不失一般性,令b=(6,0),a=(1,$\sqrt{15}$),c=(x,y)

根據題意$(\vec{c}-\vec{a})(\vec{c}-\vec{b})=0$,可得$\left|\vec{c}\right|^2-\vec{c}(\vec{a}+\vec{b})+6=0$

$x^2-7x+y^2-7\sqrt{15}y+6=0$

接著考慮$f(x,y)=\sqrt{x^2+y^2}$限制在g(x,y)=$x^2-7x+y^2-7\sqrt{15}y+6=0$上的最大值

$By-the-lagrange-Condition$

$$\nabla f(x,y)=\frac{x}{\sqrt{x^2+y^2}}i+\frac{y}{\sqrt{x^2+y^2}}j,\nabla g(x,y)=(2x-7)i+(2y-7\sqrt{15})j$$

$$\nabla f(x,y)=\lambda\nabla g(x,y)\Rightarrow\begin{cases}\dfrac{x}{\sqrt{x^2+y^2}}=\lambda(2x-7)......(1)\\ \dfrac{y}{\sqrt{x^2+y^2}}=\lambda(2y-7\sqrt{15})......(2)\end{cases}$$

$(1)/(2)\Rightarrow y=\sqrt{15}x$再帶回g(x,y)=0得到$8x^2-56x+3=0,ie,x=\dfrac{14\pm\sqrt{190}}{4}$

最後,所求$\sqrt{x^2+y^2}$=$4x=14+\sqrt{190}$最大