106-全國高中教師聯招 詳解整理

1. 單選

解：

(1) $a=\sqrt[3]{\frac{3-\sqrt{5}}{2}}+\sqrt[3]{\frac{3+\sqrt{5}}{2}}=A+B$ **where** $A=\sqrt[3]{\frac{3-\sqrt{5}}{2}}$、$B=\sqrt[3]{\frac{3+\sqrt{5}}{2}}$

⇨ $A^3=\frac{3-\sqrt{5}}{2}$ 、 $B^3=\frac{3+\sqrt{5}}{2}$ ⇨ $A^3+B^3=3$、 $A^3B^3=1$

(2) $a^3=(A+B)^3=A^3+3AB(A+B)+B^3=3+3a$ #

(3) $a^6-6a^4+9a^2+27=(3a+3)^2-6a(3a+3)+9a^2+27=36$ ###

2. 單選

解：

(*) $\frac{16}{\sin^6\theta}+\frac{1}{\cos^6\theta}=81$

(1) $[(\frac{4}{\sin^3\theta})^2+(\frac{1}{\cos^3\theta})^2](\sin^2\theta+\cos^2\theta)\geq(\frac{4}{\sin^2\theta}+\frac{1}{\cos^2\theta})^2$ #

(2) $[(\frac{2}{\sin\theta})^2+(\frac{1}{\cos\theta})^2](\sin^2\theta+\cos^2\theta)\geq(2+1)^2=9$

⇨ $(\frac{4}{\sin^2\theta}+\frac{1}{\cos^2\theta})^2\geq 9^2=81$ #

(3) $\frac{\frac{2}{\sin\theta}}{\sin\theta}=\frac{\frac{1}{\cos\theta}}{\cos\theta}$ ⇨ $\tan^2\theta=2$ ⇨ $\tan\theta=\sqrt{2}$ ###

3. 單選

解：

(1) $ab=4$ ⇨ $\log_2 ab=\log_2 a+\log_2 b=2$ & $(\log_2 a)(\log_2 b)=-1$

⇨ $\log_2 a$、$\log_2 b$是：$x^2-2x-1=0$ 的兩根 ⇨ $x=\dfrac{2\pm\sqrt{8}}{2}=1\pm\sqrt{2}$

(2) $\log_a b=\dfrac{\log_2 b}{\log_2 a}=\dfrac{1-\sqrt{2}}{1+\sqrt{2}}=-3+2\sqrt{2}$ ###

4. 單選

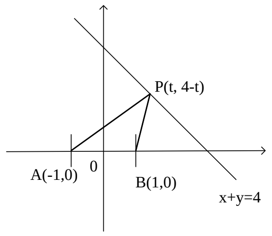
解：

(1) $\dfrac{1}{x}+\dfrac{1}{y}=\dfrac{1}{1899}$ ⇨ $xy-1899x-1899y=0$ ⇨ $(x-1899)(y-1899)=1899^2$

⇨ $(x-1899)(y-1899)=3^4\times 211^2$ ⇨ 正因數有 $(4+1)(2+1)=15$個 ###

5. 單選

解：



(1) 令 P(t, 4-t) ⇨ $\vec{AP}=(t+1, 4-t)$、$\vec{BP}=(t-1, 4-t)$

(2) $|\vec{AP}+\vec{BP}|=|(2t, 8-2t)|=\sqrt{8t^2-32t+64}=\sqrt{8(t-2)^2+32}\geq\sqrt{32}$ ###

6. 單選

解：

(1) $\begin{cases}106\leq x\leq 2017\\106\leq y\leq 2017\\8x-5y=37\end{cases}$ ⇨ $\begin{cases}x^*=74\\y^*=111\end{cases}$ ⇨ $\begin{cases}x=74+5t\\y=111+8t\end{cases}$

⇨ $\begin{cases}106\leq 74+5t\leq 2017\\106\leq 111+8t\leq 2017\end{cases}$ ⇨ $\begin{cases}7\leq t\leq 388\\0\leq t\leq 238\end{cases}$ ⇨ $7\leq t\leq 238$ ⇨ $|t|=232$ ###

