(一)設拋物線 $x^2 = 4cy$上有一定點$A(x_0, y_0)$及兩動點$B(x_1, y_1)$、$C(x_2, y_2)$，

若$\angle BAC = 90°$，則$\overline{BC}$必通過 $P(-x_0, y_0 + 4c)$。

(二)設拋物線 $y^2 = 4cx$上有一定點$A(x_0, y_0)$及兩動點$B(x_1, y_1)$、$C(x_2, y_2)$，

若$\angle BAC = 90°$，則$\overline{BC}$必通過 $P(x_0 + 4c, -y_0)$。

(三)設橢圓 $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ 上有一定點$A(x_0, y_0)$及兩動點$B(x_1, y_1)$、$C(x_2, y_2)$，

若$\angle BAC = 90°$，則$\overline{BC}$必通過 $P\left(\frac{c^2x_0}{a^2+b^2}, \frac{-c^2y_0}{a^2+b^2}\right)$。

(四)設橢圓 $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$ 上有一定點$A(x_0, y_0)$及兩動點$B(x_1, y_1)$、$C(x_2, y_2)$，

若$\angle BAC = 90°$，則$\overline{BC}$必通過 $P\left(\frac{-c^2x_0}{a^2+b^2}, \frac{c^2y_0}{a^2+b^2}\right)$。

(五)設雙曲線 $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ 上有一定點$A(x_0, y_0)$及兩動點$B(x_1, y_1)$、$C(x_2, y_2)$，

若$\angle BAC = 90°$，則$\overleftrightarrow{BC}$必通過 $P\left(\frac{c^2x_0}{a^2-b^2}, \frac{-c^2y_0}{a^2-b^2}\right)$。

(六)設雙曲線 $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$ 上有一定點$A(x_0, y_0)$及兩動點$B(x_1, y_1)$、$C(x_2, y_2)$，

若$\angle BAC = 90°$，則$\overleftrightarrow{BC}$必通過 $P\left(\frac{-c^2x_0}{a^2-b^2}, \frac{c^2y_0}{a^2-b^2}\right)$。

(一)設拋物線 $x^2=4cy$上有一定點$A(x_0, y_0)$及兩動點$B(x_1, y_1)$、$C(x_2, y_2)$，

若$\angle BAC=90°$，則$\overline{BC}$必通過 $P(-x_0, y_0+4c)$。

證明：

設$\overleftrightarrow{BC}$ ：$y=mx+d$

由$\begin{cases} y=mx+d \ldots\ldots\ldots ① \\ x^2=4cy \ldots\ldots\ldots ② \end{cases}$

①代入②可得$x^2=4c(mx+d)$

$\Rightarrow x^2-4cmx-4cd=0$

由根與係數

$\Rightarrow x_1+x_2=4cm$，$x_1x_2=-4cd \ldots\ldots\ldots ③$

由$\begin{cases} mx=y-d \ldots\ldots\ldots ④ \\ m^2x^2=4cm^2y \ldots\ldots\ldots ⑤ \end{cases}$

④代入⑤可得$(y-d)^2-4cm^2y=0$

$\Rightarrow y^2+(-2d-4cm^2)y+d^2=0$

由根與係數

$\Rightarrow y_1+y_2=2d+4cm^2$，$y_1y_2=d^2 \ldots\ldots\ldots ⑥$

由$\angle BAC=90°$

$\Rightarrow \overrightarrow{AB}\cdot\overrightarrow{AC}=0$

$\Rightarrow (x_0-x_1, y_0-y_1)\cdot(x_0-x_2, y_0-y_2)=0$

$\Rightarrow {x_0}^2-(x_1+x_2)x_0+x_1x_2+{y_0}^2-(y_1+y_2)y_0+y_1y_2=0 \ldots\ldots\ldots ⑦$

③、⑥代入⑦

$\Rightarrow {x_0}^2-4cmx_0-4cd+{y_0}^2-2dy_0-4cm^2y_0+d^2=0$
$\Rightarrow 4cy_0-4cmx_0-4cd+{y_0}^2-2dy_0+d^2-m^2{x_0}^2=0$
$\Rightarrow 4c(y_0-mx_0-d)+(y_0-d)^2-m^2{x_0}^2=0$
$\Rightarrow 4c(y_0-mx_0-d)+(y_0-d-mx_0)(y_0-d+mx_0)=0$

