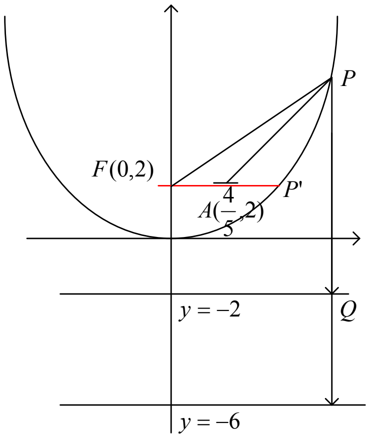
102-全國高中教師聯招 詳解整理

1. 單選

解：



(1) $x^2 = 8y$ ⇨ $c = 2$ ，$F(0,2)$ ，準線 $y = -2$

(2) $|d(P,L) - \overline{AP}| = |4 + \overline{PQ} - \overline{AP}| = |4 + \overline{PF} - \overline{AP}|$

(3) ΔPFA 中 $|\overline{PF} - \overline{AP}| < \overline{AF}$ ，當 $P = P'$ 時，$|\overline{PF} - \overline{AP}| = \overline{AF}$ 最大值為 $\frac{4}{5}$

⇨ $|4 + \overline{PF} - \overline{AP}| = 4 + \frac{4}{5} = \frac{24}{5}$ ###

2. 單選

解：

(*) **ie** $[(\sqrt{3} + \sqrt{2})^{2012}] \bmod 10 = ?$

(1) 令 $a = (\sqrt{3} + \sqrt{2})^{2012} = (5 + 2\sqrt{6})^{1006}$ 、$b = (5 - 2\sqrt{6})^{1006}$ ，又 $0 < b < 1$

⇨ $a + b = 2[5^{1006} + C_2^{1006} 5^{1004} (2\sqrt{6})^2 + \ldots + C_{1004}^{1006} 5^2 (2\sqrt{6})^{1004} + (2\sqrt{6})^{1006}] \in N$

(2) $[a + b] \bmod 10 = 2[5^{1006} + C_2^{1006} 5^{1004} (2\sqrt{6})^2 + \ldots + C_{1004}^{1006} 5^2 (2\sqrt{6})^{1004} + (2\sqrt{6})^{1006}] \bmod 10$

$= 2[5 + 0 + \ldots + 0 + 4] \bmod 10 = 8$ ⇨ $[a] \bmod 10 = 7$ ###

3. 單選

解：

(1) $\begin{cases} f(2011)=9 \\ f(2012)=9 \\ f(2013)=9 \end{cases}$ ⇨ 設 $f(x)=a(x-2011)(x-2012)(x-2013)+9$

⇨ $f(2010)=a(2010-2011)(2010-2012)(2010-2013)+9=1$

⇨ $a=\frac{8}{6}$ ⇨ $f(x)=\frac{4}{3}(x-2011)(x-2012)(x-2013)+9$

(2) $f(2014)=\frac{4}{3}(2014-2011)(2014-2012)(2014-2013)+9=17$ ###

4. 單選

解：

X	Y	$X-\mu_X$	$Y-\mu_Y$	$(X-\mu_X)^2$	$(Y-\mu_Y)^2$	$(X-\mu_X)(Y-\mu_Y)$
7	11	-2	1	4	1	-2
8	12	-1	2	1	4	-2
9	10	0	0	0	0	0
10	8	1	-2	1	4	-2
11	9	2	-1	4	1	-2
$\mu_X=9$	$\mu_Y=10$			$\sigma_X{}^2=2$ $\sigma_X=\sqrt{2}$	$\sigma_Y{}^2=2$ $\sigma_Y=\sqrt{2}$	

(1) $r_{XY}=\frac{1}{n}\sum(\frac{x_i-\mu_X}{\sigma_X})(\frac{y_i-\mu_Y}{\sigma_Y})=\frac{1}{5}(\frac{-2}{2}+\frac{-2}{2}+\frac{0}{2}+\frac{-2}{2}+\frac{-2}{2})=-0.8$ ###

