104-全國高中教師聯招 詳解整理

1. 單選

解：

(1) $a_n=\sqrt{1\times 2}+\sqrt{2\times 3}+\ldots+\sqrt{n(n+1)}=n(\sqrt{\frac{1}{n}\times\frac{2}{n}}+\sqrt{\frac{2}{n}\times\frac{3}{n}}+\ldots+\sqrt{\frac{n}{n}\times\frac{n+1}{n}})$

⇨ $n(\sqrt{\frac{1}{n}\times\frac{1}{n}}+\sqrt{\frac{2}{n}\times\frac{2}{n}}+\ldots+\sqrt{\frac{n}{n}\times\frac{n}{n}})\le a_n\le n(\sqrt{\frac{2}{n}\times\frac{2}{n}}+\sqrt{\frac{3}{n}\times\frac{3}{n}}+\ldots+\sqrt{\frac{n+1}{n}\times\frac{n+1}{n}})$

⇨ $n(\frac{n(n+1)/2}{n})\le a_n\le n(\frac{n(n+3)/2}{n})$

(2) $\frac{n+1}{2n}\le\frac{a_n}{n^2}\le\frac{n+3}{2n}$ ⇨ $\lim\frac{n+1}{2n}\le\lim\frac{a_n}{n^2}\le\lim\frac{n+3}{2n}$ ⇨ $\frac{1}{2}\le\lim\frac{a_n}{n^2}\le\frac{1}{2}$ ###

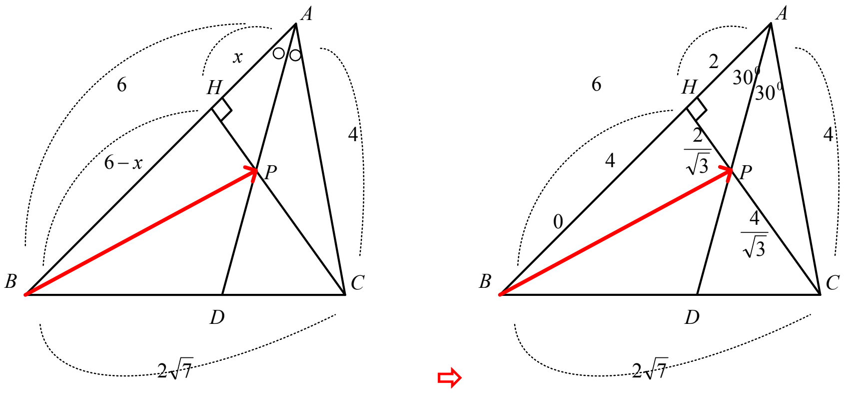
2. 單選

解：

(1) $\underbrace{888\,888}_{6個}\bmod 13=0$ ⇨ $(888888\;888888\ldots888888\;\underbrace{888\;88}_{5個})\bmod 13=7$ ###

3. 單選

解：



(1) $\overline{CH}^2=4^2-x^2=(2\sqrt{7})^2-(6-x)^2$ ⇨ $x=2$ ⇨ $\angle A=60^0$ ⇨ $\overline{HP}:\overline{PC}=1:2$ #

(2) $\overrightarrow{BP}=\frac{2}{3}\overrightarrow{BH}+\frac{1}{3}\overrightarrow{BC}=\frac{2}{3}(\frac{2}{3}\overrightarrow{BA})+\frac{1}{3}\overrightarrow{BC}=\frac{4}{9}\overrightarrow{BA}+\frac{1}{3}\overrightarrow{BC}$ ⇨ $\alpha+\beta=\frac{7}{9}$ ###

4. 單選

解：

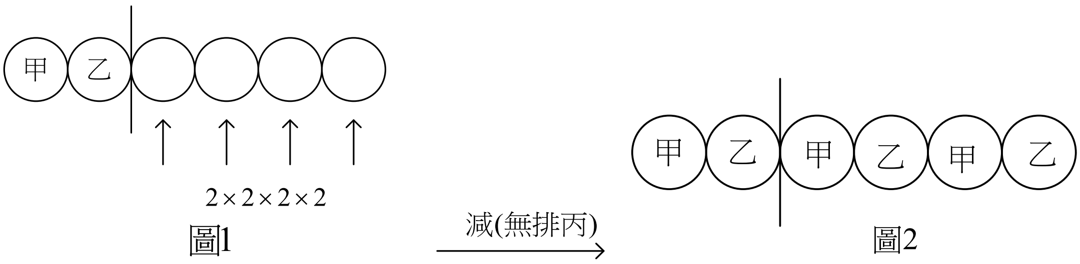
(1) $f(x)=x^3-3x^2+bx+c$ ⇨ $\begin{cases}\alpha+\beta+\gamma=3\\ \alpha\beta+\beta\gamma+\gamma\alpha=b\\ \alpha\beta\gamma=-c\\ f(-1)=1\Rightarrow -b+c=5\end{cases}$

