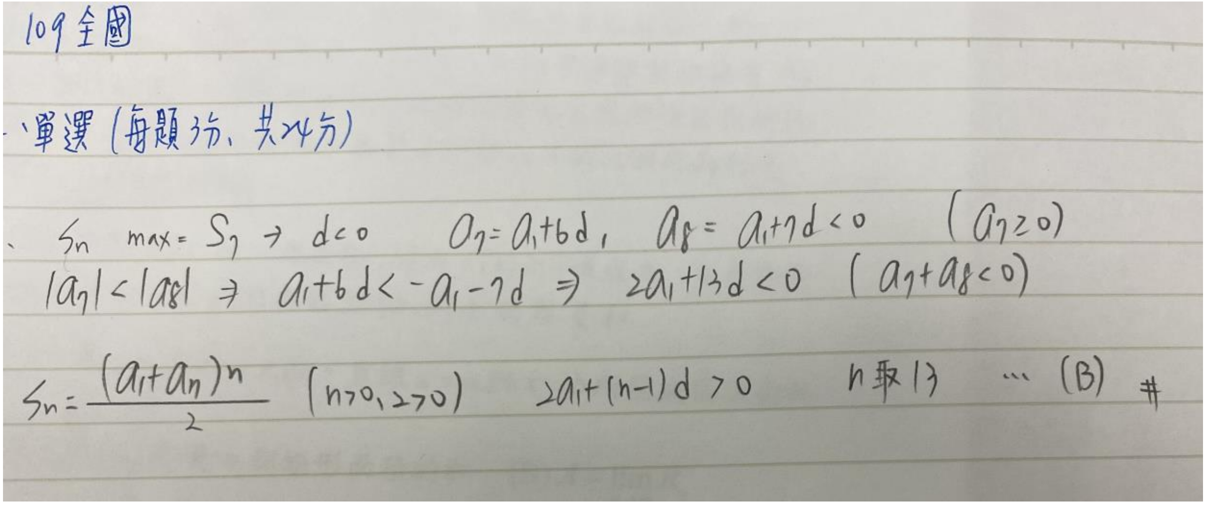
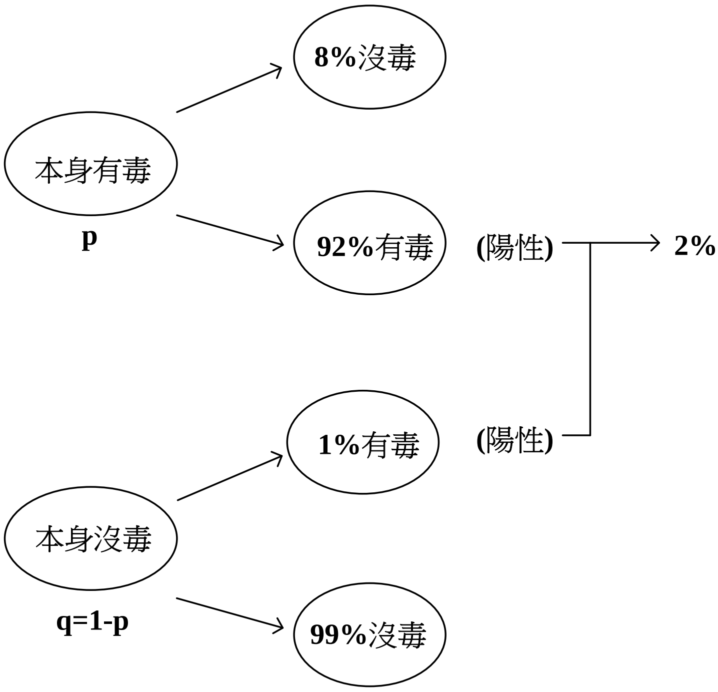
109-全國高中教師聯招 試題解

1. 單選

解：



2. 單選



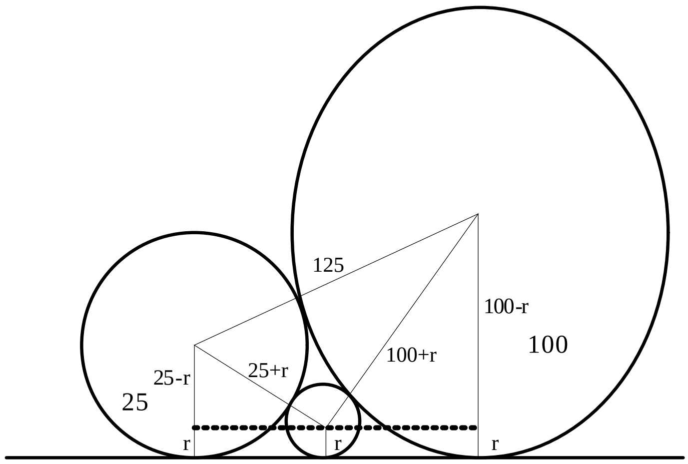
解：

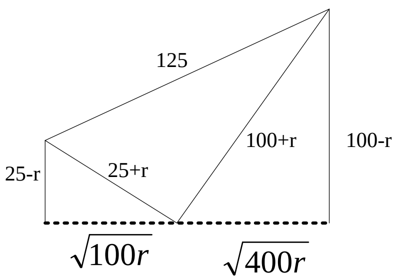
$$⇨ \frac{p\times\frac{92}{100}}{p\times\frac{92}{100}+q\times\frac{1}{100}}=\frac{2}{100} \quad ⇨ \quad p=\frac{2}{9008}=0.02\% \quad \#\#\#$$

