105-全國高中教師聯招 詳解整理

1. 單選

解：**(借解)**

發表於 2016-5-7 22:26 只看該作者

回復 8# csihcs 的帖子

單選第1題
真的要算也是差不多的方法

$n^5 \equiv n \pmod{10}$

令$n^5 = 4^5 + 5^5 + 6^5 + 7^5 + 9^5 + 11^5$

$n^5 \equiv 4+5+6+7+9+11 \equiv 2 \pmod{10}$

$4^5 < n^5 < 6 \times 11^5 < 2^5 \times 11^5$

$4 < n < 22$

$n = 12$

###

2. 單選

解：

(A) $(\cos^2 25^0 - \sin^2 25^0) = \cos 50^0 = \sin 40^0$ #

(B) $\sqrt{1+\sin 340^0} - \sqrt{1-\sin 340^0} = \sqrt{1-\sin 20^0} - \sqrt{1+\sin 20^0} = \sqrt{1-\cos 70^0} - \sqrt{1+\cos 70^0}$

$= \sqrt{1-(1-2\sin^2 35^0)} - \sqrt{1+(2\cos^2 35^0 - 1)} = \sqrt{2\sin^2 35^0} - \sqrt{2\cos^2 35^0}$

$= \sqrt{2}(\sin 35^0 - \cos 35^0) = \sqrt{2}(\cos 55^0 - \cos 35^0) = \sqrt{2}(-2\sin 45^0 \sin 10^0) = -2\sin 10^0$ #

(C) $\sin 23^0 \cos 112^0 - \sin 292^0 \sin 67^0 = -\sin 23^0 \cos 68^0 + \cos 22^0 \sin 67^0$

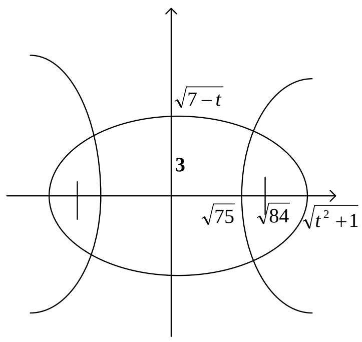
$= -\cos 67^0 \sin 22^0 + \cos 22^0 \sin 67^0 = \sin 45^0$ ###

(D) $\dfrac{2\tan 67.5^0}{1-\tan 67.5^0} = \tan 135^0$ #

3. 單選

解：

(1) $t^2 + 1 - 84 = 7 - t$ ⇨ $t^2 + t - 90 = 0$ ⇨ $t = -10, t = 9\ (\times)$ ###



4. 單選

解：

(1) $\sqrt{2016}=\sqrt{x}+\sqrt{y}$ ⇨ $12\sqrt{14}=\sqrt{x}+\sqrt{y}$ ⇨ $(x,y)=(\sqrt{14},11\sqrt{14}),,,(5\sqrt{14},7\sqrt{14})$ ###

5. 單選

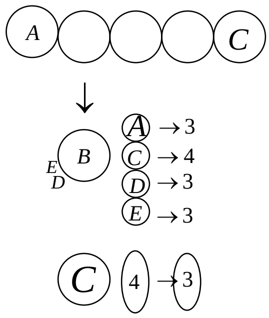
解：

(1) 原式 $f(x)=(\frac{a}{2}x^2-bx^2+\frac{c}{2}x^2)+...$ ⇨ $\frac{a}{2}-b+\frac{c}{2}\neq 0$ ⇨ $a+c\neq 2b$

⇨ $a+c=2b$ 有 **18** 組 ⇨ $P(a+c\neq 2b)=1-\frac{18}{216}=\frac{11}{12}$ ###

6. 單選

解：



(1) $(3+4+3+3)\times 3+4\times 3=51$ ###

7. 單選

解：

(*) $x^2-4ax+4a^2-4a-3b+9=0$

(1) $\begin{cases}\alpha+\beta=4a\\ \alpha\beta=4a^2-4a-3b+9\\ \alpha-\beta=2\sqrt{101}\end{cases}$

