103-全國高中教師聯招 詳解整理

1. 單選

解：

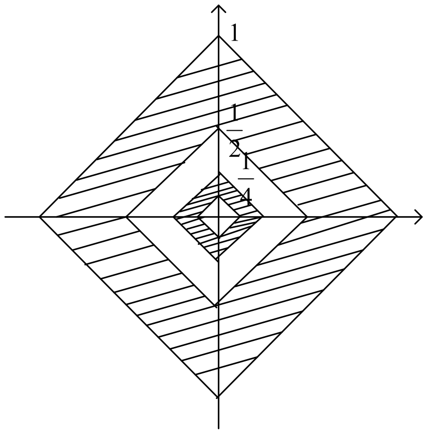
(1) $p_n = p_{n-1} \times 0.8 + q_{n-1} \times 0.6$ ⇨ $\lim p_n = \lim(p_{n-1} \times 0.8 + q_{n-1} \times 0.6)$

⇨ $p = p \times 0.8 + q \times 0.6$ ⇨ $p = \frac{3}{4}$ ###

2. 單選

解：

(1) $A = 4 \times \frac{1}{2}[1^2 - (\frac{1}{2})^2 + (\frac{1}{4})^2 - (\frac{1}{8})^2 + ...] = 2(1 - \frac{1}{4} + \frac{1}{16} - \frac{1}{64} + ...) = \frac{8}{5}$ ###



3. 單選

解：

(*) $p:\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases}$ **逆時針旋轉** ϕ ⇨ $p':\begin{cases} X = r\cos(\theta+\phi) \\ Y = r\sin((\theta+\phi) \end{cases}$ ⇨ $\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$ **，圖 1**

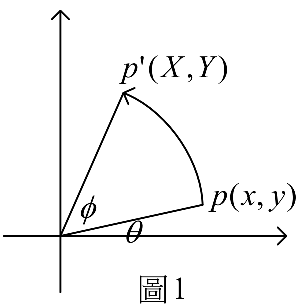


圖1

(**) $p(x,y)$ 鏡射 $y=mx$ ⇨ $p'(X,Y)$：⇨ $\begin{bmatrix} X \\ Y \end{bmatrix} = \frac{1}{1+m^2}\begin{bmatrix} 1-m^2 & 2m \\ 2m & m^2-1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$，圖 2

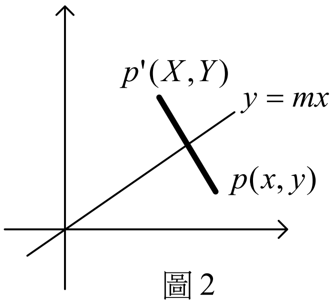


圖 2

(**) $p(x,y)$ 鏡射 $y=mx$ ⇨ $p'(X,Y)$：⇨ $\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$，圖 3

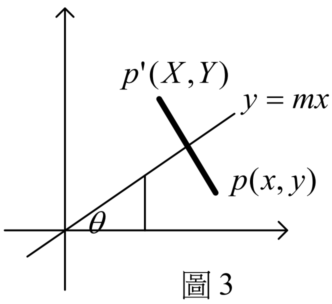


圖 3

(1) $p(x,y)$ 先旋轉 $\phi = 80^0$ ⇨ $\begin{bmatrix} X_1 \\ Y_1 \end{bmatrix} = \begin{bmatrix} \cos 80^0 & -\sin 80^0 \\ \sin 80^0 & \cos 80^0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$

(1*) 鏡射 $(\sqrt{3}-1)x-(\sqrt{3}+1)y=0$ ⇨ $m=\frac{\sqrt{3}-1}{\sqrt{3}+1}=2-\sqrt{3}=\tan\theta$

⇨ $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta} = \frac{2(2-\sqrt{3})}{1-(2-\sqrt{3})^2} = \frac{1}{\sqrt{3}}$ ⇨ $2\theta = 30^0$ #

$$⇨ \begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{bmatrix}\begin{bmatrix} X_1 \\ Y_1 \end{bmatrix} = \begin{bmatrix} \cos 30^0 & \sin 30^0 \\ \sin 30^0 & -\cos 30^0 \end{bmatrix}\begin{bmatrix} \cos 80^0 & -\sin 80^0 \\ \sin 80^0 & \cos 80^0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

$$= \begin{bmatrix} \cos 30^0 \cos 80^0 + \sin 30^0 \sin 80^0 & -\cos 30^0 \sin 80^0 + \sin 30^0 \cos 80^0 \\ \sin 30^0 \cos 80^0 - \cos 30^0 \sin 80^0 & -\sin 30^0 \sin 80^0 - \cos 30^0 \cos 80^0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

$$= \begin{bmatrix} \cos(30^0-80^0) & \sin(30^0-80^0) \\ \sin(30^0-80^0) & -\cos(30^0-80^0) \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos(-50^0) & \sin(-50^0) \\ \sin(-50^0) & -\cos(-50^0) \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} \quad \#$$