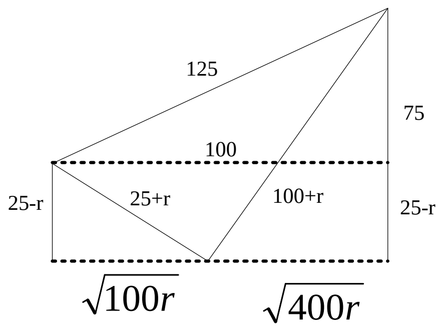
3. 單選

解：利用微積分分割。

4. 單選



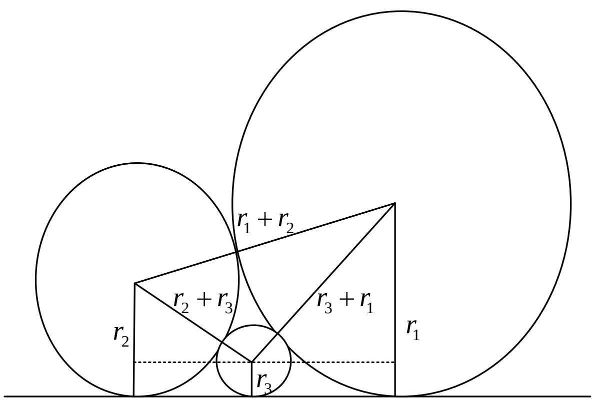




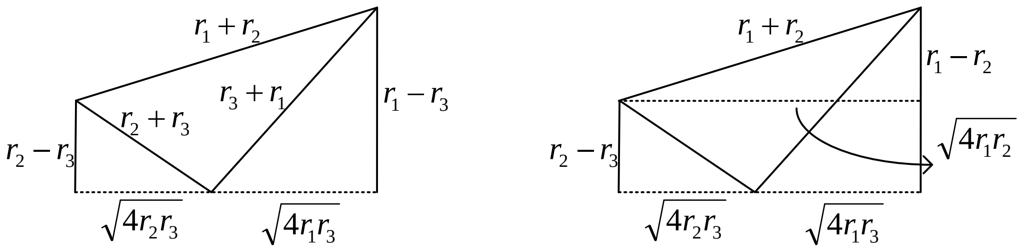
解：

$$\sqrt{100r}+\sqrt{400r}=100 \;\Rightarrow\; r=\frac{100}{9} \quad \#\#\#$$

4. 證明：



解：

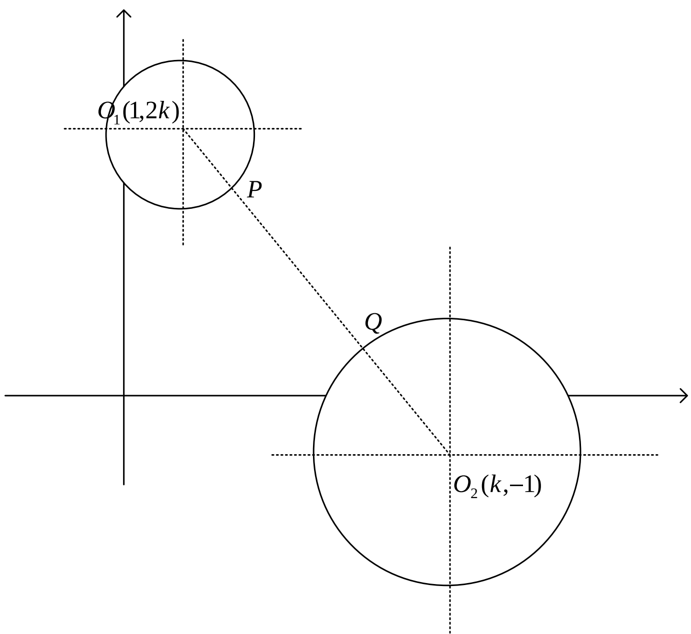


⇨ $\sqrt{4r_1r_2}=\sqrt{4r_2r_3}+\sqrt{4r_1r_3}$

⇨ $\frac{1}{\sqrt{r_3}}=\frac{1}{\sqrt{r_1}}+\frac{1}{\sqrt{r_2}}$

(⇨ $\frac{1}{\sqrt{r_3}}=\frac{1}{\sqrt{25}}+\frac{1}{\sqrt{100}}=\frac{1}{5}+\frac{1}{10}=\frac{3}{10}$ ⇨ $r_3=\frac{100}{9}$ **###)**

5. 單選



解：

(1) $A=\{z\,|\,|z-z_1|\le\sqrt{2}\}$表示在複數平面上，一點到$(1,\,2k)$的距離小於等於$\sqrt{2}$ 。

Ie, 一實圓 (圖 A)

(2) $B=\{z\,|\,|z-z_2|\le 2\sqrt{2}\}$表示在複數平面上，一點到$(k,\,-1)$的距離小於等於$2\sqrt{2}$ 。

Ie, 一實圓 (圖 B)