5. 單選

解：

$$\begin{array}{r|ll} 9 & 12345 & \\ \hline 9 & 1371 & 6 \\ \hline 9 & 152 & 3 \\ \hline 9 & 16 & 8 \\ \hline & 1 & 7 \end{array}$$

(1) $12345=1\times 9^4+7\times 9^3+8\times 9^2+3\times 9+6$ ⇨ $a+b+c+d+e=25$ ###

6. 單選

解：

(1) 令 $z=x+yi$ ⇨ $|2z-i|=|z-2i|$ ⇨ $|2(x+yi)-i|=|(x+yi-2i|$

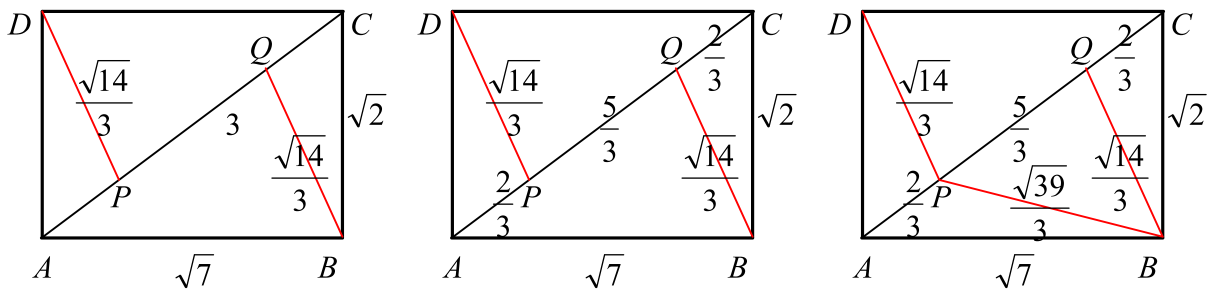
⇨ $\sqrt{(2x)^2+(2y-1)^2}=\sqrt{(x)^2+(y-2)^2}$ ⇨ $4x^2+4y^2-4y+1=x^2+y^2-4y+4$

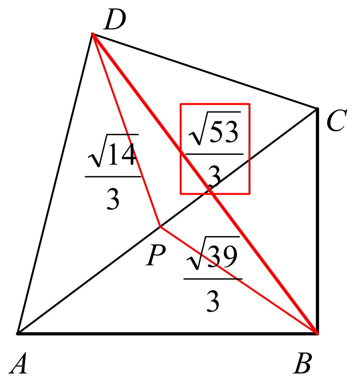
⇨ $3x^2+3y^2=3$ ⇨ $x^2+y^2=1$ **圓** ###

7. 單選

解：

(1) $\overline{AC}=3$ ⇨ $\overline{DP}=\overline{BQ}=\dfrac{\sqrt{14}}{3}$ ⇨ $\overline{AP}=\overline{CQ}=\dfrac{2}{3}$ ⇨ $\overline{PQ}=\dfrac{5}{3}$ ⇨ $\overline{BP}=\dfrac{\sqrt{39}}{3}$ ⇨ $\overline{BD}=\dfrac{\sqrt{53}}{3}$





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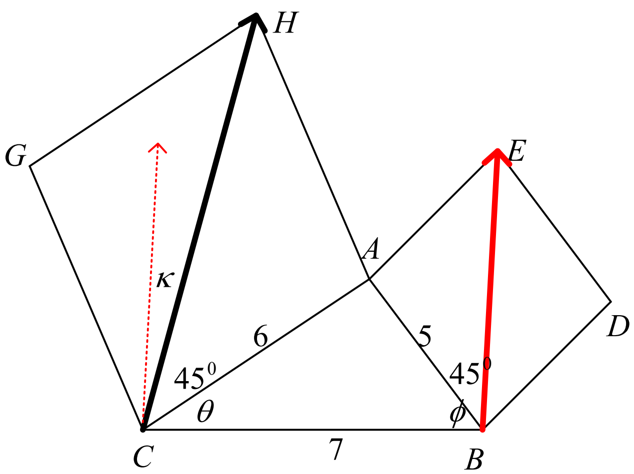
8. 單選

