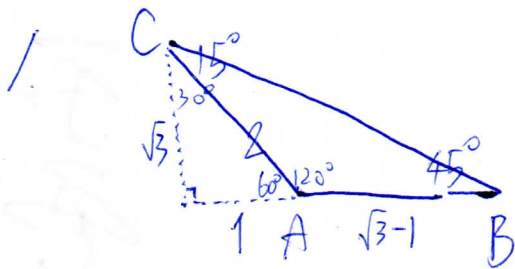
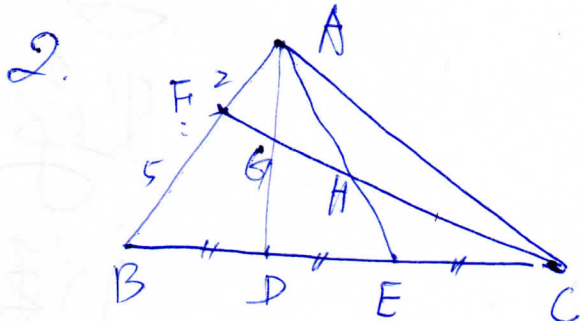


109 複選 2



$$\overline{AB} : \overline{AC} = \sqrt{3} - 1 : 2$$



$$\frac{2}{1} \times \frac{1}{2} \times \frac{\overline{FG}}{\overline{CG}} = 1$$

$$\Rightarrow \overline{FG} : \overline{CG} = 1 : 7$$

$$\frac{1}{2} \times \frac{7}{2} \times \frac{\overline{FH}}{\overline{CH}} = 1$$

$$\Rightarrow \overline{FH} : \overline{CH} = 4 : 7$$

可得 $\overline{GH} = \frac{21}{88} \overline{CF}$

$$\Delta AGH = \frac{21}{88} \Delta ACF = \frac{3}{44} \Delta ABC$$

3.

$$3^{2019} = 3 \times 3^{2018}$$

$$= 3 \times 9^{1009}$$

$$= 3 \times (10-1)^{1009}$$

$$= 3 \times \left[C_{0}^{1009} 10^0 (-1)^{1009} + C_{1}^{1009} 10^1 (-1)^{1008} + C_{2}^{1009} 10^2 (-1)^{1007} + \dots \right]$$

$$= 3 \times \left[-1 + 10090 - \frac{1009 \times 1008}{2} \times 100 + \dots \right]$$

末三位為 67

$$3^{108} = (10-1)^{54}$$

$$= C_{0}^{54} 10^0 (-1)^{54} + C_{1}^{54} 10^1 (-1)^{53} + C_{2}^{54} 10^2 (-1)^{52} + \dots$$

$$= 1 - 540 + \frac{54 \times 53}{2} \times 100 + \dots$$

末三位為 61

故 $3^{2019} - 3^{108}$ 末三位為 06

4. 可知第“奇数”次操作後

A. B
各 $\frac{1}{2}$ $\frac{1}{2}$ 24

第108次

若將B中 $\frac{1}{109}$ 倒入A. 故 B剩 $\frac{1}{2} \times \frac{108}{109} = \frac{54}{109}$

5. 令 $m-108=a^2$

$$m+21=b^2$$

$$\text{則 } b^2 - a^2 = 129$$

$$(b+a)(b-a) = 129 \times 1 \text{ 或 } 43 \times 3$$

$$(a, b) = (64, 65) \text{ 或 } (20, 23)$$

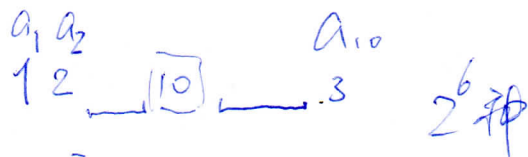
$$\text{可得 } (m, n) = (4204, 129) \text{ 或 } (508, 43)$$

$$m+n = 4333 \quad \text{或} \quad 551$$

6. $a_2 - a_1 + a_3 - a_2 + \dots + a_{10} - a_9$

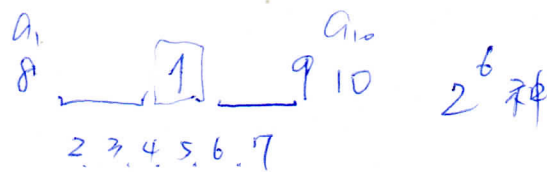
$$= a_{10} - a_1 \text{ 必須} = 2$$

只轉向一次 ① 先遞增, 再遞減



剩 4, 5, 6, 7, 8, 9

② 先遞減, 再遞增



$$\text{共 } 2 \times 2^6 = 128$$

$$1. (2) \quad a = \frac{9}{2} - \frac{25}{c} = \frac{9c-50}{2c}$$

$$9b + \frac{4}{\frac{9c-50}{2c}} = f \Rightarrow 9b + \frac{8c}{9c-50} = f \Rightarrow b = \frac{f - \frac{8c}{9c-50}}{9}$$