(3) $A\cap B=\phi$ ⇨ $\overline{O_1O_2}>\overline{O_1P}+\overline{O_2Q}$ ⇨ $\sqrt{(k-1)^2+(2k+1)^2}>3\sqrt{2}$

⇨ $5k^2+2k-16>0$ ⇨ $(5k-8)(k+2)>0$ ⇨ $k<-2\quad or\quad k>\frac{8}{5}$ ###

6. 單選

解：

(1) $\lim_{x \to 0} \frac{h(x)-h(0)}{x} = h'(0)$

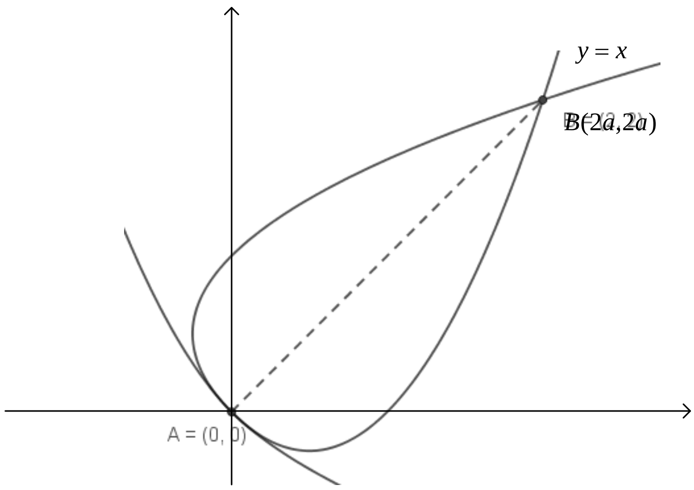
(2) $h(f(x)) = g(x+1)$ ⇨ $h'(f(x))f'(x) = g'(x+1)$ ⇨ $h'(f(-\frac{1}{3}))f'(-\frac{1}{3}) = g'(-\frac{1}{3}+1)$

⇨ $h'(0)*3 = 4$ ⇨ $h'(0) = \frac{4}{3}$ ###

7. 單選

解：

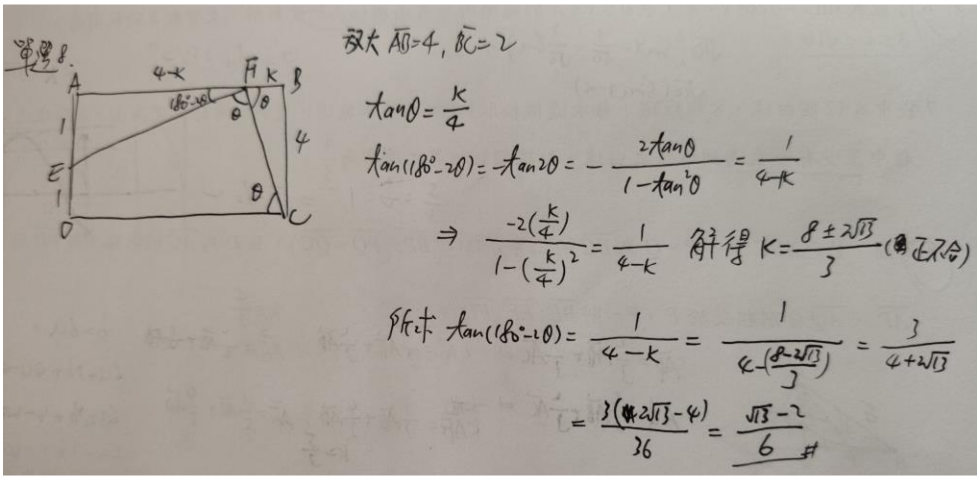
(1) $A = 2\int_0^{2a}[x - \frac{x}{a}(x-a)]dx = \frac{8}{3}a^2$ ###



8. 單選

解：

(1) $\tan\theta = \frac{k}{4}$**(錯誤，邊也錯)** ⇨ $\tan\theta = \frac{2}{k}$ ###



9. 複選題

解：

(1) $A=\begin{bmatrix}0 & -1\\ -1 & 0\end{bmatrix}$ ⇨ $A^2=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=I$ ⇨ $A^{91}=A^{71}$ (√)

(2) $B=\begin{bmatrix}\frac{1}{2} & -\frac{\sqrt{3}}{2}\\ \frac{\sqrt{3}}{2} & \frac{1}{2}\end{bmatrix}=\begin{bmatrix}\cos 60^0 & -\sin 60^0\\ \sin 60^0 & \cos 60^0\end{bmatrix}$ ⇨ $B^2=\begin{bmatrix}\cos 120^0 & -\sin 120^0\\ \sin 120^0 & \cos 120^0\end{bmatrix}$ ⇨ $B^6=I$

⇨ $B^{91}=B$, $B^{71}=B^5=\begin{bmatrix}\cos 300^0 & -\sin 300^0\\ \sin 300^0 & \cos 300^0\end{bmatrix}$ ⇨ $B^{91}=B^{71}$ (×)