7. 單選

解：

(*) $\lim\limits_{n\to\infty}\left(\sum\limits_{k=n}^{2n-1}\frac{3}{\sqrt{nk}}\right)$

(1) $\left(\sum\limits_{k=n}^{2n-1}\frac{3}{\sqrt{nk}}\right)=3[\frac{1}{\sqrt{n^2}}+\frac{1}{\sqrt{n(n+1)}}+\frac{1}{\sqrt{n(n+2)}}+\ldots+\frac{1}{\sqrt{n(2n-1)}}]$

$$=3[\frac{1}{\sqrt{n^2(1+0)}}+\frac{1}{\sqrt{n^2(1+{}^{1}\!/_{n})}}+\frac{1}{\sqrt{n^2(1+{}^{2}\!/_{n})}}+\ldots+\frac{1}{\sqrt{n^2(1+{}^{n-1}\!/_{n})}}]$$

$$=\frac{3}{n}[\frac{1}{\sqrt{(1+0)}}+\frac{1}{\sqrt{(1+{}^{1}\!/_{n})}}+\frac{1}{\sqrt{(1+{}^{2}\!/_{n})}}+\ldots+\frac{1}{\sqrt{(1+{}^{n-1}\!/_{n})}}]$$

(2) $\lim\limits_{n\to\infty}\left(\sum\limits_{k=n}^{2n-1}\frac{3}{\sqrt{nk}}\right)=3\int_0^1\frac{1}{\sqrt{1+x}}dx=6(\sqrt{2}-1)=2.46$ ###

8. 單選

解：(借)

發表於 2017-5-26 12:59 只看該作者

單選8

可先將問題轉換為白球以外的球最後取完的機率

例如:紅球最後取完，黑球倒數第二取完，綠球倒數第三取完，所以白球倒數第四取完（即白球最先取完）

紅球最後取完之機率為 $\frac{2}{12}$
黑球倒數第二取完之機率為 $\frac{2}{10}$
綠球倒數第三取完之機率為 $\frac{2}{8}$

故這種情形發生之機率為 $\frac{2}{12}\times\frac{2}{10}\times\frac{2}{8}=\frac{1}{120}$

而其他顏色的取完順序有3!=6種，故所求為$\frac{1}{20}$

[*本帖最後由 jfy281117 於 2017-5-26 13:14 編輯*]

解：

(*) $\dfrac{12!}{6!2!2!2!}=12\times11\times10\times9\times7$ #

(1) 白色第 6 次先抽完： $1\times\dfrac{6!}{2!2!2!}=90 \Leftarrow$ ○○○○○○

(2) 白色第 7 次先抽完： $3\times(\dfrac{6!}{5!}\times\dfrac{5!}{2!2!})=540 \Leftarrow$ ○○○○○R○

(3) 白色第 8 次先抽完： $3\times(\dfrac{7!}{5!}\times\dfrac{4!}{2!})=1512 \Leftarrow$ ○○○○○RB○

(4) 白色第 9 次先抽完： $\dfrac{8!}{5}\times3!=2016 \Leftarrow$ ○○○○○RBG○

(*) $P=\dfrac{90+540+1512+2016}{12\times11\times10\times9\times7}=\dfrac{4158}{12\times11\times10\times9\times7}=\dfrac{1}{20}$ ###

9. 複選題

解：

(*) $a^{\log_b c}=a+b+c=9$

(1)

a=1 ⇨ **X**

a=2 ⇨ $\log_b c=\log_2 9$ ⇨ **X**

a=3 ⇨ $\log_b c=\log_3 9=2$ ⇨ **b=2, c=4** ⇨ $a^2+bc=17$

a=4 ⇨ $\log_b c=\log_4 9=\log_2 3$ ⇨ **b=2, c=3** ⇨ $a^2+bc=2$

a=5 ⇨ $\log_b c=\log_5 9$ ⇨ **X**

a=6 ⇨ $\log_b c=\log_6 9$ ⇨ **X**

a=7 ⇨ $\log_b c=\log_7 9$ ⇨ **X**

a=8 ⇨ $\log_b c=\log_8 9$ ⇨ **X**

a=9 ⇨ $\log_b c=\log_9 9$ ⇨ **X** ###

10. 複選題

解：

(*)