(2) $p(x,y)$ 鏡射 $y=(\tan\theta)x$ ⇨ $\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$ #

(3) $\begin{bmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{bmatrix} = \begin{bmatrix} \cos(-50^0) & \sin(-50^0) \\ \sin(-50^0) & -\cos(-50^0) \end{bmatrix} = \begin{bmatrix} \cos(310^0) & \sin(310^0) \\ \sin(310^0) & -\cos(310^0) \end{bmatrix}$

⇨ $2\theta = 310^0$ ⇨ $\theta = 155^0$ ###

4. 單選

解：

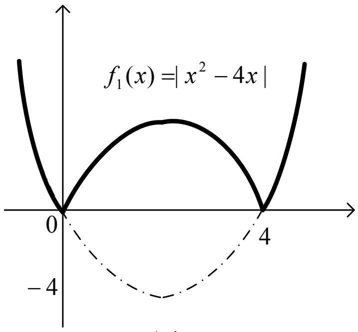


圖 1

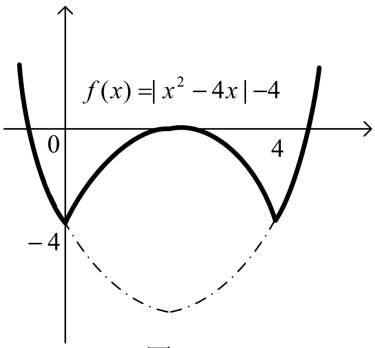


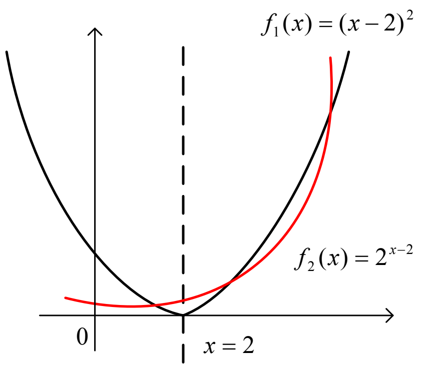
圖 2

(1) 作圖 $f_1(x) = |x^2 - 4x|$ **，圖 1**

(2) 作圖 $f(x) = |x^2 - 4x| - 4$ **，圖 2 ⇨ 最小值 -4** ###

5. 單選

解：



(1) 作圖 $f_1(x) = (x-2)^2$ **，** $f_2(x) = 2^{x-2}$ ###

6. 單選

解：

回復 9# Ellipse 的帖子

橢圓兄~我今天還在思索你的"凡德爾夢"行列式中XD

推理了橢圓兄的單選6的秒殺做法：

n個人的錯排機率爲 $\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} \ldots + (-1)^n \frac{1}{n!}$, 取極限後

$\lim_{n\to\infty} \left(\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} \ldots + (-1)^n \frac{1}{n!}\right) = \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} = e^{-1} = \frac{1}{e} \approx 0.367879$

取n=10時誤差已經很小了~故選擇0.35

原來如此，又學了一招必殺，沒想過還真的不知道XD

(借解，厲害)

(1) $A_i=\{\text{第}i\text{人抽到自己禮物}\}$ ⇨ 10人無抽到自己禮物 $=$ 全部 $-(A_1\cup A_2\cup A_3\cup\ldots\cup A_{10})$

$$|S|=10!-C_1^{10}9!+C_2^{10}8!-C_3^{7}7!+\ldots-C_9^{10}1!+C_{10}^{10}$$

(2) $p=\dfrac{|S|}{10!}=\dfrac{10!-C_1^{10}9!+C_2^{10}8!-C_3^{10}7!+\ldots+C_{10}^{10}}{10!}$

$$=1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}-\frac{1}{5!}+\frac{1}{6!}-\frac{1}{7!}+\frac{1}{8!}-\frac{1}{9!}+\frac{1}{10!}\approx 0.35$$

7. 單選

解：

(1) $\sin\alpha\cos\beta=\dfrac{1}{2}$ ⇨ $2\sin\alpha\cos\beta=1$ ⇨ $\sin(\alpha+\beta)+\sin(\alpha-\beta)=1$ **…** ①