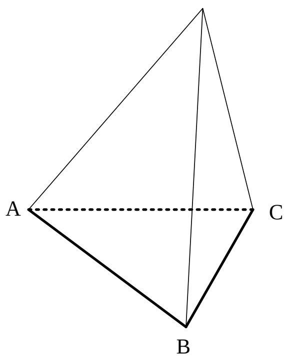
(3) $C=\begin{bmatrix}\frac{1}{2} & \frac{\sqrt{3}}{2}\\ \frac{\sqrt{3}}{2} & -\frac{1}{2}\end{bmatrix}$ ⇨ $C^2=I$ ⇨ $C^{91}=C^{71}$ (√)

(4) $D=\begin{bmatrix}-1 & 1\\ -1 & -1\end{bmatrix}$ ⇨ $D^2=\begin{bmatrix}-1 & 1\\ -1 & -1\end{bmatrix}\begin{bmatrix}-1 & 1\\ -1 & -1\end{bmatrix}=\begin{bmatrix}0 & -2\\ 2 & 0\end{bmatrix}$ ⇨ $D^3=\begin{bmatrix}2 & 2\\ -2 & 2\end{bmatrix}$

⇨ $B^5D^3=D^3B^5$ (√)

10. 複選題

解：



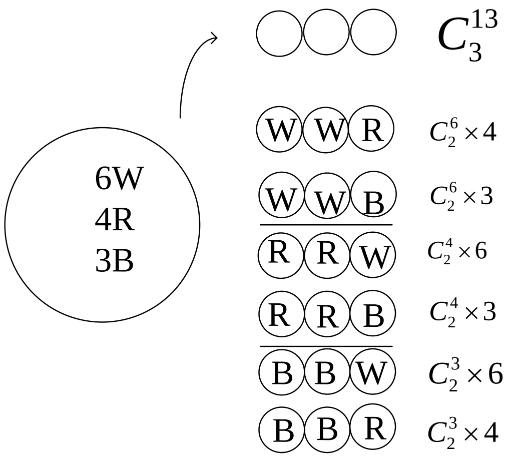
(A) ⇨ 底 **ABC** 固定 ⇨ $\frac{7\times 6\times 5}{3}=70$

x / y	72	68	64	·······	4	1
|z|	0	1	2	·······	17	18

(B)

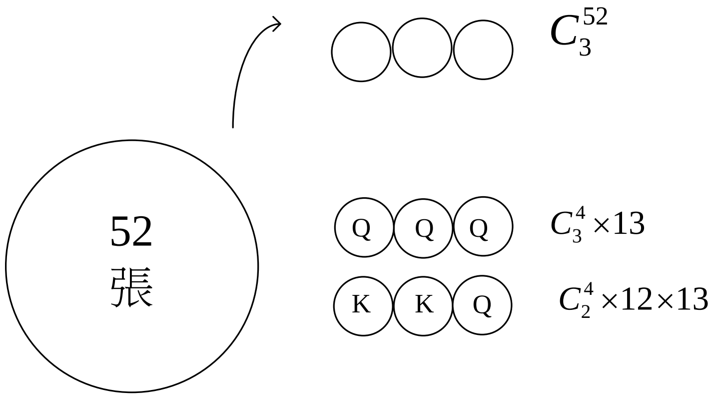
⇨ $72+(68+64+...+4)\times 2+2=1298$ (√)

(C)



⇨ $p=\dfrac{189}{286}$ (√)

(D)



⇨ $p=\dfrac{C_3^4\times13+C_2^4\times12\times13}{C_3^{52}}=\dfrac{988}{22100}$

11. 複選題

解：

(1) $(1+x-x^2)^{50}=1+ax+bx^2+...+cx^{100}$

(2) $(1+x-x^2)^{50}$ ⇨ $(1+x)^{50}$ ⇨ $ax=C_1^{50}(1)^{49}(x)^1=50x$ (√)

(3) $(1+x-x^2)^{50}$ ⇨ $[(1+x)+(-x^2)]^{50}$

$$=C_0^{50}(1+x)^{50}(-x^2)^0+C_1^{50}(1+x)^{49}(-x^2)^1+C_2^{50}(1+x)^{48}(-x^2)^2+...$$

⇨ $bx^2=C_0^{50}C_2^{50}x^2+C_1^{50}(1)^{49}(-x^2)^1=1175$ (√)

(4) $c=1$ ⇨ $a+b+c=50+1175+1=1226$ (√)

12. 複選題

Fre	k=1	k=2	k=3	k=4
Mon	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$
Pro	p	p	p	p

解：

(1) $a = E = \frac{1}{2}p + \frac{1}{4}p + \frac{1}{8}p + \frac{1}{16}p = \frac{15}{16}p$ ###