(2) $\begin{vmatrix}1+\alpha & 1 & 1\\ 1 & 1+\beta & 1\\ 1 & 1 & 1+\lambda\end{vmatrix}=\begin{vmatrix}1+\alpha & 1 & 1\\ -\alpha & \beta & 0\\ -\alpha & 0 & \lambda\end{vmatrix}=(1+\alpha)\beta\gamma+\alpha\beta+\gamma\alpha$

$=\alpha\beta\gamma+\alpha\beta+\beta\gamma+\gamma\alpha=-c+b=-5$ ###

5. 單選

解：



(1) 同一人不連兩天：$|S|=3\times2\times2\times2\times2\times2=96$ #

(2) 先排甲乙，再排其他，如圖所示：$(2\times2\times2\times2-1=15)$ ⇨ $15\times(3\times2)=90$ #

(3) $p=\dfrac{90}{96}=0.9375\approx0.94$ ###

6. 單選

解：

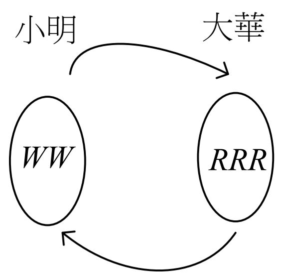
(1) $a_{n+1}=\frac{1}{3}a_n+2$ ⇨ $\lim a_{n+1}=\lim(\frac{1}{3}a_n+2)$ ⇨ $x=\frac{1}{3}x+2$ ⇨ $x=3$ ###

7. 單選

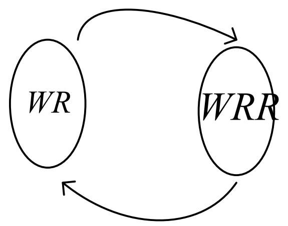
解：

(1)

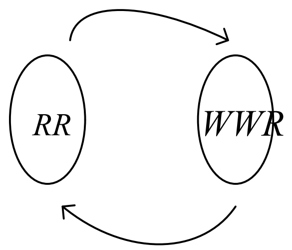
交換前　　　交換後



$$\Rightarrow \begin{aligned} &P_{小明}(WW)=0\\ &P_{小明}(WR)=1\times 1=1\\ &P_{小明}(RR)=0 \end{aligned}$$



$$\Rightarrow \begin{aligned} &P_{小明}(WW)=\frac{1}{2}\times\frac{1}{3}=\frac{1}{6}\\ &P_{小明}(WR)=\frac{1}{2}\times\frac{1}{3}+\frac{1}{2}\times\frac{2}{3}=\frac{1}{2}\\ &P_{小明}(RR)=\frac{1}{2}\times\frac{2}{3}=\frac{1}{3} \end{aligned}$$



$$\Rightarrow \begin{aligned} &P_{小明}(WW)=0\\ &P_{小明}(WR)=1\times\frac{2}{3}=\frac{2}{3}\\ &P_{小明}(RR)=1\times\frac{1}{3}=\frac{1}{3} \end{aligned}$$

(2)

$$P_1(WW)=P_0(WW)\times 0+P_0(WR)\times\frac{1}{6}+P_0(RR)\times 0$$

⇨ $$P_1(WR)=P_0(WW)\times 1+P_0(WR)\times\frac{1}{2}+P_0(RR)\times\frac{2}{3}$$

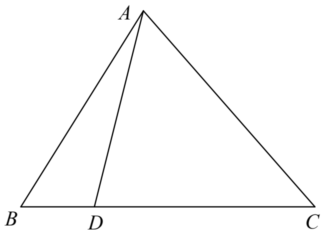
$$P_1(RR)=P_0(WW)\times 0+P_0(WR)\times\frac{1}{3}+P_0(RR)\times\frac{1}{3}$$

⇨ $$\begin{bmatrix} P_1(WW)\\ P_1(WR)\\ P_1(RR)\end{bmatrix}=\begin{bmatrix} 0 & \frac{1}{6} & 0\\ 1 & \frac{1}{2} & \frac{2}{3}\\ 0 & \frac{1}{3} & \frac{1}{3}\end{bmatrix}\begin{bmatrix} P_0(WW)\\ P_0(WR)\\ P_0(RR)\end{bmatrix}$$