解：



(1) $\vec{CH}\cdot\vec{BE}=|\overline{CH}||\overline{BE}|\cos\kappa$，$\kappa=90^0-(\theta+\phi)$ ⇨ $\cos\kappa=\cos[90^0-(\theta+\phi)]=\sin(\theta+\phi)$

(2) $\cos A=\dfrac{5^2+6^2-7^2}{2\times5\times6}=\dfrac{1}{5}$，$\cos A=\cos[180^0-(\theta+\phi)]=-\cos(\theta+\phi)$

⇨ $\cos(\theta+\phi)=-\dfrac{1}{5}$ ⇨ $\sin(\theta+\phi)=\dfrac{\sqrt{24}}{5}$

(3) $\vec{CH}\cdot\vec{BE}=|\overline{CH}||\overline{BE}|\cos\kappa=|\overline{CH}||\overline{BE}|\sin(\theta+\phi)=6\sqrt{2}\times5\sqrt{2}\times\dfrac{\sqrt{24}}{5}=24\sqrt{6}$ ###

9. 複選題

解：

(A) $E=\dfrac{3}{4}\times4\times4+\dfrac{2}{3}\times4\times3+\dfrac{1}{2}\times4\times3=26$分

(B) $P=\dfrac{3}{4}\times\dfrac{2}{3}\times\dfrac{1}{2}\times36=9$分 ###

(C) $q=\dfrac{1}{4}\times\dfrac{1}{3}\times\dfrac{1}{2}=\dfrac{1}{24}$ ⇨ $p=1-\dfrac{1}{24}=\dfrac{23}{24}$ ⇨ $E=\dfrac{23}{24}\times24=23$分

(D) $p_{甲}=\dfrac{\frac{3}{4}\times\frac{1}{3}\times\frac{1}{2}}{23/24}=\dfrac{3}{23}$ ###

10. 複選題

解：

(*) $a=(3^{50}+3^{-50})^3=3^{150}+3\cdot3^{50}+3\cdot3^{-50}+3^{-150}$ ⇨ $\log a=\log(3^{150}+3\cdot3^{50}+3\cdot3^{-50}+3^{-150})$

(A)(C) $\log3^{150}=150\log3=150\times0.4771=71.565$ ⇨ $\log3<0.565<\log4$

⇨ 有 72 位整數，首位數字為 3

(B) $(3^{150}+3\cdot3^{50}+3\cdot3^{-50}+3^{-150})\bmod 10$ ⇨ $(3^{150}+3\cdot3^{50})\bmod 10$ ⇨ $(9+7)\bmod 10=6$

(D) $\log(3^{150}+3\cdot3^{50}+3\cdot3^{-50}+3\cdot3^{-150})$ ⇨ $\log(3\cdot3^{-50}+3\cdot3^{-150})$

⇨ $\log(3\cdot3^{-50})=-23.3779=-24+0.6227$ ⇨ 小數點後，第 24 位不為 0

11. 複選題

(*) $\omega=\cos\dfrac{2\pi}{9}+i\sin\dfrac{2\pi}{9}$ ⇨ $\omega^9=1$

(A) $\omega^{2010}=\omega^3=\cos\dfrac{2\pi}{3}+i\sin\dfrac{2\pi}{3}=-\dfrac{1}{2}+\dfrac{\sqrt{3}}{2}i$ ###

(B) 令 $\omega+\omega^2+\omega^3+\omega^4=S$，$\omega^9=1$ ⇨ $1+\omega+\omega^2+\ldots+\omega^8=0$

⇨ $1+(\omega+\omega^2+\omega^3+\omega^4)+\omega^4(\omega+\omega^2+\omega^3+\omega^4)=0$ ⇨ $1+S+\omega^4 S=0$ ⇨ $S=\dfrac{-1}{1+\omega^4}$

⇨ $S=\dfrac{-1}{1+\cos\frac{8\pi}{9}+i\sin\frac{8\pi}{9}}=\dfrac{-1}{1+(2\cos^2\frac{4\pi}{9}-1)+i2\sin\frac{4\pi}{9}\cos\frac{4\pi}{9}}$