(2) $b = P(Mon > \frac{1}{3}) = P(k=1) + P(k \neq 1, k=2, k=3)$

$$= p + qp^2 = p + (1-p)p^2 \quad = -p^3 + p^2 + p$$ ###

1. 填充

解：

(1) 解 $(3^x - 9)^3 - (3 - 9^x)^3 = (3^x + 9^x - 12)^3$，

令 $A = 3^x$ ⇨ $(A-9)^3 - (3-A^2)^3 = (A + A^2 - 12)^3$

⇨ $(A-9)^3 + (A^2-3)^3 = (A + A^2 - 12)^3$

⇨ $[(A-9)+(A^2-3)][(A-9)^2 - (A-9)(A^2-3) + (A^2-3)^2] = (A + A^2 - 12)^3$

⇨ $(A^2 + A - 12)[(A-9)^2 - (A-9)(A^2-3) + (A^2-3)^2] = (A + A^2 - 12)^3$

⇨ $(A^2 + A - 12) = 0$ & $(A-9)^2 - (A-9)(A^2-3) + (A^2-3)^2 = (A + A^2 - 12)^3$

⇨ $(A^2 + A - 12) = 0$ & $(A-9)^2 - (A-9)(A^2-3) + (A^2-3)^2 = [(A-9) + (A^2-3)]^2$

⇨ $(A^2 + A - 12) = 0$ &

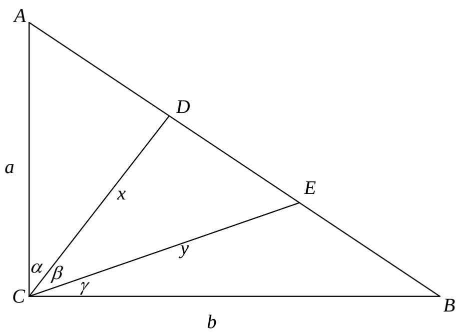
$$(A-9)^2 - (A-9)(A^2-3) + (A^2-3)^2 = (A-9)^2 + 2(A-9)(A^2-3) + (A^2-3)^2$$

⇨ $(A+4)(A-3) = 0$ & $3(A-9)(A^2-3) = 0$

⇨ $x = 1, 2, \frac{1}{2}$ ###

2. 填充

解：



(1) $\Delta ABC=\frac{1}{2}ab$ ，$\Delta ACD=\Delta CDE=\Delta CBE=\frac{1}{3}\Delta ABC$ (等底同高)

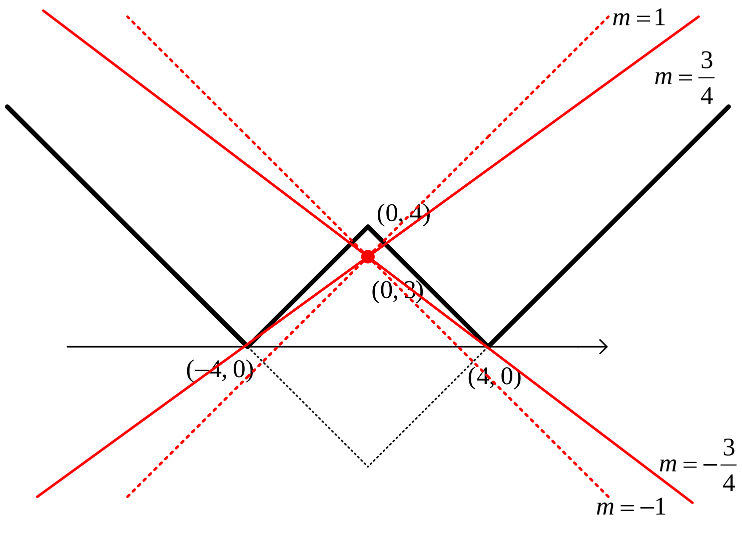
(2) $\Delta ACD=\frac{1}{2}ax\sin\alpha$，$\Delta CDE=\frac{1}{2}xy\sin\beta$，$\Delta CBE=\frac{1}{2}by\sin\gamma$

(3) $\frac{\frac{1}{2}ax\sin\alpha\cdot\frac{1}{2}by\sin\gamma}{\frac{1}{2}xy\sin\beta}=\frac{\frac{1}{3}\Delta\cdot\frac{1}{3}\Delta}{\frac{1}{3}\Delta}$ ⇨ $\frac{\frac{1}{4}abxy\sin\alpha\cdot\sin\gamma}{\frac{1}{2}xy\sin\beta}=\frac{1}{3}\Delta$

⇨ $\frac{\frac{1}{2}ab\sin\alpha\cdot\sin\gamma}{\sin\beta}=\frac{1}{3}(\frac{1}{2}ab)$ ⇨ $\frac{\sin\alpha\cdot\sin\gamma}{\sin\beta}=\frac{1}{3}$ ###

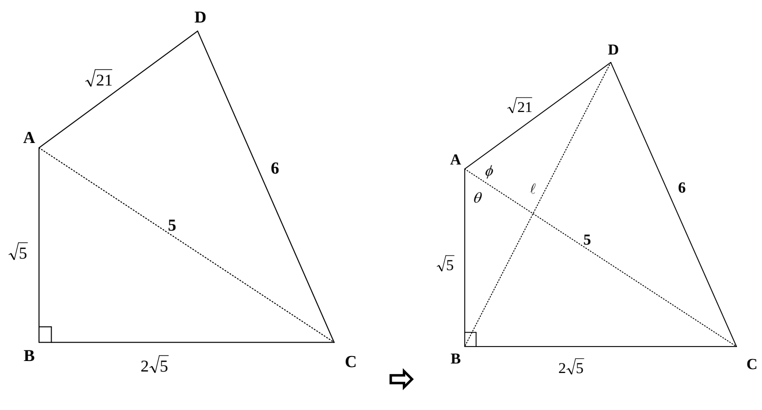
3. 填充

解：$-1<m<-\frac{3}{4}$ or $\frac{3}{4}<m<1$ ###



4. 填充

解：



(1) $\cos\theta = \frac{\sqrt{5}}{5} = \frac{1}{\sqrt{5}}$

(2) $\cos\phi = \frac{5^2 + (\sqrt{21})^2 - 6^2}{2\cdot 5\cdot\sqrt{21}} = \frac{25+21-36}{10\sqrt{21}} = \frac{10}{10\sqrt{21}} = \frac{1}{\sqrt{21}}$

