sum of its proper divisors is less than n and it is called **abundant** if this sum exceeds n.

a is the smallest abundant number. Find the value of a.

Answer：____________________

5. In a box, there are 5 red, 6 yellow and 7 white balls. If Alice randomly takes out a number of balls from the box, at least how many balls she should take in order to guarantee herself two balls of the same color?

Answer：____________________

6. Among the 900 three-digit numbers from 100 to 999, how many of them the sum of the three digits is smaller than 15?

Answer：____________________

7. If 48 volunteer places are allocated to 3 nursing homes, each nursing homes has at least one place and the allocation number of each nursing home's places is different from each other. How many different methods to allocate these people are there?

Answer：____________________

8. I have two blue socks, two red socks, and two yellow socks in a drawer. On Monday, I randomly pick two of the socks to wear. On Tuesday, I randomly pick two of the remaining socks to wear. On Wednesday, I wear the two remaining socks from the drawer.
What is the probability that I didn't wear a matching pair of socks on any of the three days?

Answer：____________________

9. When a student multiplied the number 66 by the repeating decimal $1.abab\cdots = 1.\overline{ab}$, where a and b are digits, he did not notice the notation and just multiplied 66 times $1.ab$. Later he found that his answer is 0.5 less than the correct answer. What is the 2-digit number ab?

Answer：____________________

10. A permutation is an ordered arrangement of objects. For example, 3124 is one possible permutation of the digits 1, 2, 3 and 4. If all of the permutations are listed numerically or alphabetically, we call it lexicographic order. The lexicographic permutations of 0, 1 and 2 are:

$$012 \qquad 021 \qquad 102 \qquad 120 \qquad 201 \qquad 210$$

What is the 100th lexicographic permutation of the digits 0, 1, 2, 3, 4?

Answer：____________________

11. A Pythagorean triplet is a set of three natural numbers, $a < b < c$, for which,

$$a^2 + b^2 = c^2$$

For example, $3^2 + 4^2 = 9 + 16 = 25 = 5^2$.

There exists exactly one Pythagorean triplet for which $a + b + c = 1000$.

Find the value of a.

Answer：____________________

12. If one vertex of a square is $A(6,2)$, and the equation of one diagonal is $y = -5x + 32$. What is the equations of the line containing the two adjacent sides of the square passing through point A? (The answers must be expressed in the form $ax + by = c$.)

Answer：____________________

13. How many ordered pairs of real numbers (x, y) satisfy the following system of equations?

$$x+3y=3$$
$$||x|-|y||=1$$

Answer：____________________

14. Given the polynomial $f(x)$, it leaves a remainder of $(2x+3)$ when divided by (x^2+1), it leaves a remainder of $(-4x+13)$ when divided by (x^2-1), and it leaves a remainder of $r(x)$ when divided by x^4-1. Find the value of $r(-3)$.

Answer：____________________

15. Quadratic polynomials $P(x)$ and $Q(x)$ have leading coefficients 1 and -1, respectively. The graphs of both polynomials pass through the two points $(1,5)$ and $(2,3)$. Find $P(0)+Q(0)$.

Answer：____________________

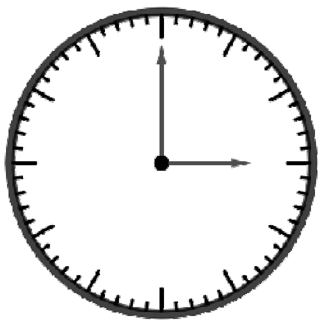
16. Given that $\omega=\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}$, find the value of $\prod_{j=1}^{5}(\omega^{2j}+\omega^{j}-2)$.

Answer：____________________

17. In a triangle $\triangle ABC$, where $\overline{AB}=6$, $\overline{BC}=2\sqrt{7}$, and $\overline{AC}=4$, point O is the circumcenter of $\triangle ABC$. If line L passes the point O and intersects $\overline{AB}$ and $\overline{AC}$ at points D and E, respectively. What is the minimum value of $\overline{AD}+\overline{AE}$.

Answer：____________________

18. The minute hand on a watch is 4 cm long and the hour hand is 3 cm long. How fast is the distance between the tips of the hands changing at three o'clock? Notice that this is a 12 hour watch.



Answer：____________________ cm per minute.

19. Let $f(x)=(1+\frac{b}{x})^x$, where $b>0$ and $x>0$. $f'(b)=?$

Answer：____________________

二、證明題

20. a、b、c、d are real positive numbers. Prove that $\frac{a}{b}+\frac{c}{d}>\frac{a+c}{b+d}$.