$\Rightarrow (y_0 - d - mx_0)(y_0 - d + mx_0 + 4c) = 0$

$\because \overleftrightarrow{BC}$不通過$A(x_0, y_0)$ $\therefore y_0 - d - mx_0 \neq 0$

即$y_0 - d + mx_0 + 4c = 0$

$\Rightarrow d = y_0 + mx_0 + 4c$代入①

$\Rightarrow y = mx + y_0 + mx_0 + 4c$
$\Rightarrow y = m(x + x_0) + y_0 + 4c$

則$\overline{BC}$必通過 $(-x_0, y_0 + 4c)$

(二)設拋物線 $y^2 = 4cx$上有一定點$A(x_0, y_0)$及兩動點$B(x_1, y_1)$、$C(x_2, y_2)$，

若$\angle BAC = 90°$，則$\overline{BC}$必通過 $P(x_0 + 4c, -y_0)$。

證明：

設$\overleftrightarrow{BC}$ ∶ $y = mx + d$

由$\begin{cases} y = mx + d \ldots\ldots\ldots ① \\ y^2 = 4cx \ldots\ldots\ldots ② \end{cases}$

①代入②可得$(mx + d)^2 = 4cx$

$\Rightarrow m^2x^2 + (2dm - 4c)x + d^2 = 0$

由根與係數

$\Rightarrow x_1 + x_2 = \dfrac{-(2dm - 4c)}{m^2}$，$x_1 x_2 = \dfrac{d^2}{m^2} \ldots\ldots\ldots ③$

由$\begin{cases} mx = y - d \ldots\ldots\ldots ④ \\ my^2 = 4cmx \ldots\ldots\ldots ⑤ \end{cases}$

④代入⑤可得$my^2 - 4cy + 4cd = 0$

由根與係數

$\Rightarrow y_1 + y_2 = \dfrac{4c}{m}$，$y_1 y_2 = \dfrac{4cd}{m} \ldots\ldots\ldots ⑥$

由$\angle BAC = 90°$

$\Rightarrow \overrightarrow{AB} \cdot \overrightarrow{AC} = 0$

$\Rightarrow (x_0 - x_1, y_0 - y_1) \cdot (x_0 - x_2, y_0 - y_2) = 0$

$\Rightarrow {x_0}^2 - (x_1 + x_2)x_0 + x_1x_2 + {y_0}^2 - (y_1 + y_2)y_0 + y_1y_2 = 0 \ldots\ldots\ldots ⑦$

③、⑥代入⑦

$\Rightarrow {x_0}^2 + \dfrac{(2dm - 4c)}{m^2}x_0 + \dfrac{d^2}{m^2} + {y_0}^2 - \dfrac{4c}{m}y_0 + \dfrac{4cd}{m} = 0$

$\Rightarrow m^2{x_0}^2 + 2dmx_0 - 4cx_0 + d^2 + m^2{y_0}^2 - 4cmy_0 + 4cdm = 0$

$\Rightarrow (m^2{x_0}^2 + 2dmx_0 + d^2) - 4cx_0 + 4cm^2x_0 - 4cmy_0 + 4cdm = 0$

$\Rightarrow (mx_0 + d)^2 - {y_0}^2 + 4cm(mx_0 - y_0 + d) = 0$

$\Rightarrow (mx_0 + d + y_0)(mx_0 + d - y_0) + 4cm(mx_0 + d - y_0) = 0$
$\Rightarrow (mx_0 + d - y_0)(mx_0 + d + y_0 + 4cm) = 0$

$\because \overleftrightarrow{BC}$不通過$A(x_0, y_0)$ $\therefore mx_0 + d - y_0 \neq 0$