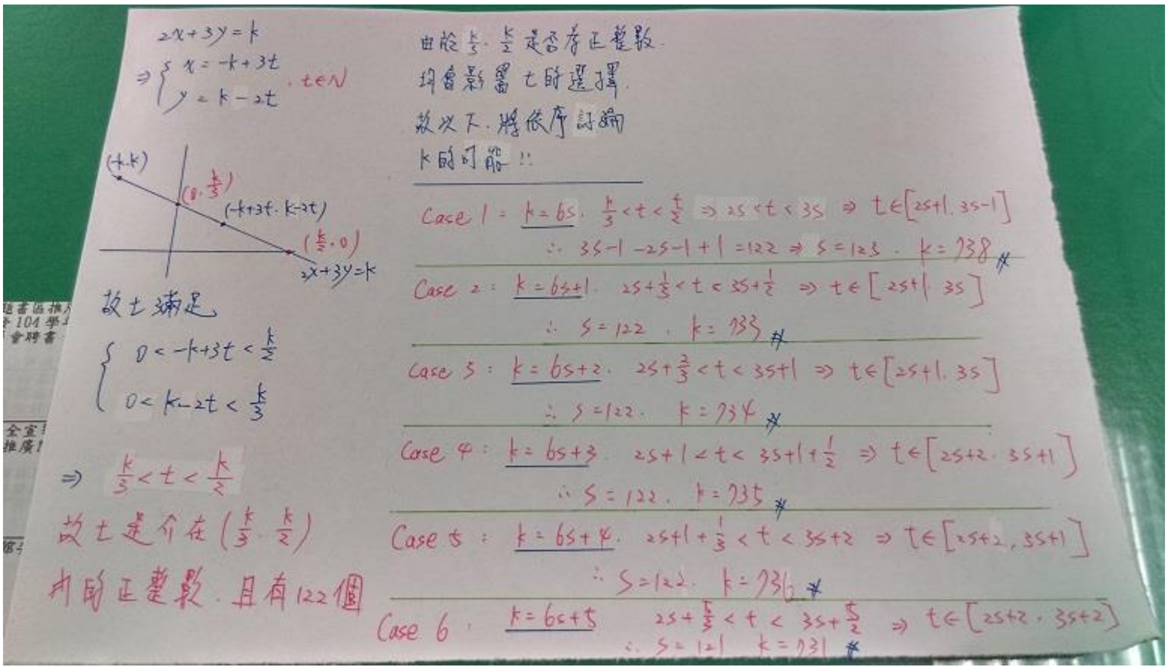
(2) $(\alpha+\beta)^2=16a^2$ ⇨ $(\alpha-\beta)^2+4\alpha\beta=16a^2$ ⇨ $404+4(4a^2-4a-3b+9)=16a^2$

⇨ $440=16a+12b$ ⇨ $4a+3b=110$ ⇨ $\begin{cases}a=110+3t\\ b=-110-4t\end{cases}$ ⇨ $-36.xx<t<-27.xx$

⇨ $t=-28,-29,\dots,-36$ ###

8. 單選

解：(借解)



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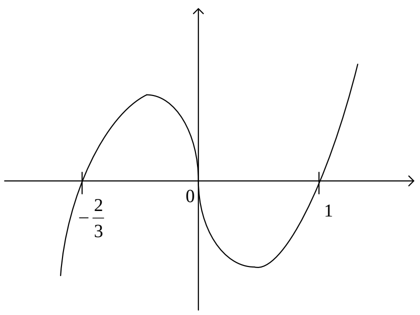
9. 複選題

解：

(*) $f(x)=3x^3-x^2-2x=x(x-1)(3x+2)$

(A) $f'(x)=9x^2-2x-2$ ⇨ $m=f'(1)=5$ ⇨ $y=5(x-1)$ #



(B) $|\int_0^1(3x-x^2-2x)dx|=\frac{7}{12}$ #

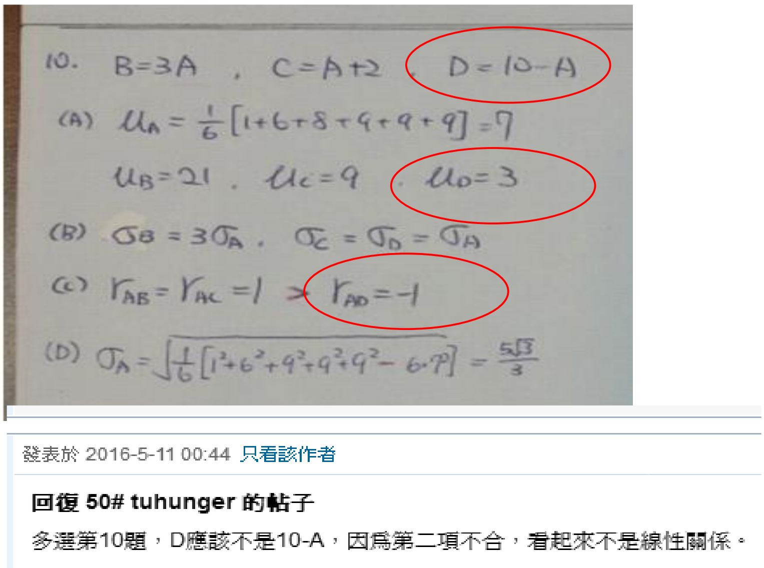
(C) $f''(x)=18x-2$ ⇨ $x=\frac{1}{9}$ ⇨ $C(\frac{1}{9},\frac{-56}{243})$ #