⇨ $$\begin{bmatrix} P_3(WW)\\ P_3(WR)\\ P_3(RR)\end{bmatrix}=\begin{bmatrix} 0 & \frac{1}{6} & 0\\ 1 & \frac{1}{2} & \frac{2}{3}\\ 0 & \frac{1}{3} & \frac{1}{3}\end{bmatrix}^3\begin{bmatrix} 1\\ 0\\ 0\end{bmatrix}=\begin{bmatrix} \frac{1}{12}\\ \frac{23}{36}\\ \frac{5}{18}\end{bmatrix}$$ ⇨ $P_3(WR)=\frac{23}{36}$ ###

8. 單選

解：



(*) $R_1:R_2:R_3=1:2:3$ ⇨ 令 $R_1=r$ 、 $R_2=2r$ 、 $R_3=3r$

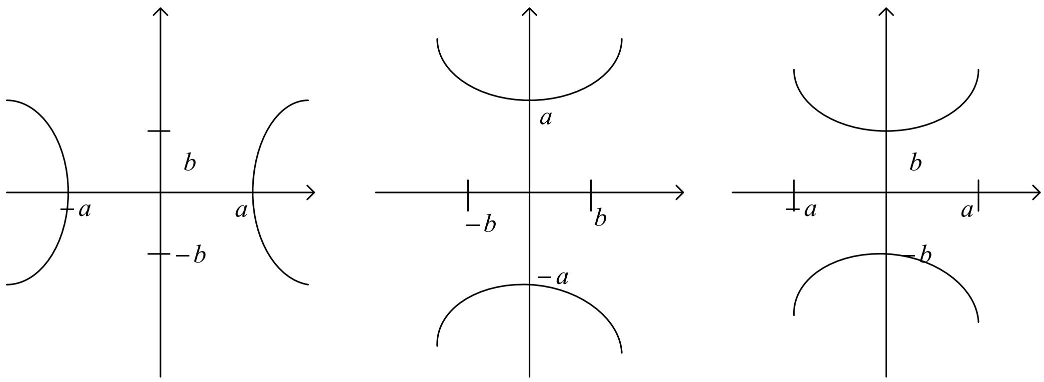
(1) $\begin{cases}\dfrac{\overline{AD}}{\sin B}=2R_1\\ \dfrac{\overline{AD}}{\sin C}=2R_2\\ \dfrac{\overline{AB}}{\sin C}=\dfrac{\overline{AC}}{\sin B}=2R_3\end{cases}$ ⇨ $\begin{cases}\dfrac{\overline{AD}}{\sin B}=2r\\ \dfrac{\overline{AD}}{\sin C}=4r\\ \dfrac{\overline{AB}}{\sin C}=\dfrac{\overline{AC}}{\sin B}=6r\end{cases}$

⇨ $\begin{cases}\overline{AD}=2r\sin B\\ \overline{AD}=4r\sin C\\ \overline{AB}=6r\sin C\\ \overline{AC}=6r\sin B\end{cases}$ (令 $\sin C=k\Rightarrow\sin B=2k$) ⇨ $\begin{cases}\overline{AD}=2r(2k)\\ \overline{AD}=4rk\\ \overline{AB}=6rk\\ \overline{AC}=12rk\end{cases}$

⇨ $\overline{AB}:\overline{AD}:\overline{AC}=6rk:4rk:12rk=3:2:6$ ###

9. 複選題

解：



(*)

(A) Γ_1 **長軸(長)**$=2a$ 、**短軸(長)**$=2b$ ；Γ_2 **長軸(長)**$=2a$ 、**短軸(長)**$=2b$

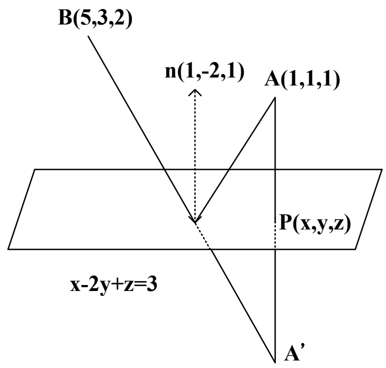
(B) Γ_1 **轉動** $90^0=\Gamma_2$ ###

(C) Γ_2 ：$c^2-a^2=b^2$ ⇨ $c^2=a^2+b^2$ ；Γ_3 ：$c^2-b^2=a^2$ ⇨ $c^2=a^2+b^2$ ###

(D) Γ_2 **漸近線**：$\dfrac{y}{a}\pm\dfrac{x}{b}=0$ ；Γ_3 **漸近線**：$\dfrac{y}{b}\pm\dfrac{x}{a}=0$

10. 複選題

解：



(1) $P:\begin{cases} x=t+1 \\ y=-2t+1 \\ z=t+1 \end{cases}$ **代入** $x-2y+z=3$ ⇨ $t=\frac{1}{2}$ ⇨ $P(\frac{3}{2},0,\frac{3}{2})$ ⇨ $A'(2,-1,2)$ #