即$mx_0 + d + y_0 + 4cm = 0$

$\Rightarrow d = -y_0 - mx_0 - 4cm$代入①

$\Rightarrow y = mx - y_0 - mx_0 - 4cm$
$\Rightarrow y + y_0 = m(x - x_0 - 4c)$

則$\overline{BC}$必通過 $(x_0 + 4c, -y_0)$

(三)設橢圓 $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ 上有三點$A(x_{0}, y_0)$、$B(x_{1}, y_1)$、$C(x_{2}, y_2)$，$\angle BAC=90°$，則$\overline{BC}$必通過 $P\left(\frac{c^2x_0}{a^2+b^2},\frac{-c^2y_0}{a^2+b^2}\right)$。

證明：

設$\overleftrightarrow{BC}$：$y=mx+d$

由$\begin{cases} y=mx+d \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots ① \\ b^2x^2+a^2y^2-a^2b^2=0 \ldots\ldots\ldots ② \end{cases}$

①代入②可得$b^2x^2+a^2(mx+d)^2-a^2b^2=0$

$\Rightarrow (a^2m^2+b^2)x^2+2a^2dmx+a^2d^2-a^2b^2=0$

由根與係數

$\Rightarrow x_1+x_2=\frac{-2a^2dm}{a^2m^2+b^2}$，$x_1x_2=\frac{a^2d^2-a^2b^2}{a^2m^2+b^2} \ldots\ldots\ldots ③$

由$\begin{cases} mx=y-d \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots ④ \\ b^2m^2x^2+a^2m^2y^2-a^2b^2m^2=0 \ldots\ldots\ldots ⑤ \end{cases}$

④代入⑤可得$b^2(y-d)^2+a^2m^2y^2-a^2b^2m^2=0$

$\Rightarrow (a^2m^2+b^2)y^2-2b^2dy+b^2d^2-a^2b^2m^2=0$

由根與係數

$\Rightarrow y_1+y_2=\frac{2b^2d}{a^2m^2+b^2}$，$y_1y_2=\frac{b^2d^2-a^2b^2m^2}{a^2m^2+b^2} \ldots\ldots\ldots ⑥$

由$\angle BAC=90°$

$\Rightarrow \overrightarrow{AB}\cdot\overrightarrow{AC}=0$

$\Rightarrow (x_0-x_1,y_0-y_1)\cdot(x_0-x_2,y_0-y_2)=0$

$\Rightarrow {x_0}^2-(x_1+x_2)x_0+x_1x_2+{y_0}^2-(y_1+y_2)y_0+y_1y_2=0 \ldots\ldots\ldots ⑦$

③、⑥代入⑦

$\Rightarrow {x_0}^2-\frac{-2a^2dmx_0}{a^2m^2+b^2}+\frac{a^2d^2-a^2b^2}{a^2m^2+b^2}+{y_0}^2-\frac{2b^2dy_0}{a^2m^2+b^2}+\frac{b^2d^2-a^2b^2m^2}{a^2m^2+b^2}=0$

$\Rightarrow a^2m^2{x_0}^2+b^2{x_0}^2+2a^2dmx_0+a^2d^2-a^2b^2+a^2m^2{y_0}^2+b^2{y_0}^2-2b^2dy_0+b^2d^2$

$-a^2b^2m^2=0$

$\Rightarrow a^2(m^2{x_0}^2+2dmx_0+d^2)+b^2({y_0}^2-2dy_0+d^2)+(b^2{x_0}^2-a^2b^2)$

$+m^2(a^2{y_0}^2-a^2b^2)=0$

$\Rightarrow a^2(mx_0+d)^2+b^2(y_0-d)^2+(-a^2{y_0}^2)+m^2(-b^2{x_0}^2)=0$

$\Rightarrow a^2((mx_0+d)^2-{y_0}^2)+b^2((y_0-d)^2-m^2{x_0}^2)=0$

$\Rightarrow a^2(mx_0+d-y_0)(mx_0+d+y_0)+b^2(y_0-d-mx_0)(y_0-d+mx_0)=0$

$\Rightarrow (mx_0+d-y_0)(a^2mx_0+a^2d+a^2y_0-b^2y_0+b^2d-b^2mx_0)=0$

$\because \overleftrightarrow{BC}$不通過$A\left(x_0, y_0\right) \therefore mx_0+d-y_0\neq 0$