(2) 令 $\cos\alpha\sin\beta=A$ ⇨ $2\cos\alpha\sin\beta=2A$ ⇨ $\sin(\alpha+\beta)-\sin(\alpha-\beta)=2A$ **…**②

(3) ①+② ⇨ $2\sin(\alpha+\beta)=2A+1$ ⇨ $\sin(\alpha+\beta)=\dfrac{2A+1}{2}$

⇨ $-1\le\dfrac{2A+1}{2}\le 1$ ⇨ $-\dfrac{3}{2}\le A\le\dfrac{1}{2}$ ###

8. 單選

解：

(1) $x^2-ax+a^2-4=0$ ⇨ $\begin{cases}\alpha+\beta=a\\ \alpha\beta=a^2-4\le 0\\ a^2-4(a^2-4)\ge 0\end{cases}$ ⇨ $-2\le a\le 2$

(2)

a	-2	-1	0	1	2
$x^2-ax+a^2-4=0$	$x^2+2x=0$	$x^2+x-3=0$	$x^2-4=0$	$x^2-x-3=0$	$x^2-2x=0$
解否？	**0, -2**	$\dfrac{-1\pm\sqrt{13}}{2}$	**-2, 2**	$\dfrac{1\pm\sqrt{13}}{2}$	**0, 2**

9. 複選題

解：

(A)

分堆	(5,0,0)	(4,1,0)	(3,2,0)	(3,1,1)	(2,2,1)
種	C_5^5**=1**	C_4^5**=5**	C_3^5**=10**	C_3^5**=10**	$C_1^5\times\dfrac{C_2^4}{2}$**=15**

(B) $3^5=243$

(C) $H_5^3=C_5^7=21$

(D) 如(A)分堆：5 組

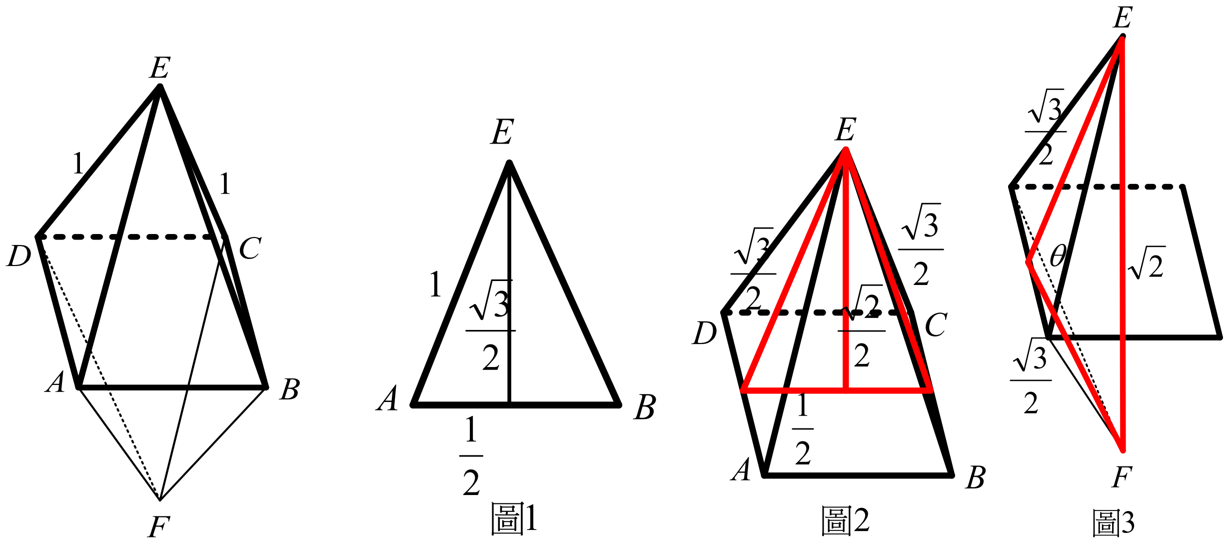
10. 複選題

解：

(1) 令 $\log a = 2 + 2\alpha$ 、 $\log b = 3 + \alpha$ ⇨ $2\log b - \log a = 4$ ⇨ $\log\frac{b^2}{a} = 4$

⇨ $\frac{b^2}{a} = 10^4$ ⇨ $b^2 = a \times 10^4$ ⇨ $b = \sqrt{a} \times 10^2$ ⇨ $\sqrt{a} = 19$ ⇨ $a = 19^2$ 、 $b = 1900$ ###