(3) $\cos(\theta+\phi) = \cos\theta\cos\phi - \sin\theta\sin\phi = \frac{1}{\sqrt{5}}\times\frac{1}{\sqrt{21}} - \frac{2}{\sqrt{5}}\times\frac{\sqrt{20}}{\sqrt{21}} = \frac{1-4\sqrt{5}}{\sqrt{105}} = \frac{(\sqrt{5})^2 + (\sqrt{21})^2 - \ell^2}{2\cdot\sqrt{5}\cdot\sqrt{21}}$

⇨ $\ell^2 = 24 + 8\sqrt{5}$ ⇨ $\ell = \sqrt{24+8\sqrt{5}} = 2 + 2\sqrt{5}$ ###

5. 填充

解：

(1) $a = \sum_{n=1}^{\infty}(\frac{1}{2})^n \cos\frac{n\pi}{3}$

$=[(\frac{1}{2})^1(\frac{1}{2}) + (\frac{1}{2})^2(-\frac{1}{2}) + (\frac{1}{2})^3(-1) + (\frac{1}{2})^4(-\frac{1}{2}) + (\frac{1}{2})^5(\frac{1}{2}) + (\frac{1}{2})^6(1)] + [\sim\sim\sim] = 0$ ###

(2) $b = \sum_{n=1}^{\infty}(\frac{1}{2})^n \sin\frac{n\pi}{3}$

$=[(\frac{1}{2})^1(\frac{\sqrt{3}}{2}) + (\frac{1}{2})^2(\frac{\sqrt{3}}{2}) + (\frac{1}{2})^3(0) + (\frac{1}{2})^4(-\frac{\sqrt{3}}{2}) + (\frac{1}{2})^5(-\frac{\sqrt{3}}{2}) + (\frac{1}{2})^6(0)] + [\sim\sim\sim]$

$= \frac{21\sqrt{3}}{64} + \ldots\ldots = \frac{\frac{21\sqrt{3}}{64}}{1-\frac{1}{64}} = \frac{\sqrt{3}}{3}$ ###

6. 填充

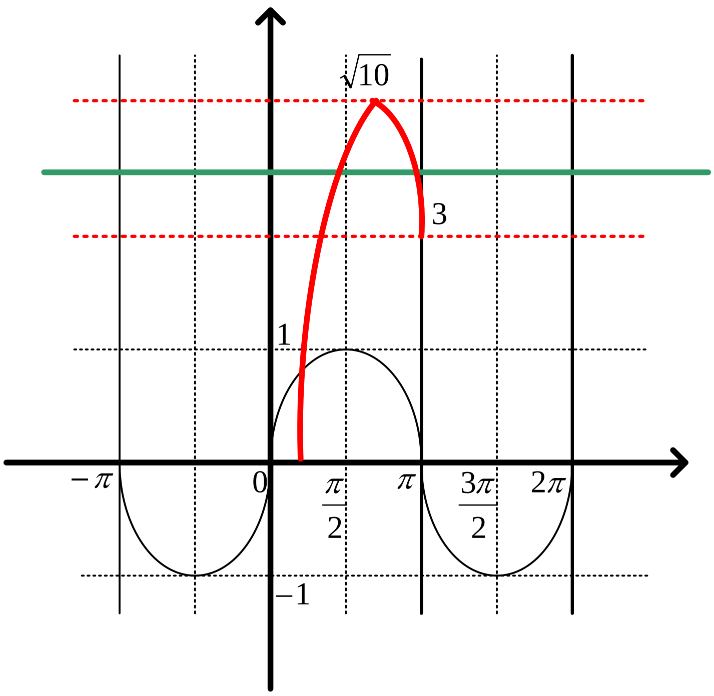
解：

(1) $y = \sin x - 3\cos x = \sqrt{10}(\frac{1}{\sqrt{10}}\sin x - \frac{3}{\sqrt{10}}\cos x)$

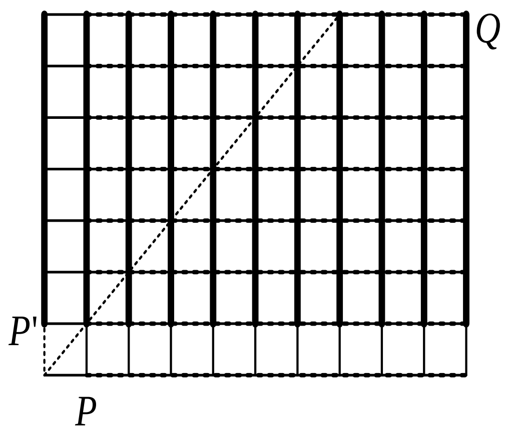
$= \sqrt{10}(\cos\theta\sin x - \sin\theta\cos x)$ (that $\cos\theta = \frac{1}{\sqrt{10}}, \sin\theta = \frac{3}{\sqrt{10}}$)