即$a^2mx_0+a^2d+a^2y_0-b^2y_0+b^2d-b^2mx_0=0$

$\Rightarrow d=\dfrac{-a^2mx_0-a^2y_0+b^2y_0+b^2mx_0}{a^2+b^2}$代入①

$\Rightarrow y=mx+\dfrac{-a^2mx_0-a^2y_0+b^2y_0+b^2mx_0}{a^2+b^2}$

$$\Rightarrow y=m\left(x-\frac{(a^2-b^2)x_0}{a^2+b^2}\right)-\frac{(a^2-b^2)y_0}{a^2+b^2}$$

$$\Rightarrow y=m\left(x-\frac{c^2x_0}{a^2+b^2}\right)-\frac{c^2y_0}{a^2+b^2}$$

則$\overline{BC}$必通過 $\left(\dfrac{c^2x_0}{a^2+b^2},\dfrac{-c^2y_0}{a^2+b^2}\right)$

(四)設橢圓 $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$ 上有一定點$A(x_0, y_0)$及兩動點$B(x_1, y_1)$、$C(x_2, y_2)$，

若$\angle BAC=90°$，則$\overline{BC}$必通過 $P\left(\frac{-c^2x_0}{a^2+b^2},\frac{c^2y_0}{a^2+b^2}\right)$。

證明：

設$\overleftrightarrow{BC}$ ：$y=mx+d$

$$由\begin{cases} y=mx+d \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots ① \\ a^2x^2+b^2y^2-a^2b^2=0 \ldots\ldots\ldots ② \end{cases}$$

①代入②可得$a^2x^2+b^2(mx+d)^2-a^2b^2=0$

$\Rightarrow (b^2m^2+a^2)x^2+2b^2dmx+b^2d^2-a^2b^2=0$

由根與係數

$\Rightarrow x_1+x_2=\frac{-2b^2dm}{b^2m^2+a^2}$ ，$x_1x_2=\frac{b^2d^2-a^2b^2}{b^2m^2+a^2}\ldots\ldots\ldots$ ③

$$由\begin{cases} mx=y-d \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots ④ \\ a^2m^2x^2+b^2m^2y^2-a^2b^2m^2=0 \ldots\ldots\ldots ⑤ \end{cases}$$

④代入⑤可得$a^2(y-d)^2+b^2m^2y^2-a^2b^2m^2=0$

$\Rightarrow (b^2m^2+a^2)y^2-2a^2dy+a^2d^2-a^2b^2m^2=0$

由根與係數

$\Rightarrow y_1+y_2=\frac{2a^2d}{b^2m^2+a^2}$ ，$y_1y_2=\frac{a^2d^2-a^2b^2m^2}{b^2m^2+a^2}\ldots\ldots\ldots$ ⑥

由$\angle BAC=90°$

$\Rightarrow \overrightarrow{AB}\cdot\overrightarrow{AC}=0$

$\Rightarrow (x_0-x_1,y_0-y_1)\cdot(x_0-x_2,y_0-y_2)=0$

$\Rightarrow {x_0}^2-(x_1+x_2)x_0+x_1x_2+{y_0}^2-(y_1+y_2)y_0+y_1y_2=0\ldots\ldots\ldots$ ⑦

③、⑥代入⑦

$\Rightarrow {x_0}^2-\frac{-2b^2dmx_0}{b^2m^2+a^2}+\frac{b^2d^2-a^2b^2}{b^2m^2+a^2}+{y_0}^2-\frac{2a^2dy_0}{b^2m^2+a^2}+\frac{a^2d^2-a^2b^2m^2}{b^2m^2+a^2}=0$

$\Rightarrow b^2m^2{x_0}^2+a^2{x_0}^2+2b^2dmx_0+b^2d^2-a^2b^2+b^2m^2{y_0}^2+a^2{y_0}^2-2a^2dy_0+a^2d^2$