X	**1**	**2**	**2**	**3**	**2**

Y	3	2	1	a	b

(1) $\mu_X = 2$ 、 $\sigma_X = \sqrt{\dfrac{(1-2)^2+(2-2)^2+(2-2)^2+(3-2)^2+(2-2)^2}{5}} = \sqrt{\dfrac{2}{5}}$ #

(2) $y = \frac{1}{2}x + 3$ ⇨ $\mu_Y = \dfrac{1}{2}\times 2 + 3 = 4$ ⇨ $\mu_Y = 4 = \dfrac{3+2+1+a+b}{5}$ ⇨ $a+b=14$ #

(3) $斜率 = \dfrac{1}{2} = \dfrac{S_{XY}}{S_{XX}} = \dfrac{\sum(x-\mu_X)(y-\mu_Y)}{\sum(x-\mu_X)^2} = \dfrac{\sum(x-\mu_X)(y-\mu_Y)}{2}$

⇨ $S_{XY} = \sum(x-\mu_X)(y-\mu_Y) = 1$ #

(4)

X- μ_X	**-1**	**0**	**0**	**1**	**0**
Y- μ_Y	**-1**	**-2**	**-3**	**a-4**	**b-4**

⇨ $S_{XY} = \sum(x-\mu_X)(y-\mu_Y) = 1 + (a-4) = 1$ ⇨ $a = 4$ ⇨ $b = 10$ ###

(5)

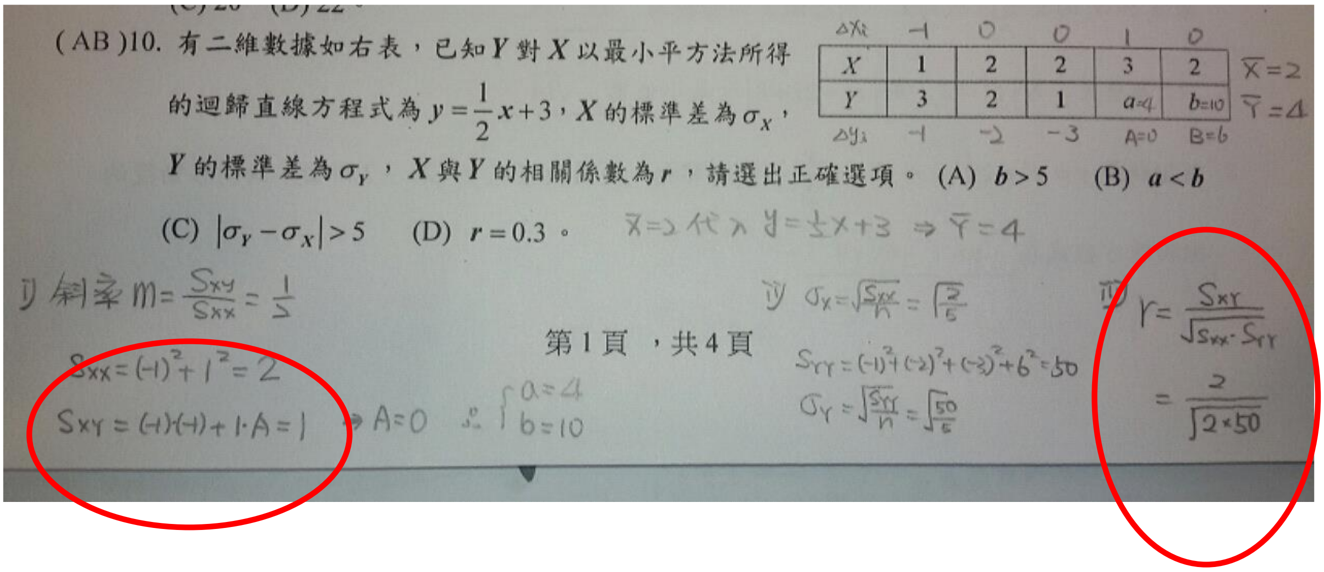
X	**1**	**2**	**2**	**3**	**2**
Y	**3**	**2**	**1**	**4**	**10**