(D) 如圖 #

10. 複選題

解：**(借解)**

(*)


10. $B=3A$, $C=A+2$, $D=10-A$

(A) $\mu_A=\frac{1}{6}[1+6+8+9+9+9]=7$

$\mu_B=21$, $\mu_C=9$, $\mu_D=3$

(B) $\sigma_B=3\sigma_A$, $\sigma_C=\sigma_D=\sigma_A$

(C) $r_{AB}=r_{AC}=1$, $r_{AD}=-1$

(D) $\sigma_A=\sqrt{\frac{1}{6}[1^2+6^2+9^2+9^2+9^2-6\cdot 7^2]}=\frac{5\sqrt{3}}{3}$

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回復 50# tuhunger 的帖子

多選第10題，D應該不是10-A，因為第二項不合，看起來不是線性關係。 ###

11. 複選題

(*) $f(x)=x^5-2px^4+x^3-3px^2+x-2$

(A) $f(1)=1-2p+1-3p+1-2=1-5p=0$ ⇨ 不存在 $p\in Z$ #

(B) $f(-1)=-1-2p-1-3p-1-2=-5-5p=0$ ⇨ $p=-1$

⇨ $f(-2)=-32+32-8+12-2-2=0$ #

(C) $f(-1)=-1-2p-1-3p-1-2=-5-5p\neq 0$ ⇨ $p\neq -1$

⇨ 檢查 $f(x)$ 所有可能因式 $(x+1)$ 、 $(x-1)$ 、 $(x+2)$ 、 $(x-2)$ 皆不是因式。#

(D) 若 $(x+1)^2$ 是因式 ⇨ $(x+1)$ 是因式 ⇨ $p=-1$

⇨ $f(x)=x^5+2x^4+x^3+3x^2+x-2=(x+1)(x^4+x^3+3x-2)$ #

12. 複選題

解：

(A) $A=\begin{bmatrix}4 & -3\\ 2 & -1\end{bmatrix}$ ⇨ $\begin{vmatrix}4-\lambda & -3\\ 2 & -1-\lambda\end{vmatrix}=0$ ⇨ $\lambda^2-3\lambda+2=0$ ⇨ $A^2-3A+2I=0$ #

(B) $A^4-7A^3+10A^2-8A+3I=(A^2-3A+2I)(A^2-4A+4I)-12A+112I$

$=-12A+112I=\begin{bmatrix}-37 & 36\\ -24 & 23\end{bmatrix}$ #

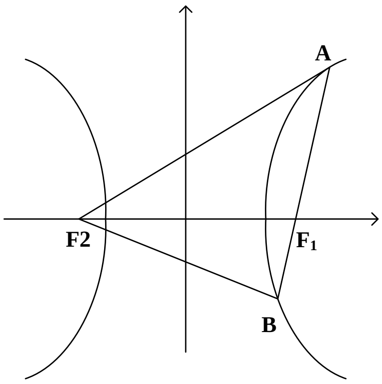
(C) $P=\begin{bmatrix}1 & 3\\ 1 & 2\end{bmatrix}$ ⇨ $P^{-1}=\begin{bmatrix}-2 & 3\\ 1 & -1\end{bmatrix}$ ⇨ $A=PBP^{-1}$

$\Rightarrow \ B = P^{-1}AP = \begin{bmatrix} -2 & 3 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 4 & -3 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$ #

(D) $A^{10} = (PBP^{-1})^{10} = PB^{10}P^{-1} = \begin{bmatrix} 1 & 3 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 2^{10} \end{bmatrix}\begin{bmatrix} -2 & 3 \\ 1 & -1 \end{bmatrix}$ #