(2) $\overrightarrow{A'B}=(3,4,0)$ ⇨ $\overleftrightarrow{A'B}:\begin{cases} \frac{x-2}{3}=\frac{y+1}{4} \\ z=2 \end{cases}$ #

(3) $L:\begin{cases} \frac{x-3}{4}=\frac{y+a}{b} \\ z=c \end{cases} \equiv \overleftrightarrow{A'B}$ ⇨

(i) $4:b=3:4$ ⇨ $b=\frac{16}{3}$ ###

(ii) L **上一點** $(3,-a,c)$ **代入** $\overleftrightarrow{A'B}:\begin{cases} \frac{x-2}{3}=\frac{y+1}{4} \\ z=2 \end{cases}$

⇨ $\frac{3-2}{3}=\frac{-a+1}{4}$ ⇨ $a=-\frac{1}{3}$ 、 $c=2$ ###

11. 複選題

學生	甲	乙	丙	丁	戊	己	庚	辛	壬	癸	平均
數學(X)	90	80	50	65	75	50	70	80	60	90	71
物理(Y)	80	70	50	70	65	55	60	75	65	90	68

(A) $\sigma_X^2=\frac{(x_1-\mu_X)^2+(x_2-\mu_X)^2+\ldots+(x_n-\mu_X)^2}{n}=\frac{(90-\mu_X)^2+(80-\mu_X)^2+\ldots}{n}$

$\sigma_{X'}^2=\frac{(80-\mu_X)^2+(90-\mu_X)^2+\ldots}{n}$ ⇨ $\sigma_X^2=\sigma_{X'}^2$ ###

(B) $r_{原}=\frac{1}{n}[(\frac{x_1-\mu_X}{\sigma_X})(\frac{y_1-\mu_Y}{\sigma_Y})+(\frac{x_2-\mu_X}{\sigma_X})(\frac{y_2-\mu_Y}{\sigma_Y})+...]$

$$=\frac{1}{n}[(\frac{90-\mu_X}{\sigma_X})(\frac{80-\mu_Y}{\sigma_Y})+(\frac{80-\mu_X}{\sigma_X})(\frac{70-\mu_Y}{\sigma_Y})+...]$$

$r_{新}=\frac{1}{n}[(\frac{80-\mu_X}{\sigma_X})(\frac{80-\mu_Y}{\sigma_Y})+(\frac{90-\mu_X}{\sigma_X})(\frac{70-\mu_Y}{\sigma_Y})+...]$

(*) **若** $r_{原}=r_{新}$ **，則** $\frac{1}{n}[(\frac{90-\mu_X}{\sigma_X})(\frac{80-\mu_Y}{\sigma_Y})+(\frac{80-\mu_X}{\sigma_X})(\frac{70-\mu_Y}{\sigma_Y})+...]$

$$=\frac{1}{n}[(\frac{80-\mu_X}{\sigma_X})(\frac{80-\mu_Y}{\sigma_Y})+(\frac{90-\mu_X}{\sigma_X})(\frac{70-\mu_Y}{\sigma_Y})+...]$$

⇨ $(\frac{90-\mu_X}{\sigma_X})(\frac{80-\mu_Y}{\sigma_Y})+(\frac{80-\mu_X}{\sigma_X})(\frac{70-\mu_Y}{\sigma_Y})=(\frac{80-\mu_X}{\sigma_X})(\frac{80-\mu_Y}{\sigma_Y})+(\frac{90-\mu_X}{\sigma_X})(\frac{70-\mu_Y}{\sigma_Y})$

⇨ $(90-\mu_X)(80-\mu_Y)+(80-\mu_X)(70-\mu_Y)=(80-\mu_X)(80-\mu_Y)+(90-\mu_X)(70-\mu_Y)$

⇨ $10(80-\mu_Y)=10(70-\mu_Y)$ ⇨ $(80-\mu_Y)=(70-\mu_Y)$ **矛盾** ⇨ $r_{原}\neq r_{新}$ ###

(C) $r_{原}=\frac{1}{n}[(\frac{x_1-\mu_X}{\sigma_X})(\frac{y_1-\mu_Y}{\sigma_Y})+(\frac{x_2-\mu_X}{\sigma_X})(\frac{y_2-\mu_Y}{\sigma_Y})+...]$

$r_{新}=\frac{1}{n}[(\frac{y_1-\mu_Y}{\sigma_Y})(\frac{x_1-\mu_X}{\sigma_X})+(\frac{y_2-\mu_Y}{\sigma_Y})(\frac{x_2-\mu_X}{\sigma_X})+...]$

⇨ $r_{原}=r_{新}$ ###

(D) $L:y=ax+b$ ⇨ $a=\frac{S_{XY}}{S_{XX}}$ ， $S_{XY}=\sum(x_i-\mu_X)(y_i-\mu_Y)$ 、 $S_{XX}=\sum(x_i-\mu_X)^2$