⇨ $\sigma_Y = \sqrt{\dfrac{(3-4)^2+(2-4)^2+(1-4)^2+(4-4)^2+(10-4)^2}{5}} = \sqrt{10}$ #

(6)

(X- μ_X **)/** σ_X	**-1/** $\sqrt{\frac{2}{5}}$	**0/** $\sqrt{\frac{2}{5}}$	**0/** $\sqrt{\frac{2}{5}}$	**1/** $\sqrt{\frac{2}{5}}$	**0/** $\sqrt{\frac{2}{5}}$
(Y- μ_Y **)/** σ_Y	**-1/** $\sqrt{10}$	**-2/** $\sqrt{10}$	**-3/** $\sqrt{10}$	**0/** $\sqrt{10}$	**6/** $\sqrt{10}$

⇨ $r = \dfrac{(\frac{1}{2})+(0)+(0)+(0)+(0)}{5} = 0.1$ ###

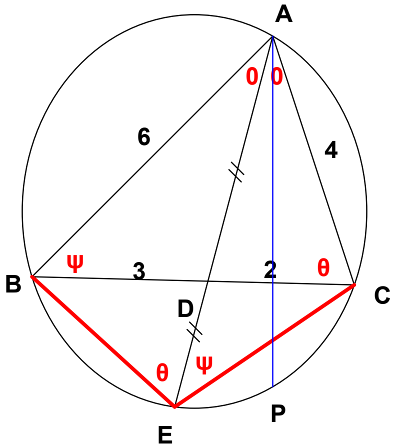


(AB)10. 有二維數據如右表，已知 Y 對 X 以最小平方法所得的迴歸直線方程式為 $y=\frac{1}{2}x+3$，X 的標準差為 σ_X，Y 的標準差為 σ_Y，X 與 Y 的相關係數為 r，請選出正確選項。 (A) $b>5$ (B) $a<b$ (C) $|\sigma_Y-\sigma_X|>5$ (D) $r=0.3$。

X	1	2	2	3	2
Y	3	2	1	a	b

第 1 頁，共 4 頁

11. 複選題



解：

(A) $\frac{6}{\sin\theta}=\frac{4}{\sin\phi}$ ⇨ $\frac{6}{\sin\angle AEB}=\frac{4}{\sin\angle AEC}$ ⇨ $\sin\angle AEB:\sin\angle AEC=3:2$ #

(B) (i) $\Delta ABD\sim\Delta CED$ ⇨ $\frac{\overline{AD}}{2}=\frac{3}{\overline{DE}}$ ⇨ $\overline{AD}\times\overline{DE}=6$ **(1)**

(ii) $\Delta ABD\sim\Delta AEC$ ⇨ $\frac{6}{\overline{AE}}=\frac{\overline{AD}}{4}$ ⇨ $\overline{AD}\times\overline{AE}=24$ **(2)**

由(1)(2)得 $\overline{AD}=3\sqrt{2}$、$\overline{DE}=\sqrt{2}$ #

(D) $\overline{AP}$ **最長就是直徑** ⇨ $\cos A=\frac{6^2+4^2-5^2}{2\times6\times4}=\frac{9}{16}$ ⇨ $\frac{5}{\sin A}=2R=\overline{AP}$

⇨ $\overline{AP}=\dfrac{5}{5\sqrt{7}/16}=\dfrac{16\sqrt{7}}{7}$ #

12. 複選題

解： (借解)

> **回復 49# nanpolend 的帖子**
>
> 第12題
>
> $AB-AC=A(B-C)=\begin{bmatrix}-3 & 1\\ 11 & 3\end{bmatrix}$
>
> $B-C=A^{-1}\begin{bmatrix}-3 & 1\\ 11 & 3\end{bmatrix}$
>
> 把四個選項的反矩陣求出來，再乘一乘，看所有元是否爲整數

###

1. 填充

解：

(*) $f(n)=1^n-2^n+3^n-4^n+\ldots-2016^n+2017^n$

(1) $f(1)=1^1-2^1+3^1-4^1+\ldots-2016^1+2017^1$

$=1^1+(-2^1+3^1)+(-4^1+\ldots+(-2016^1+2017^1)=1009$ #

(2) $f(2)=1^2-2^2+3^2-4^2+\ldots-2016^2+2017^2=1^2+(-2^2+3^2)+(-4^2+\ldots+(-2016^2+2017^2)$