$$c + \frac{16}{b} = 34 \Rightarrow b = \frac{16}{34-c}$$

$$\text{得 } f - \frac{8c}{9c-50} = \frac{9 \times 16}{34-c} \Rightarrow 2 - \frac{2c}{9c-50} = \frac{36}{34-c}$$

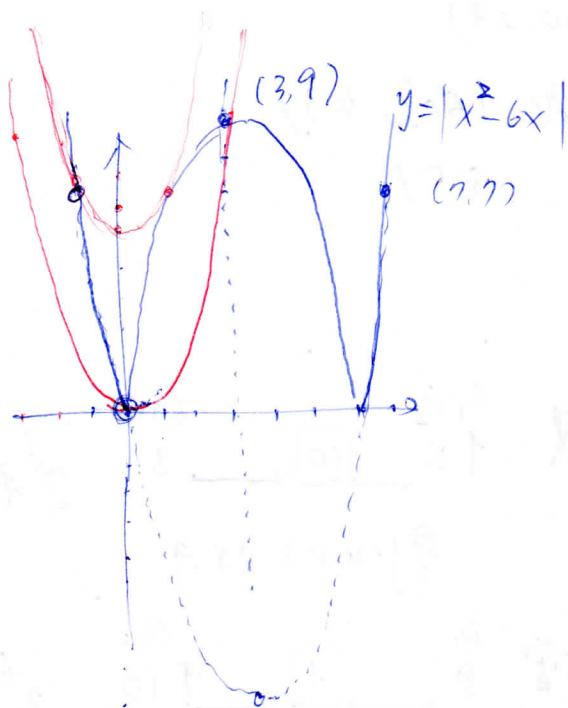
$$\frac{16c-100}{9c-50} = \frac{36}{34-c}$$

$$-(16c-100)(c-34) = 36(9c-50) \text{ 得 } c = 10$$

$$b = \frac{2}{3} \quad a = 2.$$

$$(a, b, c) = \left(2, \frac{2}{3}, 10\right)$$

2.



$$x^2 + c = -x^2 + 6x$$

$$2x^2 - 6x + c = 0$$

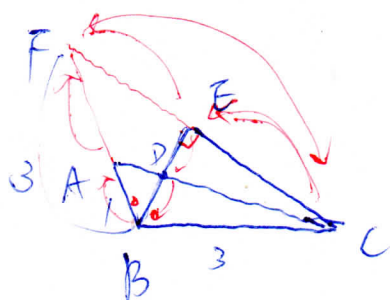
只有一切实根.

$$\Delta = 36 - 8c = 0$$

$$c = \frac{9}{2}$$

故 $0 < c < \frac{9}{2}$ 有 3 相实根

3.



$$\triangle BFE \sim \triangle BCE$$

$$\frac{1}{2} \times \frac{2}{1} \times \frac{\overline{DE}}{\overline{BD}} = 1 \quad \therefore \overline{DE} = \overline{BD}$$

$$4. \quad 108109110 \dots 144 = N$$

$$666 = 2 \times 3 \times 111 = 2 \times 9 \times 37$$

N 為 2 倍數

$$\text{數字和: } \underset{\text{百位}}{1} \times (144-107) + \underset{\text{十位}}{(1+2+3)} \times 10 + \underset{\text{十位}}{4} \times 5 + \underset{\text{個位}}{(1+2+\dots+9)} \times 3 + 0+9 + 1+2+3+4$$

$$110 \sim 119$$

$$120 \sim 129$$

$$130 \sim 139$$

$$140, 141, 142, 143, 144$$

$$= 37 + 60 + 20 + 135 + 27 = 279 \text{ 為 3 倍數}$$

111 的倍數 必為 37 倍數

$$\text{三個數字一節總和} = \frac{(108+144) \times 37}{2} \text{ 為 37 倍數}$$

故 N 為 666 倍數