1. 填充

解：



(*)

(1) $\overline{AF_2} - \overline{AF_1} = 2a = 6$, $\overline{BF_2} - \overline{BF_1} = 2a = 6$ $\Rightarrow$ $\overline{AF_2} + \overline{BF_2} - \overline{AF_1} - \overline{BF_1} = 12$ $\Rightarrow$ $\overline{AF_2} + \overline{BF_2} - 15 = 12$

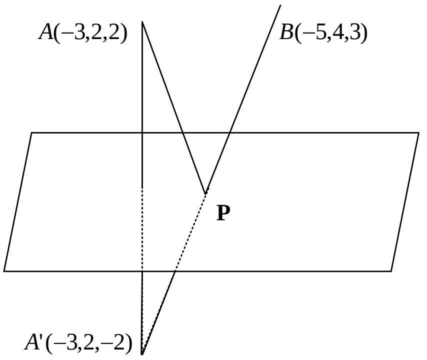
$\Rightarrow$ $\overline{AF_2} + \overline{BF_2} = 27$ $\Rightarrow$ $\overline{AF_2} + \overline{BF_2} + \overline{AB} = 27 + 15 = 42$ ###

2. 填充

解：

(1) 令 $A(-3,2,2)$、$B(-5,4,3)$、$P(x,y,0)$，則

$$\sqrt{(x+3)^2 + (y-2)^2 + 4} + \sqrt{(x+5)^2 + (y-4)^2 + 9} = \overline{AP} + \overline{BP}$$ #

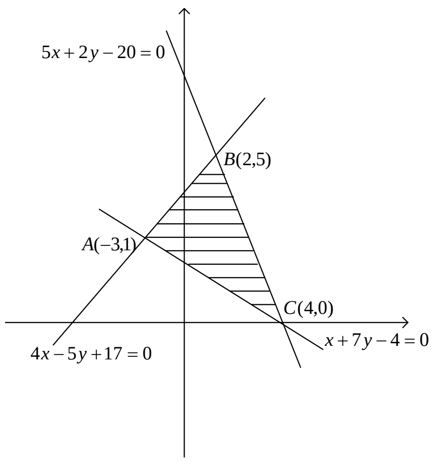


(2) ⇨ $\min\{\overline{AP}+\overline{BP}\}=\overline{BA'}=\sqrt{33}$ ###

(3) 令 $\vec{BP}=t\vec{BA'}$ ⇨ $P:\begin{cases}2t-5\\-2t+4\\-5t+3=0\end{cases}$ ⇨ $t=\dfrac{3}{5}$ ⇨ $P(-\dfrac{19}{5},\dfrac{14}{5},0)$ ###

3. 填充

解：



* $f(x,y)=ax-y$

(1) $\begin{aligned}f(A)&=-3a-1\\f(B)&=2a-5\\f(C)&=4a-1\end{aligned}$ ⇨ $\begin{aligned}4a-1&\le-3a-1\\4a-1&\le 2a-5\end{aligned}$ ⇨ $\begin{aligned}&a\le 0\\&a\le-\dfrac{5}{2}\end{aligned}$ ⇨ $a\le-\dfrac{5}{2}$ ###

4. 填充

解：

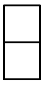
$$*\begin{cases} a_1 = 2 \\ a_{n+1} = 2a_n - 1 \end{cases}$$

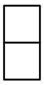
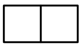
(1) $a_{n+1} = 2a_n - 1$ ⇨ $(a_{n+1} - 1) = 2(a_n - 1)$ ⇨ $\begin{matrix} a_2 - 1 = 2(a_1 - 1) \\ a_3 - 1 = 2(a_2 - 1) \\ \cdots \\ a_n - 1 = 2(a_{n-1} - 1) \end{matrix}$ ⇨ $a_n = 2^{n-1} + 1 > 1000$

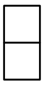
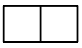
⇨ $n = 11$ ###