$=1+5+9+\ldots+4033=1009\times 2017$ #

(3) $f(3)=1^3-2^3+3^3-4^3+\ldots-2016^3+2017^3$

$=(1^3+2^3+3^3+4^3+\ldots+2016^3+2017^3)-2(2^3+4^3+\ldots+2016^3)$

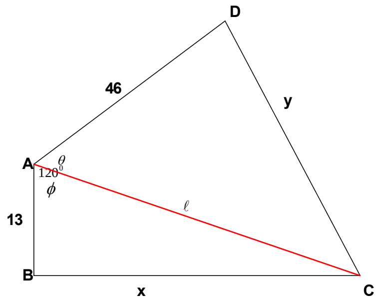
$=(\dfrac{2017\times 2018}{2})^2-2\times 2^3(1^3+2^3+\ldots+1008^3)=2017^2\times 1009^2-2\times 2^3(\dfrac{1008\times 1009}{2})^2$

$=2017^2\times 1009^2-2016^2\times 1009^2=4033\times 1009^2$ #

(4) $\dfrac{f(1)f(2)}{f(3)}=\dfrac{1009\times 1009\times 2017}{4033\times 1009^2}=\dfrac{2017}{4033}$ ###

2. 填充

解：



(1) ⇨ $\begin{cases} \ell^2 = x^2 + 13^2 \\ \ell^2 = y^2 + 46^2 \end{cases}$ ⇨ $\begin{cases} \cos\theta = \dfrac{46}{\ell}, \sin\theta = \dfrac{y}{\ell} \\ \cos\phi = \dfrac{13}{\ell}, \sin\phi = \dfrac{x}{\ell} \end{cases}$

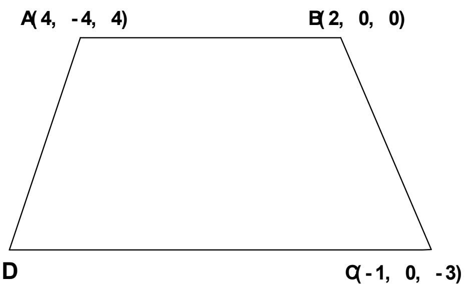
(2) $\cos 120^0 = \cos(\theta+\phi) = \cos\theta\cos\phi - \sin\theta\sin\phi \;= \dfrac{46\times 13}{\ell^2} - \dfrac{xy}{\ell^2}$

(3) $-\dfrac{1}{2} = \dfrac{46\times 13}{\ell^2} - \dfrac{xy}{\ell^2}$ ⇨ $13\times 46 - xy = -\dfrac{1}{2}\ell^2$ ⇨ $13\times 46 - \sqrt{(\ell^2-13^2)(\ell^2-46^2)} = -\dfrac{1}{2}\ell^2$

⇨ $\ell = 62$ ###

3. 填充

解：



(1) $\vec{AB} = (-2, 4, -4)$ **，令** $\vec{CD} = t(-1, 2, -2)$ ⇨ $D = (-t-1, 2t, -2t-3)$