$=\dfrac{\cos\pi+i\sin\pi}{2\cos\frac{4\pi}{9}(\cos\frac{4\pi}{9}+i\sin\frac{4\pi}{9})}=\dfrac{1}{2\cos\frac{4\pi}{9}}(\cos\frac{5\pi}{9}+i\sin\frac{5\pi}{9})$

⇨ $\mathrm{Re}(S)=\dfrac{\cos\frac{5\pi}{9}}{2\cos\frac{4\pi}{9}}=\dfrac{-\cos\frac{4\pi}{9}}{2\cos\frac{4\pi}{9}}=-\dfrac{1}{2}$ ###

(C) 令 $\omega+\omega^3+\omega^5+\omega^7=S$ ⇨ $1+\omega+\omega^2+...+\omega^8=0$

⇨ $1+(\omega+\omega^3+\omega^5+\omega^7)+\omega(\omega+\omega^3+\omega^5+\omega^7)=0$ ⇨ $1+S+\omega S=0$ ⇨ $S=\dfrac{-1}{1+\omega}$

⇨ $S=\dfrac{-1}{1+\cos\frac{2\pi}{9}+i\sin\frac{2\pi}{9}}=\dfrac{-1}{1+(2\cos^2\frac{\pi}{9}-1)+i2\sin\frac{\pi}{9}\cos\frac{\pi}{9}}$

$=\dfrac{\cos\pi+i\sin\pi}{2\cos\frac{\pi}{9}(\cos\frac{\pi}{9}+i\sin\frac{\pi}{9})}=\dfrac{1}{2\cos\frac{\pi}{9}}(\cos\frac{8\pi}{9}+i\sin\frac{8\pi}{9})$

⇨ $\mathrm{Im}(S)=\dfrac{\sin\frac{8\pi}{9}}{2\cos\frac{\pi}{9}}=\dfrac{\sin\frac{\pi}{9}}{2\cos\frac{\pi}{9}}=\dfrac{\tan\frac{\pi}{9}}{2}$ ###

(D) $\omega^9=1$ ⇨ $x^9-1=0$

⇨ $x^9-1=(x-1)(1+x+x^2+...+x^8)=(x-1)(x-\omega)(x-\omega^2)...(x-\omega^8)$

⇨ $1+x+x^2+...+x^8=(x-\omega)(x-\omega^2)...(x-\omega^8)$

⇨ $(1-\omega)(1-\omega^2)...(1-\omega^8)=9$ ###

12. 複選題

解：

(1) $\dfrac{27^{100}}{5^{200}}=(\dfrac{27}{25})^{100}$ ⇨ $\log(\dfrac{27}{25})^{100}=100\log(\dfrac{27}{25})\approx 3.33=3+0.33$

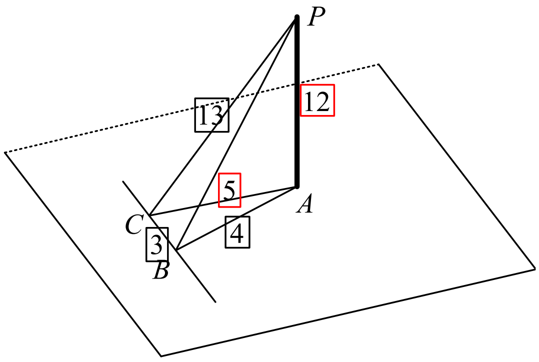
⇨ $\dfrac{27^{100}}{5^{200}}=(2)a_3a_2a_1.(\ )(\ )...(\ )$ ###

(2) $\dfrac{27^{100}}{5^{200}}=1.08^{100}=a_4aaa.b_1bb...b_{200}=2a_3a_2a_1.b_1bb...6$ ###