(*) $S_{X'Y}=\sum[(x_i+5)-(\mu_X+5)](y_i-\mu_Y)=\sum(x_i-\mu_X)(y_i-\mu_Y)=S_{XY}$

⇨ $S_{X'Y}=S_{XY}$ **，同理** $S_{X'X}=S_{XX}$ ⇨ $a'=\frac{S_{X'Y}}{S_{X'X}}=a$ ###

12. 複選題

解：

11	12	13	14	15	16
21	22	23	24	25	26
31	32	33	34	35	36
41	42	43	44	45	46
51	52	53	54	55	56
61	62	63	64	65	66

圖1

11	12	13	14	15	16
21	22	23	24	25	26
31	32	33	34	35	36
41	42	43	44	45	46
51	52	53	54	55	56
61	62	63	64	65	66

圖2

11	12	13	14	15	16
21	22	23	24	25	26
31	32	33	34	35	36
41	42	43	44	45	46
51	52	53	54	55	56
61	62	63	64	65	66

圖3

(A) $(a<b<c)$ ⇨ **利用(圖 1)：**

a	1	1,2	1,2,3	1,2,3,4
(b<c)個	4	3	2	1

⇨ $p=\dfrac{1\times4+2\times3+3\times2+4\times1}{216}=\dfrac{20}{216}=\dfrac{5}{54}$ ###

(B) $(a\le b\le c)$ ⇨ **利用(圖 2)：**

a	1	1,2	1,2,3	1,2,3,4	1,2,3,4,5	1,2,3,4,5,6
$(b\le c)$個	6	5	4	3	2	1

⇨ $p=\dfrac{1\times6+2\times5+3\times4+4\times3+5\times2+6\times1}{216}=\dfrac{56}{216}=\dfrac{7}{27}$ ###

(C) $(a+b+c=11)$ ⇨ **利用(圖 3)：**

a	1	2	3	4	5	6
(b+c)個	3	4	5	6	5	4

⇨ $p=\dfrac{3+4+5+6+5+4}{216}=\dfrac{27}{216}=\dfrac{1}{8}$ ###

(D) $(a-b)(b-c)=0$，**先算** $(a-b)(b-c)\ne0$ ⇨ **有 6×5×5 個**

⇨ $p=1-\dfrac{6\times5\times5}{216}=\dfrac{11}{36}$ ###

1. 填充

解：

(*) $a+\log_2 3$ 、 $a+\log_4 3$ 、 $a+\log_8 3$ **等比**

(1) $(a+\log_4 3)^2=(a+\log_2 3)(a+\log_8 3)$ ⇨ $(a+\frac{1}{2}A)^2=(a+A)(a+\frac{1}{3}A)$ (令$A=\log_2 3$)

⇨ $a^2+aA+\frac{1}{4}A^2=a^2+\frac{4}{3}aA+\frac{1}{3}A^2$ ⇨ $\frac{1}{3}aA+\frac{1}{12}A^2=0$

⇨ $4aA+A^2=0$ ⇨ $a=-\frac{1}{4}A$

(2) $a+\log_2 3$ 、 $a+\log_4 3$ 、 $a+\log_8 3$ ⇨ $-\frac{1}{4}A+A$ 、 $-\frac{1}{4}A+\frac{1}{2}A$ 、 $-\frac{1}{4}A+\frac{1}{3}A$

⇨ $\frac{3}{4}A$ 、 $\frac{1}{4}A$ 、 $\frac{1}{12}A$ ⇨ $r=\frac{1}{3}$ ###

2. 填充

解：

(1) $\lim\frac{1}{n^2}\sum\sqrt{n^2-k^2}=\lim\frac{1}{n^2}\sum n\sqrt{1-(\frac{k}{n})^2}=\lim\frac{1}{n}\sum\sqrt{1-(\frac{k}{n})^2}=\int_0^1\sqrt{1-x^2}\,dx$

(2) 令 $x=\sin\theta$ ⇨ $dx=\cos\theta\,d\theta$

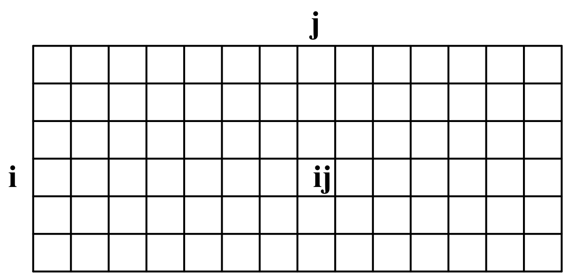
⇨ $\int_0^1\sqrt{1-x^2}\,dx=\int\sqrt{1-\sin^2\theta}\cos\theta\,d\theta=\int\cos^2\theta\,d\theta$