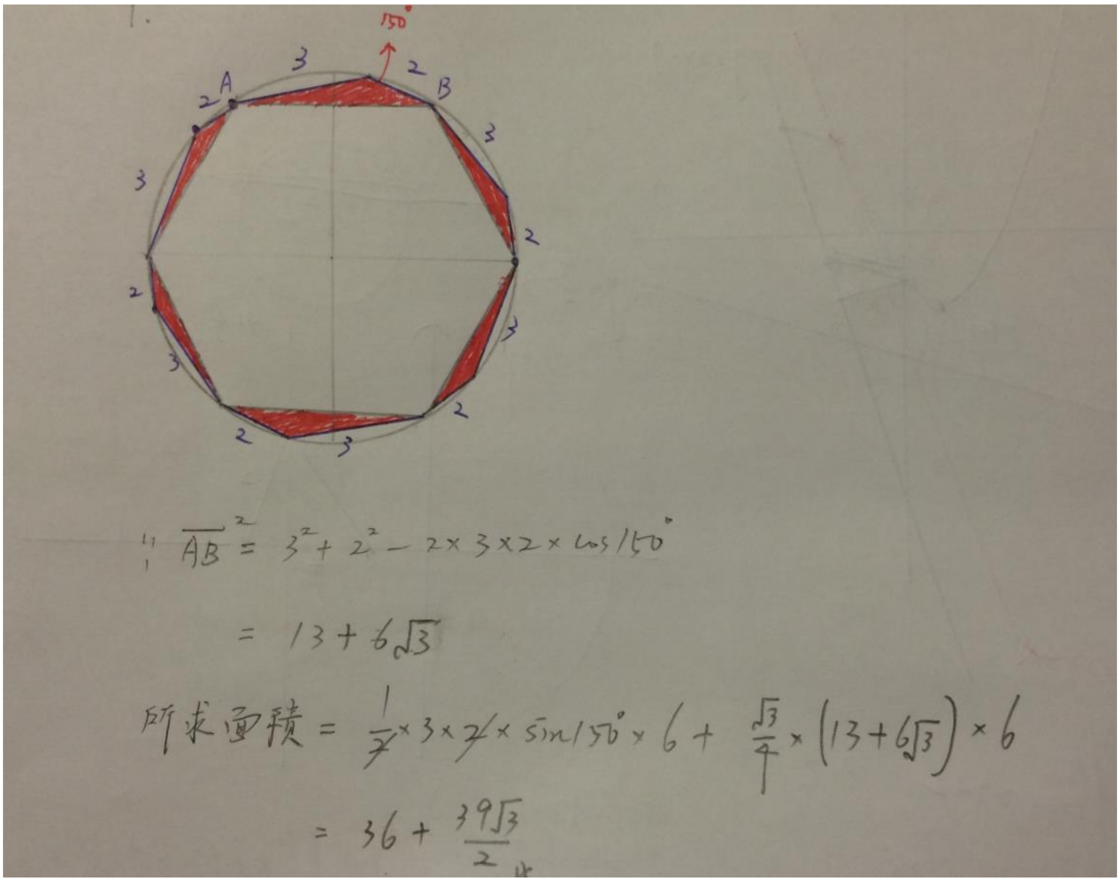
(2) $\overline{BC} = 3\sqrt{2}$ ⇨ $\overline{AD} = \sqrt{(-t-5)^2 + (2t+4)^2 + (-2t-7)^2} = 3\sqrt{2}$ ⇨ $t = -2, t = -4$

(3) **(i)** $t = -2$ ⇨ $\overline{AB} = \overline{CD}$ **平行四邊形**

(ii) $t=-4$ ⇨ $D(3,-8,5)$ ###

4. 填充

解：(借解)



###

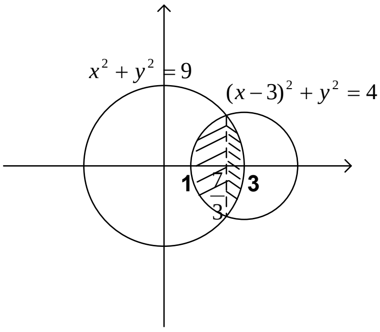
5. 填充

解：

(1) $$\sum_{k=1}^{\infty}\frac{1}{k^3+8k^2+15k}=\sum_{k=1}^{\infty}\frac{1}{k(k+3)(k+5)}=\frac{1}{2}\sum_{k=1}^{\infty}\frac{1}{k}\left(\frac{1}{k+3}-\frac{1}{k+5}\right)$$
$$=\frac{1}{2}\left[\sum_{k=1}^{\infty}\frac{1}{k(k+3)}-\sum_{k=1}^{\infty}\frac{1}{k(k+5)}\right]=\frac{139}{1800}$$ ###

6. 填充

解：



(*) 即如圖兩圓，對 **x** 軸旋轉所得體積。

(1) $V_1=\int_{7/3}^{3}\pi(9-x^2)dx=\pi(9x-\frac{1}{3}x^3)\Big|_{7/3}^{3}=\frac{100}{81}\pi$ #