$-a^2b^2m^2=0$
$\Rightarrow b^2(m^2{x_0}^2+2dmx_0+d^2)+a^2({y_0}^2-2dy_0+d^2)+(a^2{x_0}^2-a^2b^2)$
$+m^2(b^2{y_0}^2-a^2b^2)=0$
$\Rightarrow b^2(mx_0+d)^2+a^2(y_0-d)^2+(-b^2{y_0}^2)+m^2(-a^2{x_0}^2)=0$
$\Rightarrow b^2((mx_0+d)^2-{y_0}^2)+a^2((y_0-d)^2-m^2{x_0}^2)=0$
$\Rightarrow b^2(mx_0+d-y_0)(mx_0+d+y_0)+a^2(y_0-d-mx_0)(y_0-d+mx_0)=0$
$\Rightarrow (mx_0+d-y_0)(b^2mx_0+b^2d+b^2y_0-a^2y_0+a^2d-a^2mx_0)=0$

$\because \overleftrightarrow{BC}$不通過$A(x_0,y_0)$ $\therefore mx_0+d-y_0\neq 0$

即$b^2mx_0+b^2d+b^2y_0-a^2y_0+a^2d-a^2mx_0=0$

$\Rightarrow d=\dfrac{a^2mx_0+a^2y_0-b^2y_0-b^2mx_0}{a^2+b^2}$代入①

$$\Rightarrow y=mx+\frac{a^2mx_0+a^2y_0-b^2y_0-b^2mx_0}{a^2+b^2}$$

$$\Rightarrow y=m\left(x+\frac{(a^2-b^2)x_0}{a^2+b^2}\right)+\frac{(a^2-b^2)y_0}{a^2+b^2}$$

$$\Rightarrow y=m\left(x+\frac{c^2x_0}{a^2+b^2}\right)+\frac{c^2y_0}{a^2+b^2}$$

則$\overline{BC}$必通過 $\left(\dfrac{-c^2x_0}{a^2+b^2},\dfrac{c^2y_0}{a^2+b^2}\right)$

(五)設雙曲線 $\dfrac{x^2}{a^2}-\dfrac{y^2}{b^2}=1$ 上有一定點$A(x_0,y_0)$及兩動點$B(x_1,y_1)$、$C(x_2,y_2)$，

若$\angle BAC=90°$，則$\overleftrightarrow{BC}$必通過 $P\left(\dfrac{c^2x_0}{a^2-b^2},\dfrac{-c^2y_0}{a^2-b^2}\right)$。

證明：

設$\overleftrightarrow{BC}$：$y=mx+d$

由$\begin{cases} y=mx+d \dots\dots\dots\dots\dots\dots\dots\dots ① \\ b^2x^2-a^2y^2-a^2b^2=0 \dots\dots\dots ② \end{cases}$

①代入②可得$b^2x^2-a^2(mx+d)^2-a^2b^2=0$

$\Rightarrow (-a^2m^2+b^2)x^2-2a^2dmx-a^2d^2-a^2b^2=0$

由根與係數

$\Rightarrow x_1+x_2=\dfrac{2a^2dm}{-a^2m^2+b^2}$，$x_1x_2=\dfrac{-a^2d^2-a^2b^2}{-a^2m^2+b^2}\dots\dots\dots ③$

由$\begin{cases} mx=y-d \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots ④ \\ b^2m^2x^2-a^2m^2y^2-a^2b^2m^2=0 \dots\dots\dots ⑤ \end{cases}$

④代入⑤可得$b^2(y-d)^2-a^2m^2y^2-a^2b^2m^2=0$

$\Rightarrow (-a^2m^2+b^2)y^2-2b^2dy+b^2d^2-a^2b^2m^2=0$

由根與係數

$\Rightarrow y_1+y_2=\dfrac{2b^2d}{-a^2m^2+b^2}$，$y_1y_2=\dfrac{b^2d^2-a^2b^2m^2}{-a^2m^2+b^2}\dots\dots\dots ⑥$