11. 複選題



(B) $\cos\theta = \frac{(\frac{\sqrt{3}}{2})^2 + (\frac{\sqrt{3}}{2})^2 - (\sqrt{2})^2}{2(\frac{\sqrt{3}}{2})(\frac{\sqrt{3}}{2})} = -\frac{1}{3}$ ⇨ $\theta = \cos^{-1}(-\frac{1}{3})$ **(圖 3)**

(C) 表面積=$\frac{1 \times \frac{\sqrt{3}}{2}}{2} \times 8 = 2\sqrt{3}$ **(圖 1)**

(D) 體積=$\frac{1 \times 1 \times \frac{\sqrt{2}}{2}}{3} \times 2 = \frac{\sqrt{2}}{3}$ **(圖 2)**

12. 複選題

解：

(*) $(3+\sqrt{2})^n = a_n + b_n\sqrt{2}$

(**) $(3+\sqrt{2})^n = \sum_{k=0}^{n} C_k^n (3)^{n-k} (\sqrt{2})^k$

(1) 設 n 是偶數：

$$(3+\sqrt{2})^n=[C_n^n 3^n+C_{n-2}^n(3)^{n-2}(\sqrt{2})^2+\ldots+C_2^n(3)^2(\sqrt{2})^{n-2}+C_0^n(\sqrt{2})^n]$$

$$+[C_{n-1}^n(3^{n-1})(\sqrt{2})^1+C_{n-3}^n(3)^{n-3}(\sqrt{2})^3+\ldots+C_3^n(3)^3(\sqrt{2})^{n-3}+C_1^n(3)(\sqrt{2})^{n-1}]$$

$$=[C_n^n 3^n+C_{n-2}^n(3)^{n-2}(\sqrt{2})^2+\ldots+C_2^n(3)^2(\sqrt{2})^{n-2}+C_0^n(\sqrt{2})^n]$$

$$+[C_{n-1}^n(3^{n-1})+C_{n-3}^n(3)^{n-3}(\sqrt{2})^2+\ldots+C_3^n(3)^3(\sqrt{2})^{n-4}+C_1^n(3)(\sqrt{2})^{n-2}]\sqrt{2}$$

⇨ $$\begin{cases} a_n=C_n^n 3^n+C_{n-2}^n(3)^{n-2}(\sqrt{2})^2+\ldots+C_2^n(3)^2(\sqrt{2})^{n-2}+C_0^n(\sqrt{2})^n \\ b_n=C_{n-1}^n(3)^{n-1}+C_{n-3}^n(3)^{n-3}(\sqrt{2})^2+\ldots+C_3^n(3)^3(\sqrt{2})^{n-4}+C_1^n(3)(\sqrt{2})^{n-2} \end{cases}$$

(A) $(3-\sqrt{2})^n=\sum_{k=0}^{n}C_k^n(3)^{n-k}(-\sqrt{2})^k$

$$=[C_n^n 3^n+C_{n-2}^n(3)^{n-2}(-\sqrt{2})^2+\ldots+C_2^n(3)^2(-\sqrt{2})^{n-2}+C_0^n(-\sqrt{2})^n]$$

$$+[C_{n-1}^n(3^{n-1})(-\sqrt{2})^1+C_{n-3}^n(3)^{n-3}(-\sqrt{2})^3+\ldots+C_3^n(3)^3(-\sqrt{2})^{n-3}+C_1^n(3)(-\sqrt{2})^{n-1}]$$

$$=[C_n^n 3^n+C_{n-2}^n(3)^{n-2}(\sqrt{2})^2+\ldots+C_2^n(3)^2(\sqrt{2})^{n-2}+C_0^n(\sqrt{2})^n]$$

$$-[C_{n-1}^n(3^{n-1})+C_{n-3}^n(3)^{n-3}(\sqrt{2})^2+\ldots+C_3^n(3)^3(\sqrt{2})^{n-4}+C_1^n(3)(\sqrt{2})^{n-2}]\sqrt{2}$$

$$=a_n-b_n\sqrt{2}$$ ###

(B) $a_n=\dfrac{(3+\sqrt{2})^n+(3-\sqrt{2})^n}{2}$ ， $b_n=\dfrac{(3+\sqrt{2})^n-(3-\sqrt{2})^n}{2\sqrt{2}}$

⇨ $\lim\dfrac{a_n}{b_n}=\lim\dfrac{[(3+\sqrt{2})^n+(3-\sqrt{2})^n]\sqrt{2}}{(3+\sqrt{2})^n-(3-\sqrt{2})^n}=\sqrt{2}$ ###