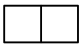
5. 填充

解：

(1) 6 個直 ⇨ **1** 種

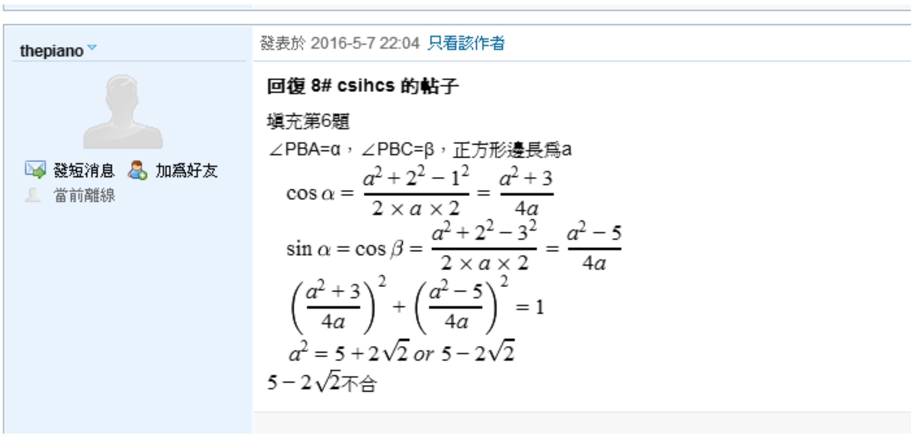
(2) 4 個直2 個橫 ⇨ **5** 種

(3) 2 個直4 個橫 ⇨ **4** 種

(4) 6 個橫 ⇨ **1** 種

6. 填充

解： (借解)



thepiano

發短消息 加爲好友

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回復 8# csihcs 的帖子

填充第6題

∠PBA=α，∠PBC=β，正方形邊長爲a

$$\cos\alpha = \frac{a^2 + 2^2 - 1^2}{2 \times a \times 2} = \frac{a^2 + 3}{4a}$$

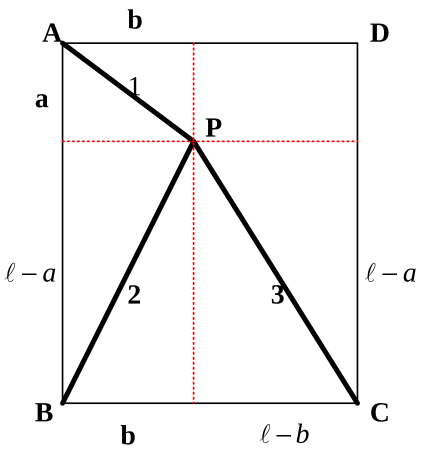
$$\sin\alpha = \cos\beta = \frac{a^2 + 2^2 - 3^2}{2 \times a \times 2} = \frac{a^2 - 5}{4a}$$

$$\left(\frac{a^2 + 3}{4a}\right)^2 + \left(\frac{a^2 - 5}{4a}\right)^2 = 1$$

$$a^2 = 5 + 2\sqrt{2} \text{ or } 5 - 2\sqrt{2}$$

$5 - 2\sqrt{2}$不合

另解：



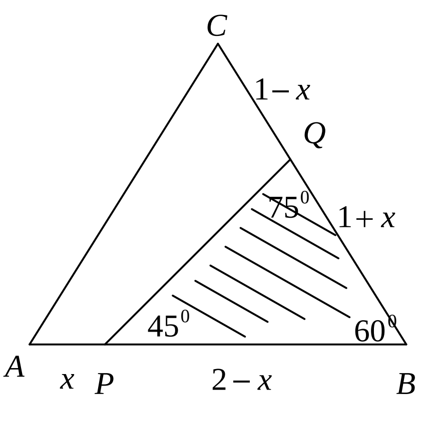
(1) $\begin{cases} a^2+b^2=1 \\ (\ell-a)^2+b^2=4 \\ (\ell-a)^2+(\ell-b)^2=9 \end{cases}$ ⇨ $\begin{matrix}(2)-(1) \\ (3)-(2)\end{matrix}\begin{cases}\ell^2-2a\ell=3 \\ \ell^2-2b\ell=5\end{cases}$ ⇨ $\begin{cases}\ell^2-3=2a\ell \\ \ell^2-5=2b\ell\end{cases}$

⇨ $(\ell^2-3)^2+(\ell^2-5)^2=4\ell^2(a^2+b^2)$ ⇨ $(\ell^2-3)^2+(\ell^2-5)^2=4\ell^2$