$= \sqrt{10}\sin(x-\theta)$

(1) ⇨ $3 \le k < \sqrt{10}$ ###



7. 填充



⇨

解：

(1) 先一步(**W**) → P；P → Q

(2) P → Q 不可碰到虛線 = (P → Q) − (P 碰到線)

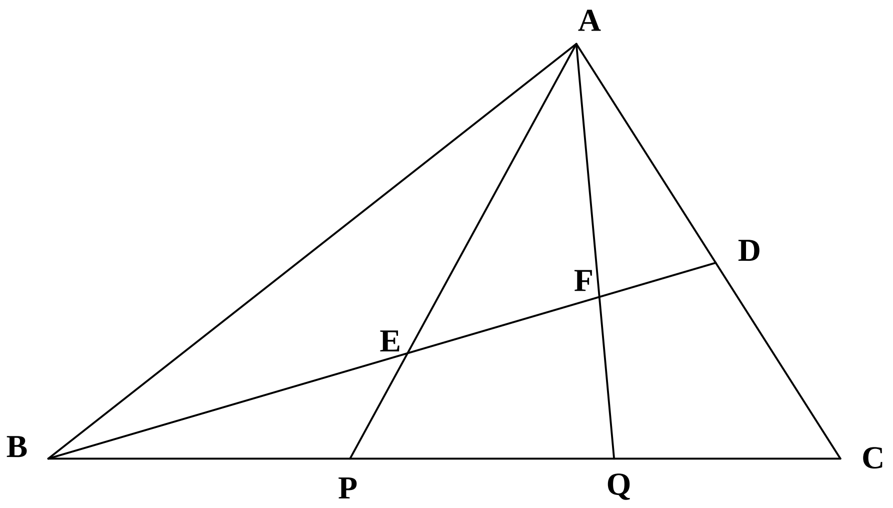
(3) (P 碰到線) = (P' 碰到線) = $\frac{19!}{12!\,7!}$

(4) (P → Q) − (P 碰到線) = $\frac{19!}{11!\,8!}-\frac{19!}{12!\,7!}$

(5) 所求機率 = $1-(\frac{19!}{11!\,8!}-\frac{19!}{12!\,7!})/\frac{20!}{12!\,8!}=\frac{4}{5}$ ###

8. 填充

解：



(1) $\overline{AP}=\frac{2}{3}\overline{AB}+\frac{1}{3}\overline{AC}$

Let $\overline{AE}=t\overline{AP}$ ⇨ $\overline{AE}=\frac{2t}{3}\overline{AB}+\frac{t}{3}\overline{AC}$ ⇨ $\overline{AE}=\frac{2t}{3}\overline{AB}+\frac{2t}{3}\overline{AD}$ ⇨ $\frac{2t}{3}+\frac{2t}{3}=1$ ⇨ $t=\frac{3}{4}$

⇨ $\overline{AE}=\frac{1}{2}\overline{AB}+\frac{1}{2}\overline{AD}$ ⇨ $\overline{BE}:\overline{DE}=1:1$

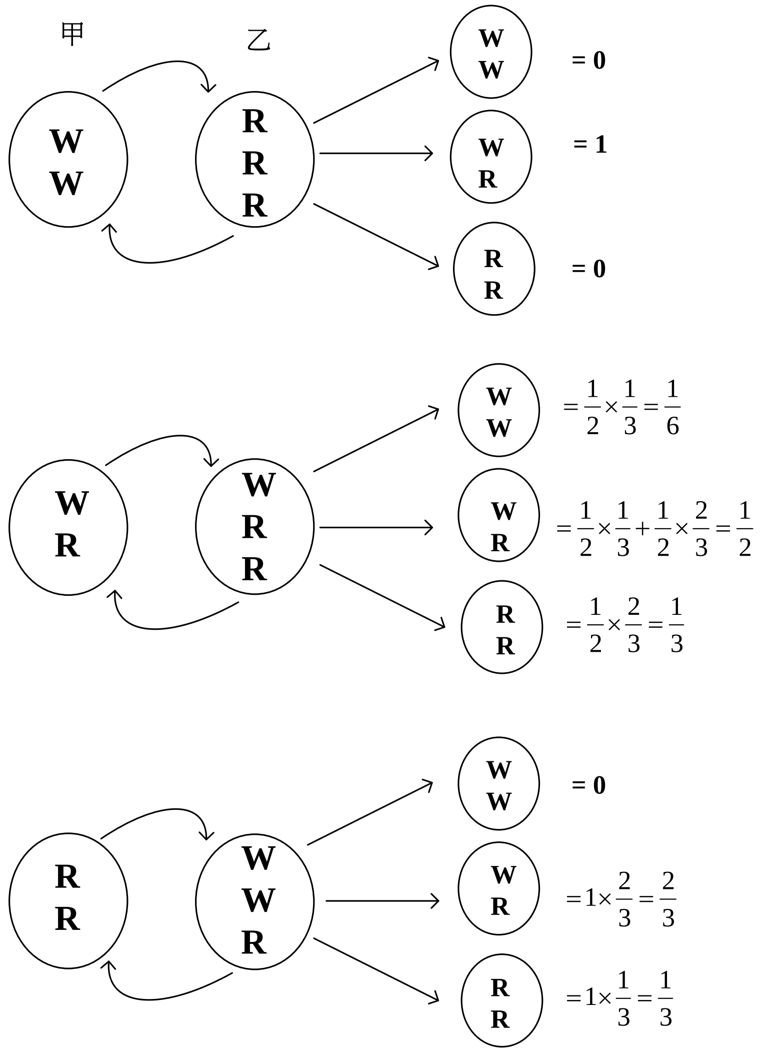
(2) $\overline{AQ}=\frac{1}{3}\overline{AB}+\frac{2}{3}\overline{AC}$