1. 填充

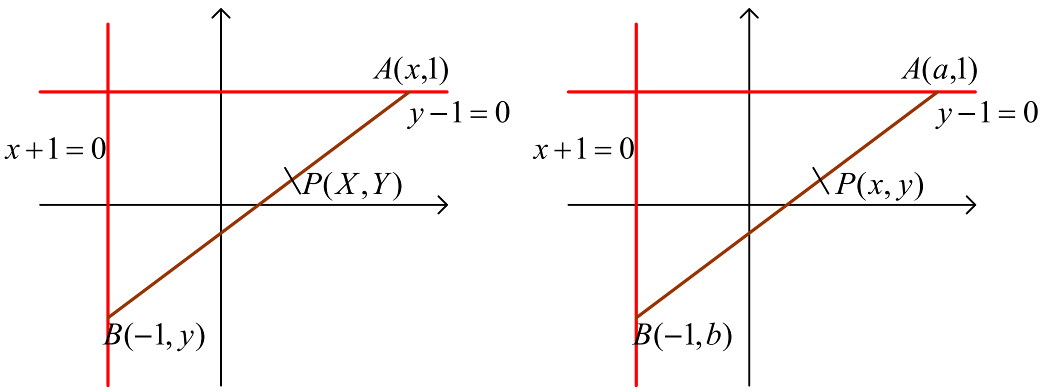
解：

(1)



2. 填充

解：



(1) 令 $A(a,1)$、$B(-1,b)$、$P(x,y)$ ⇨ $P=\dfrac{3A+2B}{2+3}=\dfrac{3(a,1)+2(-1,b)}{5}=\dfrac{(3a-2,3+2b)}{5}$

$$\Rightarrow \begin{cases} x=\dfrac{3a-2}{5} \\ y=\dfrac{3+2b}{5} \end{cases} \Rightarrow \begin{cases} a=\dfrac{5x+2}{3} \\ b=\dfrac{5y-3}{2} \end{cases}$$

(2) $\overline{AB}=\sqrt{(a+1)^2+(b-1)^2}=5$ ⇨ $(a+1)^2+(b-1)^2=25$

$$\Rightarrow (\frac{5x+2}{3}+1)^2+(\frac{5y-3}{2}-1)^2=25 \Rightarrow (\frac{5x+5}{3})^2+(\frac{5y-5}{2})^2=25$$

$$\Rightarrow \frac{(x+1)^2}{9}+\frac{(y-1)^2}{4}=1$$ ###

3. 填充

解：

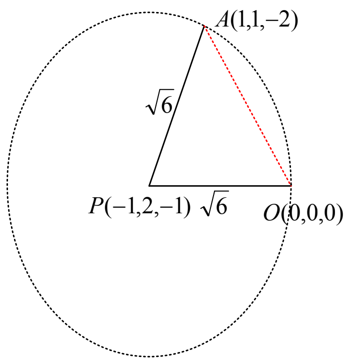
(1) **設十位數為 A，個位數為 B** ⇨ $\begin{cases} 2A>3B \\ B+4>2A \\ B\neq 0 \end{cases}$ ⇨ $B+4>2A>3B$

$\Rightarrow\ 2B<4\ \Rightarrow\ B<2\ \Rightarrow\ B=1$

(2) $B+4>2A>3B\ \Rightarrow\ 5>2A>3\ \Rightarrow\ A=2\ \Rightarrow$ $\boxed{\textbf{AB}}$ **=21** ###

4. 填充

解：



(1) $x^2+y^2+z^2+2x-4y+2z=0\ \Rightarrow\ (x+1)^2+(y-2)^2+(z+1)^2=6$

$\Rightarrow\ P(-1,2,-1)$ 、 $r=\sqrt{6}$

(2) $A(1,1,-2)$ 、 $O(0,0,0)\Rightarrow\ \overline{AO}=\sqrt{6}\ \Rightarrow\ \Delta AOP$ **正三角形**$\Rightarrow\ \angle APO=60^0$

(3) $\overset{\frown}{AO}=r\theta=\sqrt{6}\times\frac{\pi}{3}=\frac{\sqrt{6}\pi}{3}$ ###