(C) $(3+\sqrt{2})^{n+1}=a_{n+1}+b_{n+1}\sqrt{2}$ ⇨ $(3+\sqrt{2})^n(3+\sqrt{2})=(a_n+b_n\sqrt{2})(3+\sqrt{2})$

$$=(3a_n+2b_n)+(a_n+3b_n)\sqrt{2}$$

⇨ $a_{n+1}=3a_n+2b_n$ ， $b_{n+1}=a_n+3b_n$ ###

(D) $m_n=\dfrac{b_{n+1}-b_n}{a_{n+1}-a_n}=\dfrac{a_n+3b_n-b_n}{3a_n+2b_n-a_n}=\dfrac{a_n+2b_n}{2a_n+2b_n}$ ⇨ $\lim\dfrac{a_n+2b_n}{2a_n+2b_n}=\lim\dfrac{\frac{a_n}{b_n}+2}{2\frac{a_n}{b_n}+2}=\dfrac{\sqrt{2}+2}{2\sqrt{2}+2}$ #

(2) 設 n 是奇數亦同。

1. 填充

解：

(1) $A=\begin{bmatrix}\cos\frac{\pi}{2} & -\sin\frac{\pi}{2}\\ \sin\frac{\pi}{2} & \cos\frac{\pi}{2}\end{bmatrix}\begin{bmatrix}\cos\frac{\pi}{3} & \sin\frac{\pi}{3}\\ -\sin\frac{\pi}{3} & \cos\frac{\pi}{3}\end{bmatrix}=\begin{bmatrix}\cos\frac{\pi}{2} & -\sin\frac{\pi}{2}\\ \sin\frac{\pi}{2} & \cos\frac{\pi}{2}\end{bmatrix}\begin{bmatrix}\cos(-\frac{\pi}{3}) & -\sin(-\frac{\pi}{3})\\ \sin(-\frac{\pi}{3}) & \cos(-\frac{\pi}{3})\end{bmatrix}$

$$=\begin{bmatrix}\cos(\frac{\pi}{2}-\frac{\pi}{3}) & -\sin(\frac{\pi}{2}-\frac{\pi}{3})\\ \sin(\frac{\pi}{2}-\frac{\pi}{3}) & \cos(\frac{\pi}{2}-\frac{\pi}{3})\end{bmatrix}=\begin{bmatrix}\cos\frac{\pi}{6} & -\sin\frac{\pi}{6}\\ \sin\frac{\pi}{6} & \cos\frac{\pi}{6}\end{bmatrix}$$

(2) $A^n=\begin{bmatrix}\cos\frac{n\pi}{6} & -\sin\frac{n\pi}{6}\\ \sin\frac{n\pi}{6} & \cos\frac{n\pi}{6}\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ ⇨ $n=12$ ###

2. 填充

解：

(*) 兩個俄國人 A_1,A_2、三個美國人 B_1,B_2,B_3、兩個臺灣人 C_1,C_2

(1) 先排 C_1,C_2，再分組(A_1,A_2)與(B_1,B_2,B_3)後排入 $\Delta\ C_1\ \Delta\ C_2\ \Delta$ 。

(2) 分組 A_1,A_2 與 B_1,B_2,B_3 ：

- **(i)** 分(A_1,A_2)(B_1,B_2,B_3)兩組，排入 $\Delta\ C_1\ \Delta\ C_2\ \Delta$ ⇨ (3x2)x(2!)x(3!)x(2!)=144
- **(ii)** 分(A_1)(A_2)(B_1,B_2,B_3)三組，排入 $\Delta\ C_1\ \Delta\ C_2\ \Delta$ ⇨ (3x2x1)x(3!)x(2!)=72
- **(iii)** 分(A_1,A_2)(B_1)(B_2,B_3)三組，排入 $\Delta\ C_1\ \Delta\ C_2\ \Delta$ ⇨ (3x2x1)x(2!)x(2!)x(2!)x3=144

(3) **Total=144+72+144=360** ###

3. 填充

解：

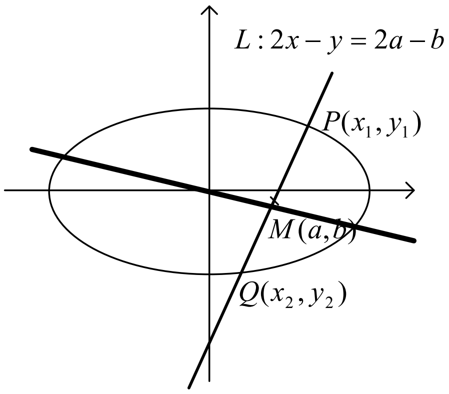
(1) $A=\begin{bmatrix}6 & 6 & 6\\ -3 & -3 & -3\\ -5 & -5 & -5\end{bmatrix}$ ⇨ $A^2=-2A$ ⇨ $A^2+2A=0$