由$\angle BAC=90°$

$\Rightarrow \overrightarrow{AB}\cdot\overrightarrow{AC}=0$

$\Rightarrow (x_0-x_1,y_0-y_1)\cdot(x_0-x_2,y_0-y_2)=0$

$\Rightarrow {x_0}^2-(x_1+x_2)x_0+x_1x_2+{y_0}^2-(y_1+y_2)y_0+y_1y_2=0\dots\dots\dots ⑦$

③、⑥代入⑦

$\Rightarrow {x_0}^2-\dfrac{2a^2dmx_0}{-a^2m^2+b^2}+\dfrac{-a^2d^2-a^2b^2}{-a^2m^2+b^2}+{y_0}^2-\dfrac{2b^2dy_0}{-a^2m^2+b^2}+\dfrac{b^2d^2-a^2b^2m^2}{-a^2m^2+b^2}=0$

$\Rightarrow -a^2m^2{x_0}^2+b^2{x_0}^2-2a^2dmx_0+-a^2d^2-a^2b^2-a^2m^2{y_0}^2+b^2{y_0}^2-2b^2dy_0$

$+b^2d^2-a^2b^2m^2=0$

$\Rightarrow -a^2({m^2x_0}^2+2dmx_0+d^2)+b^2({y_0}^2-2dy_0+d^2)+(b^2{x_0}^2-a^2b^2)$

$+m^2(-a^2{y_0}^2-a^2b^2)=0$

$\Rightarrow -a^2(mx_0+d)^2+b^2(y_0-d)^2+a^2{y_0}^2-m^2b^2{x_0}^2=0$

$\Rightarrow a^2({y_0}^2-(mx_0+d)^2)+b^2((y_0-d)^2-m^2{x_0}^2)=0$

$\Rightarrow a^2(y_0-mx_0-d)(y_0+mx_0+d)+b^2(y_0-d-mx_0)(y_0-d+mx_0)=0$

$\Rightarrow (y_0-mx_0-d)(a^2mx_0+a^2d+a^2y_0+b^2y_0-b^2d+b^2mx_0)=0$

$\because \overleftrightarrow{BC}$不通過$A\left(x_0,y_0\right)$ $\therefore y_0-mx_0-d\neq 0$

即$a^2mx_0+a^2d+a^2y_0+b^2y_0-b^2d+b^2mx_0=0$

$\Rightarrow d=\dfrac{-a^2mx_0-a^2y_0-b^2y_0-b^2mx_0}{a^2-b^2}$代入①

$\Rightarrow y=mx+\dfrac{-a^2mx_0-a^2y_0-b^2y_0-b^2mx_0}{a^2-b^2}$

$\Rightarrow y=m\left(x-\dfrac{(a^2+b^2)x_0}{a^2-b^2}\right)-\dfrac{(a^2+b^2)y_0}{a^2-b^2}$

$\Rightarrow y=m\left(x-\dfrac{c^2x_0}{a^2-b^2}\right)-\dfrac{c^2y_0}{a^2-b^2}$

則$\overleftrightarrow{BC}$必通過$\left(\dfrac{c^2x_0}{a^2-b^2},\dfrac{-c^2y_0}{a^2-b^2}\right)$

(六)設雙曲線 $\frac{y^2}{a^2}-\frac{x^2}{b^2}=1$ 上有一定點$A(x_0,y_0)$及兩動點$B(x_1,y_1)$ 、$C(x_2,y_2)$，

若$\angle BAC=90°$，則$\overleftrightarrow{BC}$必通過 $P\left(\frac{-c^2x_0}{a^2-b^2},\frac{c^2y_0}{a^2-b^2}\right)$ 。

證明：

設$\overleftrightarrow{BC}$ ：$y=mx+d$

由$\begin{cases} y=mx+d \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{①} \\ b^2y^2-a^2x^2-a^2b^2=0 \ldots\ldots\ldots \text{②} \end{cases}$

①代入②可得$b^2(mx+d)^2-a^2x^2-a^2b^2=0$

$\Rightarrow (b^2m^2-a^2)x^2+2b^2dmx+b^2d^2-a^2b^2=0$

由根與係數

$\Rightarrow x_1+x_2=\frac{-2b^2dm}{b^2m^2-a^2}$ ，$x_1x_2=\frac{b^2d^2-a^2b^2}{b^2m^2-a^2}\ldots\ldots\ldots$ ③