⇨ $\ell^4-10\ell^2+17=0$ ⇨ $\ell^2=5\pm2\sqrt{2}$ **(**負不合**)** ###

7. 填充

解：



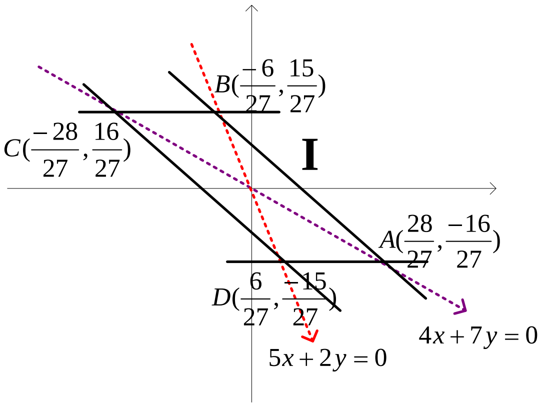
(1) $\dfrac{1+x}{\sin 45^0}=\dfrac{2-x}{\sin 75^0}$ ⇨ $\dfrac{1+x}{1/\sqrt{2}}=\dfrac{2-x}{\sqrt{6}+\sqrt{2}/4}$ ⇨ $\dfrac{\sqrt{6}+\sqrt{2}}{4}(1+x)=\dfrac{1}{\sqrt{2}}(2-x)$

⇨ $x=\dfrac{3-\sqrt{3}}{3+\sqrt{3}}=2-\sqrt{3}$ #

(2) $\Delta BPQ=\dfrac{1}{2}(\sqrt{3})(3-\sqrt{3})\sin 60^0=\dfrac{3}{4}(3-\sqrt{3})$ ###

8. 填充

解：



(1) 解 $\begin{cases}\dfrac{|4x+7y|}{3}+\dfrac{|5x+2y|}{4}=1\\ 4x+7y=0\\ 5x+2y=0\end{cases}$ ⇨ $A=(\dfrac{28}{27},\dfrac{-16}{27})$、$B=(\dfrac{-6}{27},\dfrac{15}{27})$、C、D

(2) $ABCD$面積 $=4\times\dfrac{1}{2}\begin{vmatrix}\dfrac{28}{27} & \dfrac{-16}{27}\\ \dfrac{-6}{27} & \dfrac{15}{27}\end{vmatrix}=\dfrac{8}{9}$ ###

9. 填充

解：

(1) $\begin{bmatrix}1 & 2\\ 3 & 2\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}+\begin{bmatrix}0 & 2\\ 3 & 1\end{bmatrix}=I+B$

(2) $B^2=\begin{bmatrix}0 & 2\\ 3 & 1\end{bmatrix}\begin{bmatrix}0 & 2\\ 3 & 1\end{bmatrix}=\begin{bmatrix}6 & 2\\ 3 & 7\end{bmatrix}=\begin{bmatrix}6 & 0\\ 0 & 6\end{bmatrix}+\begin{bmatrix}0 & 2\\ 3 & 1\end{bmatrix}=6I+B$ #

(3) $\begin{bmatrix}1&2\\3&2\end{bmatrix}^5=(I+B)^5=I^5+5B+10B^2+10B^3+5B^4+B^5=409I+205B$ #

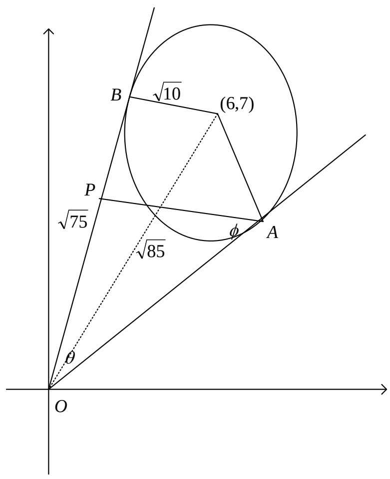
(4) $A=\begin{bmatrix}1&2\\3&2\end{bmatrix}^5\begin{bmatrix}1&2&3\\3&2&1\end{bmatrix}=(409I+205B)\begin{bmatrix}1&2&3\\3&2&1\end{bmatrix}=409I\begin{bmatrix}1&2&3\\3&2&1\end{bmatrix}+205B\begin{bmatrix}1&2&3\\3&2&1\end{bmatrix}$

$$=409\begin{bmatrix}1&2&3\\3&2&1\end{bmatrix}+205B\begin{bmatrix}1&2&3\\3&2&1\end{bmatrix}$$