(2) $(A+I)^2=A^2+2A+I^2=I$ ⇨ $(A+I)^{2014}=I^{1007}=I$ ###

4. 填充

解：

23	數學科	填充題第 4 題	2x+9y=0	更正為 $\begin{cases}x=9t\\ y=-2t\end{cases}$, $-\frac{1}{\sqrt{10}}<t<\frac{1}{\sqrt{10}}$



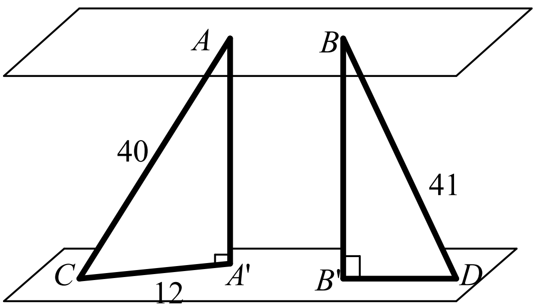
(1) 設 $M(a,b)$ ⇨ $L:2x-y=2a-b$ 如圖

(2) 解 $\begin{cases} 2x-y=2a+b \\ \dfrac{x^2}{9}+\dfrac{y^2}{4}=1 \end{cases}$ ⇨ $40x^2-36(2a-b)x+9(2a-b)^2-1=0$ ⇨ $x_1+x_2=\dfrac{36(2a-b)}{40}$

(3) $\dfrac{x_1+x_2}{2}=a$ ⇨ $\dfrac{36(2a-b)}{40}=2a$ ⇨ $2a+9b=0$ ⇨ $L':2x+9y=0$ ###

5. 填充

解：



(1) $\overline{AA'}=\sqrt{40^2-12^2}=\overline{BB'}$ ⇨ $\overline{B'D}^2=41^2-(\sqrt{40^2-12^2})^2=225$ ⇨ $\overline{B'D}=15$ ###

6. 填充

解：

(1) $a^{\log ax}=b^{\log bx}$ ⇨ $\log(a^{\log ax})=\log(b^{\log bx})$ ⇨ $(\log ax)\log a=(\log bx)\log b$

⇨ $\log a\cdot\log a+\log x\cdot\log a=\log b\cdot\log b+\log x\cdot\log b$

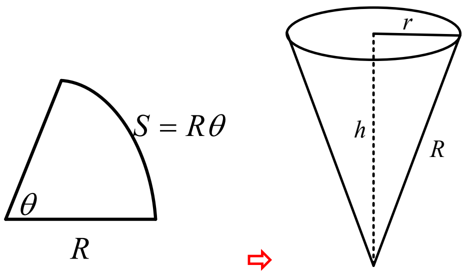
⇨ $\log x(\log a-\log b)=(\log b+\log a)(\log b-\log a)$

⇨ $\log x=-(\log b+\log a)$ ⇨ $\log x=\log(ab)^{-1}$

(2) $(ab)^{\log abx}=(ab)^{\log ab+\log x}=(ab)^{\log ab+\log(ab)^{-1}}=(ab)^{\log 1}=(ab)^0=1$ ###

7. 填充

解：



(1) $2\pi r = R\theta$ ⇨ $r = \dfrac{R\theta}{2\pi}$

(2) $V = \dfrac{1}{3}(\pi r^2)h = \dfrac{1}{3}(\pi \dfrac{R^2\theta^2}{4\pi^2})\sqrt{R^2 - r^2} = \dfrac{R^3\theta^2}{24\pi^2}\sqrt{4\pi^2 - \theta^2}$

(3) $V' = \dfrac{R^3}{24\pi^2}(2\theta \cdot \sqrt{4\pi^2 - \theta^2} + \theta^2 \cdot \dfrac{-2\theta}{2\sqrt{4\pi^2 - \theta^2}})$

(4) $V' = 0$ ⇨ $\theta = \sqrt{\dfrac{8}{3}}\pi = \dfrac{2\sqrt{6}}{3}\pi$ ###

1. 計算

解：

(1) $\begin{cases} x + y + z = 2 \\ 2x + 3y + z = 6 \\ 3x + 4y + 2z = 8 \end{cases}$ ⇨ $x + 2y = 4$ ⇨ $\begin{cases} x = 4 - 2t \\ y = t \\ z = t - 2 \end{cases}$

