計算3

條件:$a\geq\frac{-3}{2},b\geq\frac{-3}{2},c\geq\frac{-3}{2},d\geq\frac{-3}{2},e\geq\frac{-3}{2},f\geq\frac{-3}{2},a^5+b^5+c^5+d^5+e^5+f^5=2$

求$a^2+b^2+c^2+d^2+e^2+f^2<n,n$是最小正整數，求$n$?

$pf:$

令$f(a,b,c,d,e,f)=a^2+b^2+c^2+d^2+e^2+f^2,$

$g(a,b,c,d,e,f)=a^5+b^5+c^5+d^5+e^5+f^5-2$

$$\begin{cases}\nabla f(a,b,c,d,e,f)=2a\overrightarrow{x_1}+2b\overrightarrow{x_2}+2c\overrightarrow{x_3}+2d\overrightarrow{x_4}+2e\overrightarrow{x_5}+2f\overrightarrow{x_6}\\ \nabla g(a,b,c,d,e,f)=5a^4\overrightarrow{x_1}+5b^4\overrightarrow{x_2}+5c^4\overrightarrow{x_3}+5d^4\overrightarrow{x_4}+5e^4\overrightarrow{x_5}+5f^4\overrightarrow{x_6}\end{cases}$$

因為$\nabla f=\lambda\nabla g\Rightarrow\frac{2a}{5a^4}=\frac{2b}{5b^4}=\frac{2c}{5c^4}=\frac{2d}{5d^4}=\frac{2e}{5e^4}=\frac{2f}{5f^4}=\lambda$

$\Rightarrow a=b=c=d=e=f=(\frac{2}{5\lambda})^{\frac{1}{3}}$帶入$a\geq\frac{-3}{2},b\geq\frac{-3}{2},c\geq\frac{-3}{2},d\geq\frac{-3}{2},e\geq\frac{-3}{2},f\geq\frac{-3}{2}$

$\Rightarrow\lambda\leq\frac{-16}{135}$......(1)

再考慮$a^5+b^5+c^5+d^5+e^5+f^5=2$,將$a=b=c=d=e=f=(\frac{2}{5\lambda})^{\frac{1}{3}}$帶入

得到$\lambda=\frac{2}{5}\times3^{\frac{3}{5}}$和(1)矛盾,所以只要考慮$\lambda\leq\frac{-16}{135}$

考慮$a^2+b^2+c^2+d^2+e^2+f^2=(\frac{2}{5\lambda})^{\frac{2}{3}}\times6\leq\frac{27}{2}$,所以$n=14$

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