$=\int\frac{\cos 2\theta+1}{2}d\theta=\frac{1}{2}(\frac{1}{2}\sin 2\theta+\theta)=\frac{1}{2}(\sin\theta\cos\theta+\theta)$

$=\frac{1}{2}(x\sqrt{1-x^2}+\sin^{-1}x)|_0^1=\frac{1}{4}\pi$ ###

3. 填充

解：



(1) $R_{ij} = 29(i-1) + j$

(2) $C_{ij} = 17(j-1) + i$

(3) $R_{ij} = C_{ij}$ ⇨ $29(i-1) + j = 17(j-1) + i$ ⇨ $7i = 4j + 3$ #

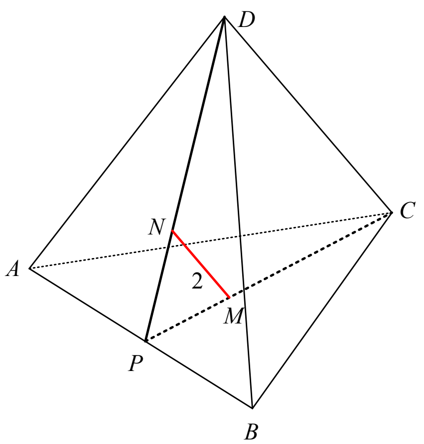
⇨

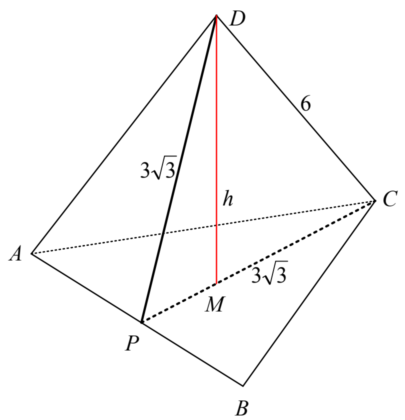
i	1	5	9	13	17
j	1	8	15	22	29
R_{ij}	1	124	247	370	493

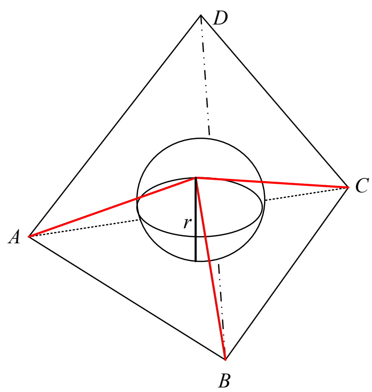
(4) Sum=1+124+247+370+493=1235 ###

4. 填充

解：







(1) $\overline{PM} : \overline{MC} = 1:2$ ⇨ $\overline{MN} : \overline{CD} = 1:3$ ⇨ $\overline{CD} = 6$ ⇨ $h = 2\sqrt{6}$ #

(2) 令ΔABC面積=S ⇨ $ABC - D$體積 $= \dfrac{Sh}{3}$ #

(3) $ABC - O$體積 $= \dfrac{Sr}{3}$ ⇨ $ABC - D$體積 $= \dfrac{Sr}{3} \times 4$ #

(4) $\dfrac{Sr}{3} \times 4 = \dfrac{Sh}{3}$ ⇨ $4r = h = 2\sqrt{6}$ ⇨ $r = \dfrac{\sqrt{6}}{2}$ #

(5) O體積 $= \dfrac{4}{3}\pi r^3 = \dfrac{4}{3}\pi(\dfrac{\sqrt{6}}{2})^3 = \sqrt{6}\pi$ ###

5. 填充

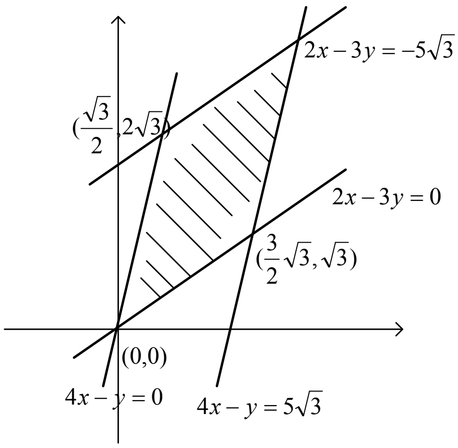
解：

(*) $\overset{\rightharpoonup}{OP}=(3\cos\alpha+\sin\beta,2\cos\alpha+4\sin\beta)$

(1) 令 $\begin{cases}x=3\cos\alpha+\sin\beta\\ y=2\cos\alpha+4\sin\beta\end{cases}$ ⇨ $\begin{cases}\cos\alpha=\dfrac{4x-y}{10}\\ \sin\beta=\dfrac{-2x+3y}{10}\end{cases}$