⇨ $x^2 + y^2 + z^2 = (4 - 2t)^2 + t^2 + (t - 2)^2 = 6t^2 - 20t + 20 = 6(t - \dfrac{5}{3})^2 + \dfrac{10}{3}$ ###

2. 計算

解：

(*) $f(x) = x^2 - 2mx + 2m + 3 = (x - m)^2 + (2m + 3 - m^2)$

(1) $m < 0$ ⇨ $f(0) = 2m + 3 > 0$ ⇨ $m > -\dfrac{3}{2}$ ⇨ $-\dfrac{3}{2} < m < 0$ **…** ①

(2) $0 \le m \le 4$ ⇨ $f(m) = (2m + 3 - m^2) > 0$ ⇨ $-1 < m < 3$ **…** ②

(3) $m > 4$ ⇨ $f(4) = 19 - 6m > 0$ ⇨ $m < \dfrac{19}{6}$ (×)

(4) ①∪② ⇨ $-\dfrac{3}{2} < m < 3$ ###

3. 計算

解：

(*) **假設 2000 可以整除** $n^2 + 103n + 2014$ ⇨ $(n^2 + 103n + 2014) \bmod 2000 = 0$

(1) $(n^2 + 103n + 2014) \bmod 2^4 \times 5^3 = 0$ ⇨ $(n^2 + 103n + 2014) \bmod 5 = 0$

⇨ $(n^2 + 3n + 4) \bmod 5 = 0$

(2) 令 $\begin{cases} n=5q \\ n=5q+1 \\ n=5q+2 \\ n=5q+3 \\ n=5q+4 \end{cases}$ 分別代入 $(n^2+3n+4) \bmod 5 \neq 0$

⇨ **2000 不能整除** $(n^2+103n+2014)$ ###

(3. 單選)

解：

(1) $p:\begin{cases} x=r\cos\theta \\ y=r\sin\theta \end{cases}$ **逆時針旋轉** ϕ ⇨ $p':\begin{cases} X=r\cos(\theta+\phi) \\ Y=r\sin((\theta+\phi) \end{cases}$ ⇨ $\begin{bmatrix} X \\ Y \end{bmatrix}=\begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$，**圖 1**

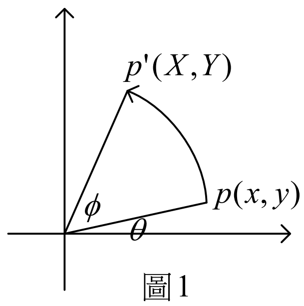


圖1

(2) $p(x,y)$ **鏡射** $y=mx$ ⇨ $p'(X,Y)$ **：**⇨ $\begin{bmatrix} X \\ Y \end{bmatrix}=\dfrac{1}{1+m^2}\begin{bmatrix} 1-m^2 & 2m \\ 2m & m^2-1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$，**圖 2**

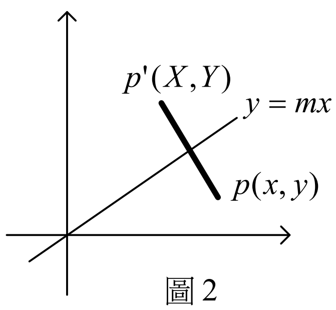


圖2

(3) $p(x,y)$ **先旋轉** $\phi=80^0$ ⇨ $\begin{bmatrix} X_1 \\ Y_1 \end{bmatrix}=\begin{bmatrix} \cos 80^0 & -\sin 80^0 \\ \sin 80^0 & \cos 80^0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$

⇨ **再鏡射** $(\sqrt{3}-1)x-(\sqrt{3}+1)y=0$，$(m=\dfrac{\sqrt{3}-1}{\sqrt{3}+1}=2-\sqrt{3})$

⇨ $\begin{bmatrix} X \\ Y \end{bmatrix}=\dfrac{1}{1+m^2}\begin{bmatrix} 1-m^2 & 2m \\ 2m & m^2-1 \end{bmatrix}\begin{bmatrix} X_1 \\ Y_1 \end{bmatrix}$

$$=\frac{1}{1+m^2}\begin{bmatrix} 1-m^2 & 2m \\ 2m & m^2-1 \end{bmatrix}\begin{bmatrix} \cos 80^0 & -\sin 80^0 \\ \sin 80^0 & \cos 80^0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$ #