(2) $V_2=\int_{1}^{7/3}\pi(4-(x-3)^2)dx=\pi(4x-\frac{1}{3}(x-3)^3)\Big|_{1}^{7/3}=\frac{224}{81}\pi$ #

(3) $V=V_1+V_2=4\pi$ ###

7. 填充

解：

(1) $|z^2-2z+8|=|z||z+\frac{8}{z}-2|=2|2(\cos\theta+i\sin\theta)+\frac{8}{2(\cos\theta+i\sin\theta)}-2|$

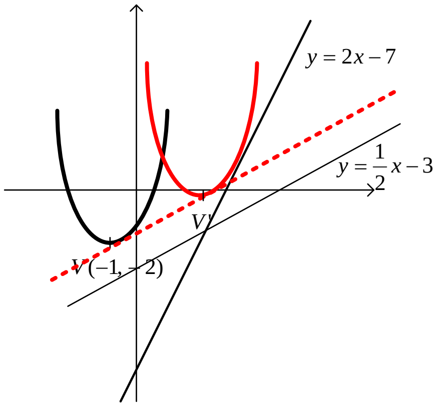
$=2|2(\cos\theta+i\sin\theta)+4(\cos\theta-i\sin\theta)-2|=2|6\cos\theta-2-2i\sin\theta|$

$=2\sqrt{(6\cos\theta-2)^2+4\sin^2\theta}=2\sqrt{32\cos^2\theta-24\cos\theta+8}$

$=2\sqrt{32(\cos\theta-\frac{3}{8})^2+\frac{7}{2}}\geq 2\sqrt{\frac{7}{2}}=\sqrt{14}$ ###

8. 填充

解：



(1) $y=x^2+2x-1=(x+1)^2-2$ ⇨ $V(-1,-2)$ #

(2) $y=\frac{1}{2}x-3$ ⇨ $m=\frac{1}{2}$ ⇨ 令$V'(-1+2t,-2+t)$⇨ $y'=(x+1-2t)^2-2+t$ #

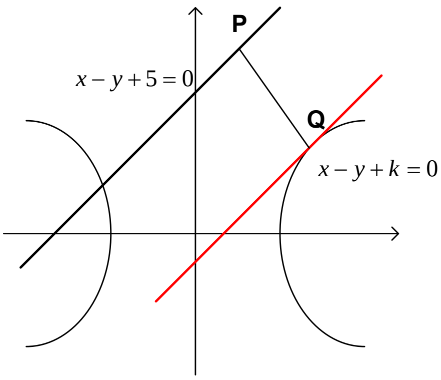
(3) $\begin{cases} y'=(x+1-2t)^2-2+t \\ y=2x-7 \end{cases}$ ⇨ $(x+1-2t)^2-2+t=2x-7$ ⇨ $x^2-4tx+(4t^2-3t+6)=0$

⇨ $b^2-4ac=0$ ⇨ $16t^2-4(4t^2-3t+6)=0$ ⇨ $t=2$ #

(4) $y'=(x+1-2t)^2-2+t$ ⇨ $y'=(x-3)^2$ ###

9. 填充

解：



(*) **P、Q 位置如圖所示。**

(1) 解 $\begin{cases} x-y+k=0 \\ x=\sqrt{\frac{9}{4}y^2+9} \end{cases}$ ⇨ $k=\pm\sqrt{5}$ ⇨ $\begin{cases} x-y+5=0 \\ x-y-\sqrt{5}=0 \end{cases}$ ⇨ $\overline{PQ}=\frac{5+\sqrt{5}}{\sqrt{2}}=\frac{5\sqrt{2}+\sqrt{10}}{2}$ ###

1. 計算

解：

(1) $(\sqrt{2}-1)^5 = C_5^5(\sqrt{2})^5 - C_4^5(\sqrt{2})^4 + C_3^5(\sqrt{2})^3 - C_2^5(\sqrt{2})^2 + C_1^5(\sqrt{2}) - C_0^5$

$= 4\sqrt{2} - 20 + 20\sqrt{2} - 20 + 5\sqrt{2} - 1 = 29\sqrt{2} - 41 = \sqrt{1682} - \sqrt{1681}$ ###