由$\begin{cases} mx=y-d \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{④} \\ b^2m^2y^2-a^2m^2x^2-a^2b^2m^2=0 \ldots\ldots\ldots \text{⑤} \end{cases}$

④代入⑤可得$b^2m^2y^2-a^2(y-d)^2-a^2b^2m^2=0$

$\Rightarrow (b^2m^2-a^2)y^2+2a^2dy-a^2d^2-a^2b^2m^2=0$

由根與係數

$\Rightarrow y_1+y_2=\frac{-2a^2d}{b^2m^2-a^2}$ ，$y_1y_2=\frac{-a^2d^2-a^2b^2m^2}{b^2m^2-a^2}\ldots\ldots\ldots$ ⑥

由$\angle BAC=90°$

$\Rightarrow \overrightarrow{AB}\cdot\overrightarrow{AC}=0$

$\Rightarrow (x_0-x_1,y_0-y_1)\cdot(x_0-x_2,y_0-y_2)=0$

$\Rightarrow {x_0}^2-(x_1+x_2)x_0+x_1x_2+{y_0}^2-(y_1+y_2)y_0+y_1y_2=0\ldots\ldots\ldots$ ⑦

③、⑥代入⑦

$\Rightarrow {x_0}^2-\frac{-2b^2dmx_0}{b^2m^2-a^2}+\frac{b^2d^2-a^2b^2}{b^2m^2-a^2}+{y_0}^2-\frac{-2a^2dy_0}{b^2m^2-a^2}+\frac{-a^2d^2-a^2b^2m^2}{b^2m^2-a^2}=0$

$\Rightarrow b^2m^2{x_0}^2-a^2{x_0}^2+2b^2dmx_0+b^2d^2-a^2b^2+b^2m^2{y_0}^2-a^2{y_0}^2+2a^2dy_0$

$-a^2d^2-a^2b^2m^2=0$

$\Rightarrow b^2(m^2{x_0}^2+2dmx_0+d^2)-a^2({y_0}^2-2dy_0+d^2)+(-a^2{x_0}^2-a^2b^2)$

$+m^2(b^2{y_0}^2-a^2b^2)=0$

$\Rightarrow b^2(mx_0+d)^2-a^2(y_0-d)^2-b^2{y_0}^2+m^2a^2{x_0}^2=0$

$\Rightarrow -b^2({y_0}^2-(mx_0+d)^2)-a^2((y_0-d)^2-m^2{x_0}^2)=0$

$\Rightarrow b^2(y_0-mx_0-d)(y_0+mx_0+d)+a^2(y_0-d-mx_0)(y_0-d+mx_0)=0$

$\Rightarrow (y_0-mx_0-d)(b^2mx_0+b^2d+b^2y_0+a^2y_0-ad+a^2mx_0)=0$

$\because \overleftrightarrow{BC}$不通過$A(x_0,y_0)$ $\therefore y_0-mx_0-d\neq 0$

即$b^2mx_0+b^2d+b^2y_0+a^2y_0-a^2d+a^2mx_0=0$

$\Rightarrow d=\dfrac{a^2mx_0+a^2y_0+b^2y_0+b^2mx_0}{a^2-b^2}$代入①

$\Rightarrow y=mx+\dfrac{a^2mx_0+a^2y_0+b^2y_0+b^2mx_0}{a^2-b^2}$

$\Rightarrow y=m\left(x+\dfrac{(a^2+b^2)x_0}{a^2-b^2}\right)+\dfrac{(a^2+b^2)y_0}{a^2-b^2}$

$\Rightarrow y=m\left(x+\dfrac{c^2x_0}{a^2-b^2}\right)+\dfrac{c^2y_0}{a^2-b^2}$

則$\overleftrightarrow{BC}$必通過 $\left(\dfrac{-c^2x_0}{a^2-b^2},\dfrac{c^2y_0}{a^2-b^2}\right)$