(3*) $p(x,y)$ **鏡射** $y=(\tan\theta)x$ ⇨ $\begin{bmatrix} X \\ Y \end{bmatrix}=\dfrac{1}{1+\tan^2\theta}\begin{bmatrix} 1-\tan^2\theta & 2\tan\theta \\ 2\tan\theta & \tan^2\theta-1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$ #

(4) $\dfrac{1}{1+\tan^2\theta}\begin{bmatrix} 1-\tan^2\theta & 2\tan\theta \\ 2\tan\theta & \tan^2\theta-1 \end{bmatrix}=\dfrac{1}{1+m^2}\begin{bmatrix} 1-m^2 & 2m \\ 2m & m^2-1 \end{bmatrix}\begin{bmatrix} \cos 80^0 & -\sin 80^0 \\ \sin 80^0 & \cos 80^0 \end{bmatrix}$

⇨ $\dfrac{1}{1+\tan^2\theta}\begin{bmatrix} 1-\tan^2\theta & 2\tan\theta \\ 2\tan\theta & \tan^2\theta-1 \end{bmatrix}=\dfrac{1}{8-4\sqrt{3}}\begin{bmatrix} 4\sqrt{3}-6 & 4-2\sqrt{3} \\ 4-2\sqrt{3} & 6-4\sqrt{3} \end{bmatrix}\begin{bmatrix} \cos 80^0 & -\sin 80^0 \\ \sin 80^0 & \cos 80^0 \end{bmatrix}$

⇨ $\begin{bmatrix} \dfrac{1-\tan^2\theta}{1+\tan^2\theta} & \dfrac{2\tan\theta}{1+\tan^2\theta} \\ \dfrac{2\tan\theta}{1+\tan^2\theta} & \dfrac{\tan^2\theta-1}{1+\tan^2\theta} \end{bmatrix}=\begin{bmatrix} \dfrac{4\sqrt{3}-6}{8-4\sqrt{3}}\cos 80^0+\dfrac{1}{2}\sin 80^0 & \dfrac{4\sqrt{3}-6}{8-4\sqrt{3}}(-\sin 80^0)+\dfrac{1}{2}\cos 80^0 \\ \dfrac{1}{2}\cos 80^0+\dfrac{6-4\sqrt{3}}{8-4\sqrt{3}}\sin 80^0 & \dfrac{1}{2}(-\sin 80^0)+\dfrac{6-4\sqrt{3}}{8-4\sqrt{3}}\cos 80^0 \end{bmatrix}$

⇨ $\begin{cases} \dfrac{1-\tan^2\theta}{1+\tan^2\theta}=\dfrac{4\sqrt{3}-6}{8-4\sqrt{3}}\cos 80^0+\dfrac{1}{2}\sin 80^0 \\ \dfrac{2\tan\theta}{1+\tan^2\theta}=\dfrac{1}{2}\cos 80^0+\dfrac{6-4\sqrt{3}}{8-4\sqrt{3}}\sin 80^0 \end{cases}$ ②÷①

⇨ $\dfrac{2\tan\theta}{1-\tan^2\theta}=\dfrac{(4-2\sqrt{3})\cos 80^0+(6-4\sqrt{3})\sin 80^0}{(4\sqrt{3}-6)\cos 80^0+(4-2\sqrt{3})\sin 80^0}$

⇨ $\tan 2\theta=\dfrac{(2-\sqrt{3})\cos 80^0+(3-2\sqrt{3})\sin 80^0}{(2\sqrt{3}-3)\cos 80^0+(2-\sqrt{3})\sin 80^0}=\dfrac{(2-\sqrt{3})\cos 80^0+(\sqrt{3}-2)\sqrt{3}\sin 80^0}{(2-\sqrt{3})\sqrt{3}\cos 80^0+(2-\sqrt{3})\sin 80^0}$

⇨ $\tan 2\theta=\dfrac{\cos 80^0-\sqrt{3}\sin 80^0}{\sqrt{3}\cos 80^0+\sin 80^0}=\dfrac{2(\frac{1}{2}\cos 80^0-\frac{\sqrt{3}}{2}\sin 80^0)}{2(\frac{\sqrt{3}}{2}\cos 80^0+\frac{1}{2}\sin 80^0)}$

⇨ $\tan 2\theta=\dfrac{\sin 30^0\cos 80^0-\cos 30^0\sin 80^0}{\cos 30^0\cos 80^0+\sin 30^0\sin 80^0}=\dfrac{\sin(30^0-80^0)}{\cos(30^0-80^0)}=\tan(-50^0)$

⇨ $\tan 2\theta=\tan(310^0)$ ⇨ $\theta=155^0$ ###