(2) $\begin{cases}\dfrac{\pi}{6}\le\alpha\le\dfrac{\pi}{2}\\ 0\le\beta\le\dfrac{\pi}{3}\end{cases}$ ⇨ $\begin{cases}0\le\cos\alpha\le\dfrac{\sqrt{3}}{2}\\ 0\le\sin\beta\le\dfrac{\sqrt{3}}{2}\end{cases}$ ⇨ $\begin{cases}0\le\dfrac{4x-y}{10}\le\dfrac{\sqrt{3}}{2}\\ 0\le\dfrac{-2x+3y}{10}\le\dfrac{\sqrt{3}}{2}\end{cases}$ ⇨ $\begin{cases}0\le4x-y\le5\sqrt{3}\\ -5\sqrt{3}\le2x-3y\le0\end{cases}$

⇨ **如圖所示**



(3) 面積$=\left|\begin{vmatrix}\dfrac{\sqrt{3}}{2} & 2\sqrt{3}\\ \dfrac{3}{2}\sqrt{3} & \sqrt{3}\end{vmatrix}\right|=\dfrac{15}{2}$ ###

6. 填充

解：

(1) 總數： $C_2^9=36$

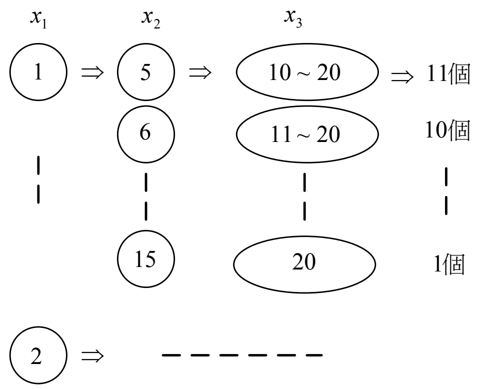
(2) 共有：(15)(16)(54)(59)(48)|(69)(62)(98)(97)(83)|(27)(73)

=6+7+9+14+12+15+8+17+16+11+9+10=134

(3) $E=\dfrac{134}{36}=\dfrac{67}{18}$ ###

7. 填充

解：



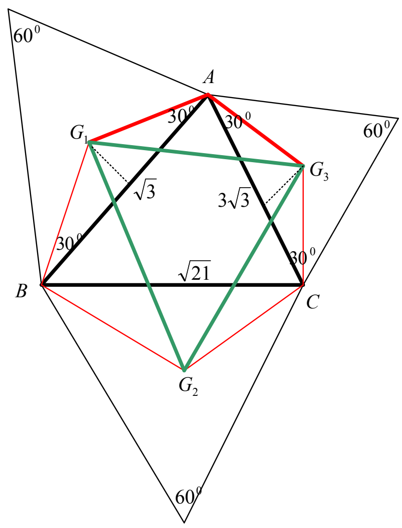
(1) 總數 $= C_3^{20} = 1140$

(2) (x_1,x_2,x_3)共有 $(11+10+...+2+1)+(10+9+...+2+1)+...+(2+1)+(1)=286$

(3) $p=\dfrac{286}{1140}=\dfrac{143}{570}$ ###

8. 填充

解：



(1) $\cos A=\dfrac{(\sqrt{3})^2+(3\sqrt{3})^2-(\sqrt{21})^2}{2\cdot\sqrt{3}\cdot 3\sqrt{3}}=\dfrac{1}{2}$ ⇨ $\angle A=60^0$ #

(2) $\overline{AG_1}=1$ 、 $\overline{AG_3}=3$ #

(3) $\cos\angle G_1AG_3=\cos 120^0=\dfrac{1^2+3^2-(\overline{G_1G_3})^2}{2\cdot 1\cdot 3}$ ⇨ $\overline{G_1G_3}=\sqrt{13}$ #

(4) $\Delta G_1G_2G_3=(\dfrac{\sqrt{3}}{4}a^2)=\dfrac{13\sqrt{3}}{4}$ ###

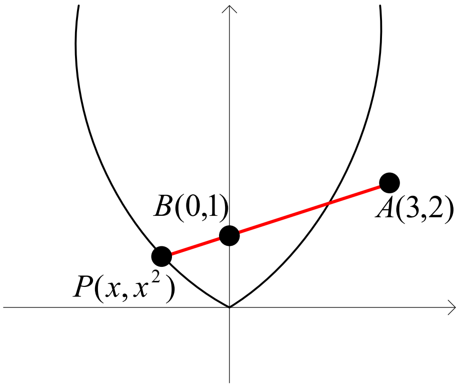
9. 填充

解：

(1) $f(x)=\sqrt{x^4-3x^2-6x+13}-\sqrt{x^4-x^2+1}=\sqrt{(x^2-2)^2+(x-3)^2}-\sqrt{(x^2-1)^2+x^2}$ #