(5) 元總和 $=409(6+6)+205(2\times6+3\times6+1\times6)=12288$ ###

10. 填充

解：



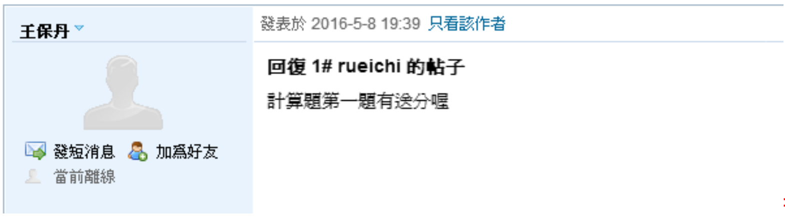
(1) $\dfrac{\overline{OP}}{\sin\phi}=\dfrac{\overline{AP}}{\sin\theta}$ ⇨ $\dfrac{\overline{OP}}{\overline{AP}}=\dfrac{\sin\phi}{\sin\theta}$ **(1*)**

(2) $\sin\dfrac{\theta}{2}=\dfrac{\sqrt{10}}{\sqrt{85}}$、$\cos\dfrac{\theta}{2}=\dfrac{\sqrt{75}}{\sqrt{85}}$ ⇨ $\sin\theta=2\sin\dfrac{\theta}{2}\cos\dfrac{\theta}{2}=2\times\dfrac{\sqrt{10}}{\sqrt{85}}\times\dfrac{\sqrt{75}}{\sqrt{85}}=\dfrac{2\sqrt{750}}{85}$

(1*) $\dfrac{\overline{OP}}{\overline{AP}}=\dfrac{\sin\phi}{\sin\theta}=\dfrac{\sin\phi}{2\sqrt{750}/85}=\dfrac{85\sin\phi}{2\sqrt{750}}=\dfrac{85}{2\sqrt{750}}=\dfrac{17\sqrt{30}}{60}$ (當 $\sin\phi=1$ 為最大值) ###

1. 計算

解：( )



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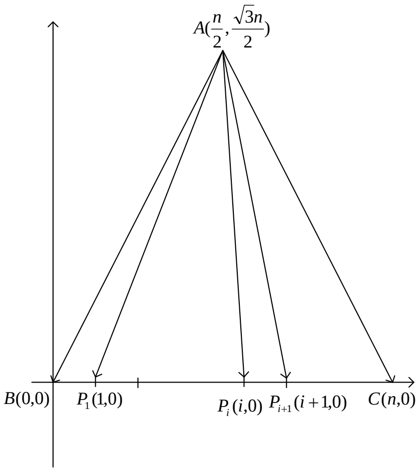
2. 計算

解：



(1) $\vec{AB}\cdot\vec{AP_1}=(\frac{1}{n})(-\frac{n}{2},-\frac{\sqrt{3}n}{2})\cdot(\frac{1}{n})(1-\frac{n}{2},-\frac{\sqrt{3}n}{2})$

(2) $\vec{AP_i}\cdot\vec{AP_{i+1}}=(\frac{1}{n})(i-\frac{n}{2},-\frac{\sqrt{3}n}{2})\cdot(\frac{1}{n})(i+1-\frac{n}{2},-\frac{\sqrt{3}n}{2})$

(3) $S_n=\vec{AB}\cdot\vec{AP_1}+\vec{AP_1}\cdot\vec{AP_2}+...+\vec{AP_{n-1}}\cdot\vec{AC}=(\frac{1}{n^2})\sum_{i=0}^{n-1}[(i-\frac{n}{2})(i+1-\frac{n}{2})+(-\frac{\sqrt{3}n}{2})(-\frac{\sqrt{3}n}{2})]$

$$=(\frac{1}{n^2})\sum_{i=0}^{n-1}[i(i+1)-\frac{n}{2}(2i+1)+\frac{n^2}{4}+\frac{3n^2}{4}]=(\frac{1}{n^2})\sum_{i=0}^{n-1}[(i^2+i)-\frac{n}{2}(2i+1)+\frac{n^2}{4}+\frac{3n^2}{4}]$$

$$=(\frac{1}{n^2})[\frac{(n-1)(n)(2n-1)}{6}+\frac{(n-1)(n)}{2}-\frac{n}{2}((n-1)(n)+n)+n^3]$$

(4) $\lim_{n\to\infty}\frac{S_n}{n^2}=\frac{2}{6}+0-\frac{1}{2}+1=\frac{5}{6}$ ###