5. 填充

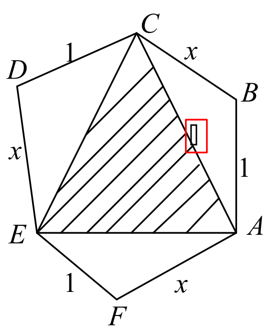
解：

(1) $(x+\frac{2}{x}+2)^5$

$\Rightarrow$ **常數項：** $\frac{5!}{2!2!1!}(x)^2(\frac{2}{x})^2(2)^1+\frac{5!}{1!1!3!}(x)^1(\frac{2}{x})^1(2)^3+\frac{5!}{0!0!5!}(x)^0(\frac{2}{x})^0(2)^5$**=592** ###

6. 填充

解：



(1) ΔACE 正三角形 ⇨ ΔACE面積 $=\dfrac{\sqrt{3}}{4}\square^2$

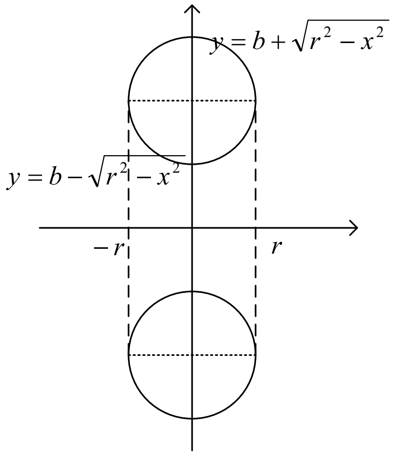
(2) $\angle ABC=120^0$ ⇨ $\cos 120^0=\dfrac{1^2+x^2-\square^2}{2\times 1\times x}$ ⇨ $x^2+x+1=\square^2$

(3) $\Delta ABC\times 3:\Delta ACE=3:7$ ⇨ $\dfrac{1}{2}\times 1\times x\sin 120^0\times 3:\dfrac{\sqrt{3}}{4}\square^2=3:7$ ⇨ $7x=\square^2$

⇨ $7x=x^2+x+1$ ⇨ $x^2-6x+1=0$ ⇨ $\alpha+\beta=6$ ###

7. 填充

解：



(*) $x^2+(y-b)^2=r^2$ ⇨ $y=b\pm\sqrt{r^2-x^2}$

(1) $V_1=\int_{-r}^{r}\pi(b+\sqrt{r^2-x^2})^2dx$

(2) $V_2=\int_{-r}^{r}\pi(b-\sqrt{r^2-x^2})^2dx$

(3) $V=V_1-V_2=\int_{-r}^{r}4b\pi(\sqrt{r^2-x^2})dx\ =8b\pi\int_0^r(\sqrt{r^2-x^2})dx=2b\pi^2r^2$ ###

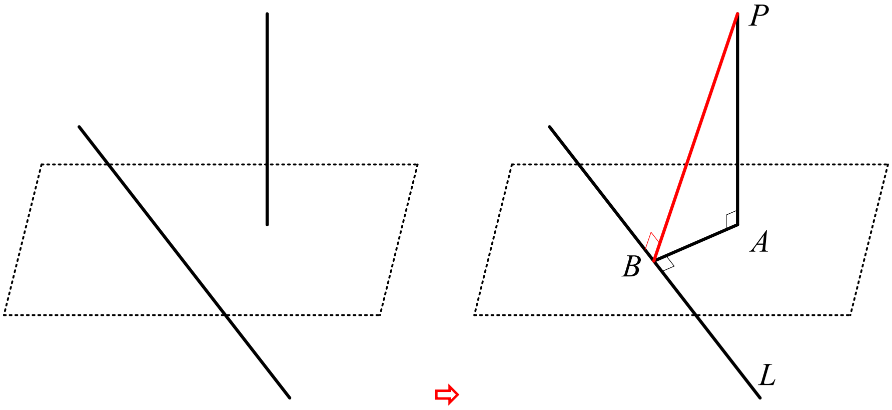
(**) $\int_0^r(\sqrt{r^2-x^2})dx=\int_0^{\frac{\pi}{2}}(\sqrt{r^2-(r\sin\theta)^2})r\cos\theta d\theta=\int_0^{\frac{\pi}{2}}r^2\cos^2\theta d\theta$