(*) 考慮 $(\sqrt{2}-1)^n = \sqrt{m+1} - \sqrt{m}$

(2) (i) n=1 ⇨ $(\sqrt{2}-1)^1 = \sqrt{2} - \sqrt{1}$ ⇨ $m=1$ **成立**

(ii) n=2 ⇨ $(\sqrt{2}-1)^2 = 3 - 2\sqrt{2} = \sqrt{9} - \sqrt{8}$ ⇨ $m=8$ **成立**

(iii) 設 n=2016 ⇨ $(\sqrt{2}-1)^{2016} = \sqrt{m+1} - \sqrt{m}$ **，存在** $m \in N$ **成立**

當 n=2017 ⇨ $(\sqrt{2}-1)^{2017} = (\sqrt{2}-1)^{2016}(\sqrt{2}-1) = (\sqrt{m+1} - \sqrt{m})(\sqrt{2}-1)$

$= \sqrt{2m+2} - \sqrt{m+1} - \sqrt{2m} + \sqrt{m} \quad = (\sqrt{2m+2} + \sqrt{m}) - (\sqrt{2m} + \sqrt{m+1})$

令 $A = \sqrt{2m+2} + \sqrt{m}$、 $B = \sqrt{2m} + \sqrt{m+1}$

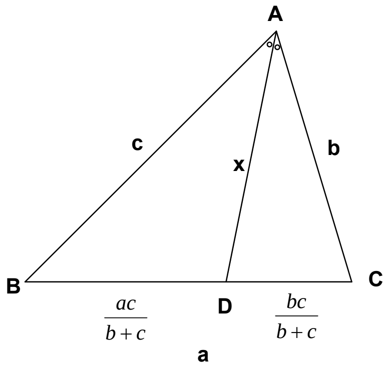
⇨ $A^2 - B^2 = (2m+2+m+2\sqrt{2m^2+2m}) - (2m+2m+1+2\sqrt{2m^2+2m}) = 1$

即存在 $B^2 = m^* \in N$、 $A^2 = m^* + 1 \in N$

使得 $(\sqrt{2}-1)^{2017} = A - B = \sqrt{m^*+1} - \sqrt{m^*}$ **亦成立** ###

2. 計算

解：



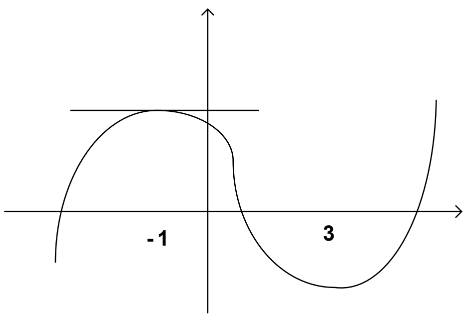
(*)

(1) $\cos B = \dfrac{c^2 + (\frac{ac}{b+c})^2 - x^2}{2c(\frac{ac}{b+c})} = \dfrac{c^2 + a^2 - b^2}{2ca}$

⇨ $x^2 = bc - \dfrac{a^2bc}{(b+c)^2}$ ⇨ $x^2 = bc - (\dfrac{ba}{b+c})(\dfrac{ca}{(b+c})$ ⇨ $\overline{AD} = \sqrt{\overline{AB}\times \overline{AC} - \overline{BD}\times\overline{CD}}$ ###

3. 計算

解：



(*) $f(x) = x^3 - ax^2 + bx + c$

(1) $f'(x) = 3x^2 - 2ax + b$ ⇨ $\Delta = 4a^2 - 12b \ge 0$ ⇨ $a^2 \ge 3b$ ###

(2) $f'(x) = 3x^2 - 2ax + b$ ⇨ $(x+1)(x-3) = 0$ ⇨ $x^2 - 2x - 3 = 0$
⇨ $3x^2 - 6x - 9 = 0$ ⇨ $a = 3, b = -9$

(3) $f(x) = x^3 - ax^2 + bx + c = x^3 - 3x^2 - 9x + c$
(i) $f(-1) = -1 - 3 + 9 + c > 0$ ⇨ $c > -5$
(ii) $f(3) = 27 - 27 - 27 + c < 0$ ⇨ $c < 27$ ⇨ $-5 < c < 27$ ###