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$$(1+x)^{200} = a_0x^0 + a_1x^1 + \dots + a_{200}x^{200}$$

$$\text{求 } C_3^n + \dots + C_{198}^n = ?$$

$$\text{令 } f(x) = (1+x)^{200}$$

$$\begin{cases} f(-1) = a_0 + a_1(-1) + a_2(-1)^2 + \dots + a_{200}(-1)^{200} \\ f\left(\frac{1+\sqrt{3}i}{2}\right) = a_0 + a_1\left(\frac{1+\sqrt{3}i}{2}\right) + a_2\left(\frac{1+\sqrt{3}i}{2}\right)^2 + \dots + a_{200}\left(\frac{1+\sqrt{3}i}{2}\right)^{200} \\ f\left(\frac{1-\sqrt{3}i}{2}\right) = a_0 + a_1\left(\frac{1-\sqrt{3}i}{2}\right) + a_2\left(\frac{1-\sqrt{3}i}{2}\right)^2 + \dots + a_{200}\left(\frac{1-\sqrt{3}i}{2}\right)^{200} \end{cases}$$
$$\Rightarrow \left(\frac{3+\sqrt{3}i}{2}\right)^{200} + \left(\frac{3-\sqrt{3}i}{2}\right)^{200} = 3a_0 - 3a_3 + 3a_6 - \dots$$