(2) 令 $P(x,x^2)$、$A(3,2)$、$B(0,1)$ ⇨ 如圖所示

$$\sqrt{(x^2-2)^2+(x-3)^2}-\sqrt{(x^2-1)^2+x^2}=\overline{PA}-\overline{PB}=\overline{AB}=\sqrt{10}$$ ###



1. 計算

解：

(1) $a^3+b^3+c^3-3abc=(a+b)^3-3ab(a+b)+c^3-3abc$

$=(a+b+c)^3-3(a+b)c(a+b+c)-3ab(a+b+c)$

$=(a+b+c)[(a+b+c)^2-3(a+b)c-3ab]$

$=(a+b+c)[a^2+b^2+c^2+2ab+2bc+2ca-3ac-3bc-3ab]$

$=(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$ ###

(2) $a^3+b^3+c^3-3abc=(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

$$=\frac{1}{2}(a+b+c)[(a-b)^2+(b-c)^2+(c-a)^2]\geq 0$$

令 $A=a^3$、$B=b^3$、$C=c^3$，

$a^3+b^3+c^3-3abc\geq 0$ ⇨ $A+B+C-3\sqrt[3]{A}\sqrt[3]{B}\sqrt[3]{C}\geq 0$ ⇨ $\dfrac{A+B+C}{3}\geq\sqrt[3]{ABC}$ ###

(3) $\dfrac{(a+1)+(b+2)+(c+3)}{3}\geq\sqrt[3]{(a+1)(b+2)(c+3)}$ ⇨ $\dfrac{(a+b+c)+6}{3}\geq\sqrt[3]{(a+1)(b+2)(c+3)}$

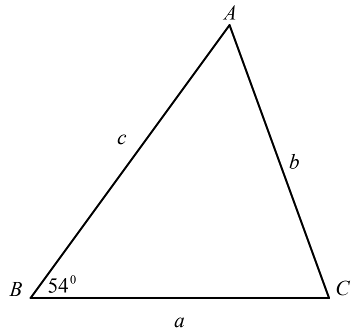
⇨ $\dfrac{18+6}{3}\geq\sqrt[3]{(a+1)(b+2)(c+3)}$ ⇨ $8\geq\sqrt[3]{(a+1)(b+2)(c+3)}$ ⇨ $512\geq(a+1)(b+2)(c+3)$

⇨ $(a+1)(b+2)(c+3)$ **最大值** 為 **512**。

⇨ $\begin{cases}a+1=b+2=c+3\\ a+b+c=18\end{cases}$，$a=7, b=6, c=5$ ###

2. 計算

解：



(1) $\overline{BC}^2-\overline{AB}^2=\overline{AC}\times\overline{AB}$ ⇨ $(2R\sin A)^2-(2R\sin C)^2=(2R\sin B)\times(2R\sin C)$

⇨ $\sin^2 A-\sin^2 C=\sin B\times\sin C$ ⇨ $\dfrac{1-\cos 2A}{2}-\dfrac{1-\cos 2C}{2}=\sin B\times\sin C$

⇨ $\dfrac{\cos 2C-\cos 2A}{2}=\sin B\times\sin C$ ⇨ $\dfrac{-2\sin(C+A)\sin(C-A)}{2}=\sin B\times\sin C$

⇨ $-\sin B\times\sin(C-A)=\sin B\times\sin C$ ⇨ $\sin(C-A)=-\sin C=\sin(-C)$

⇨ $\begin{cases} C-A=-C \\ A+C=126^0 \end{cases}$ ⇨ $\angle C=42^0$ ###

3. 計算

解：

(*) $\begin{cases} x+\dfrac{1}{y}=4 \\ y+\dfrac{1}{z}=1 \\ z+\dfrac{1}{x}=\dfrac{7}{3} \end{cases}$

(1) 相加：$x+y+z+\dfrac{1}{x}+\dfrac{1}{y}+\dfrac{1}{z}=\dfrac{22}{3}$ #

(2) 相乘：$(x+\dfrac{1}{y})(y+\dfrac{1}{z})(z+\dfrac{1}{x})=\dfrac{28}{3}$ ⇨ $xyz+x+y+z+\dfrac{1}{x}+\dfrac{1}{y}+\dfrac{1}{z}+\dfrac{1}{xyz}=\dfrac{28}{3}$

⇨ $xyz+\dfrac{1}{xyz}=2$ ⇨ $A+\dfrac{1}{A}=2$ ⇨ $A=1$ (令 $A=xyz$) ###