$=r^2\int_0^{\frac{\pi}{2}}\dfrac{1+\cos 2\theta}{2}d\theta=\dfrac{r^2}{2}(\theta+\dfrac{1}{2}\sin 2\theta)\Big|_0^{\frac{\pi}{2}}=\dfrac{r^2}{2}(\dfrac{\pi}{2})=\dfrac{\pi r^2}{4}$

[令 $x=r\sin\theta\ \ dx=r\cos\theta\, d\theta$]

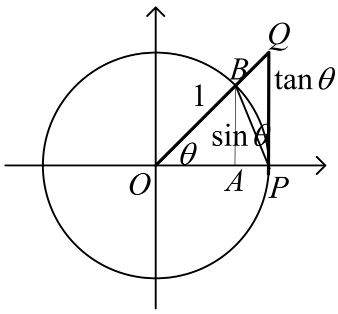
1. 計算

解：



2. 計算

解：



(1) 設一圓 O，半徑為 1，如圖：

(2) $\Delta OPB \le \overset{\frown}{OPB} \le \Delta OPQ$ ⇨ $\frac{1}{2}\sin\theta \le \frac{1}{2}r^2\theta \le \frac{1}{2}\tan\theta$ ⇨ $\sin\theta \le \theta \le \tan\theta$

⇨ $\cos\theta \le \frac{\sin\theta}{\theta} \le 1$ ⇨ $\lim\limits_{\theta\to 0}\cos\theta \le \lim\limits_{\theta\to 0}\frac{\sin\theta}{\theta} \le \lim\limits_{\theta\to 0}1$ ⇨ $1 \le \lim\limits_{\theta\to 0}\frac{\sin\theta}{\theta} \le 1$

⇨ $\lim\limits_{\theta\to 0}\frac{\sin\theta}{\theta} = 1$ ###

3. 計算

解：

(1) 無解或無限解，則 $\Delta = 0$ ⇨ $\begin{vmatrix} 1 & 1 & a \\ 2 & -a & 1 \\ 3 & 1 & 3 \end{vmatrix} = 0$ ⇨ $a = -1$、$a = \frac{4}{3}$

(2) $a = -1$ ⇨ $\begin{cases} x + y - z = 0 \\ 2x + y + z = 3 \\ 3x + y + 3z = 6 \end{cases}$ ⇨ **無限解，相交一直線。**

(3) $a=\frac{4}{3}$ ⇨ $\begin{cases} x+y+\frac{4}{3}z=0 \\ 2x-\frac{4}{3}y+z=3 \\ 3x+y+3z=6 \end{cases}$ ⇨ **無解，各兩兩相交一直線。**

4. 計算

解：

(1) $\cos\frac{2\pi}{7}=\frac{r^2+r^2-a^2}{2\cdot r\cdot r}$ ⇨ $a=2r\sin\frac{\pi}{7}$

(2) 同理 ⇨ $y=2r\sin\frac{2\pi}{7}$ 、 $x=2r\sin\frac{3\pi}{7}$

(3) $\frac{1}{x}+\frac{1}{y}=\frac{1}{2r\sin\frac{3\pi}{7}}+\frac{1}{2r\sin\frac{2\pi}{7}}=\frac{\sin\frac{2\pi}{7}+\sin\frac{3\pi}{7}}{2r\sin\frac{3\pi}{7}\sin\frac{2\pi}{7}}=\frac{2\sin\frac{5\pi}{14}\cos\frac{\pi}{14}}{2r\sin\frac{3\pi}{7}\sin\frac{2\pi}{7}}$

$=\frac{2\sin\frac{5\pi}{14}\cos\frac{\pi}{14}}{2r\sin\frac{6\pi}{14}\sin\frac{4\pi}{14}}=\frac{2\sin\frac{5\pi}{14}\cos\frac{\pi}{14}}{2r\cos\frac{\pi}{14}\cdot 2\sin\frac{2\pi}{14}\cos\frac{2\pi}{14}}=\frac{\sin\frac{5\pi}{14}}{2r\sin\frac{2\pi}{14}\sin\frac{5\pi}{14}}$

$=\frac{1}{2r\sin\frac{2\pi}{14}}=\frac{1}{a}$ ###