Let $\overline{AF}=t\overline{AQ}$ ⇨ $\overline{AF}=\frac{t}{3}\overline{AB}+\frac{2t}{3}\overline{AC}$ ⇨ $\overline{AF}=\frac{t}{3}\overline{AB}+\frac{4t}{3}\overline{AD}$ ⇨ $\frac{t}{3}+\frac{4t}{3}=1$ ⇨ $t=\frac{3}{5}$

⇨ $\overline{AF}=\frac{1}{5}\overline{AB}+\frac{4}{5}\overline{AD}$ ⇨ $\overline{AB}:\overline{AD}=4:1$

(3) ⇨ $\overline{BE}:\overline{EF}:\overline{DF}=5:3:2$ ###

9. 填充



解：

(2) $P(WW) = P(WW)\times 0 + P(WR)\times\frac{1}{6} + P(RR)\times 0$

(3) $P(WR) = P(WW)\times 1 + P(WR)\times\frac{1}{2} + P(RR)\times\frac{2}{3}$

(4) $P(RR) = P(WW)\times 0 + P(WR)\times\frac{1}{3} + P(RR)\times\frac{1}{3}$

(5) $\Rightarrow \begin{bmatrix} P(WW) \\ P(WR) \\ P(RR) \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{6} & 0 \\ 1 & \frac{1}{2} & \frac{2}{3} \\ 0 & \frac{1}{3} & \frac{1}{3} \end{bmatrix} \begin{bmatrix} P(WW) \\ P(WR) \\ P(RR) \end{bmatrix} \Rightarrow X_1 = AX_0 \Rightarrow X_{n+1} = AX_n$

(6) If stabilie ⇨ $X_n = AX_n$ ⇨ $\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{6} & 0 \\ 1 & \frac{1}{2} & \frac{2}{3} \\ 0 & \frac{1}{3} & \frac{1}{3} \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix}$ (and $a+b+c=1$)

(7) ⇨ $\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0.1 \\ 0.6 \\ 0.4 \end{bmatrix}$ ###

1. 計算

解：

(6) 令 $X = \frac{\alpha - 1}{2\alpha + 3}$ ⇨ $2\alpha X + 3X = \alpha - 1$ ⇨ $\alpha(2X - 1) = -1 - 3X$ ⇨ $\alpha = \frac{1+3X}{1-2X}$ 代入方程式

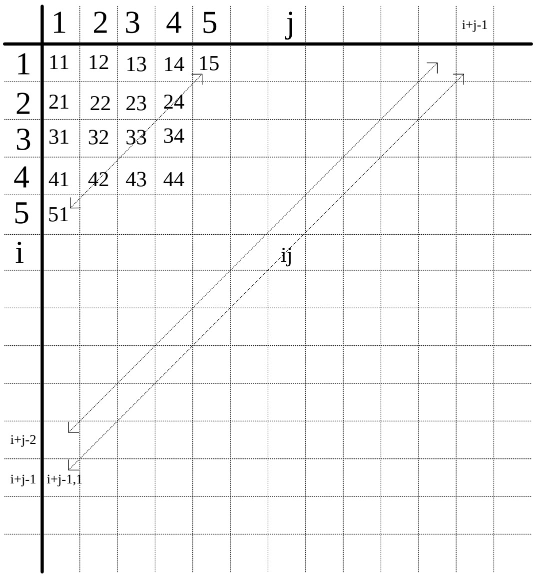
(7) 方程式 $x^3 - 2x^2 - 3x + 1 = 0$ ⇨ $(\frac{1+3X}{1-2X})^3 - 2(\frac{1+3X}{1-2X})^2 - 3(\frac{1+3X}{1-2X}) + 1 = 0$

⇨ $(1+3X)^3 - 2(1+3X)^2(1-2X) - 3(1+3X)(1-2X)^2 + (1-2X)^3 = 0$

⇨ $(27X^3 + 27X^2 + 9X + 1) - 2(-18X^3 - 3X^2 + 4X + 1)$
$- 3(12X^3 - 8X^2 - X + 1) + (1 - 6X + 12X^2 - 8X^3) = 0$

⇨ $19X^3 + 69X^2 - 2X - 3 = 0$ ###

2. 計算



解：

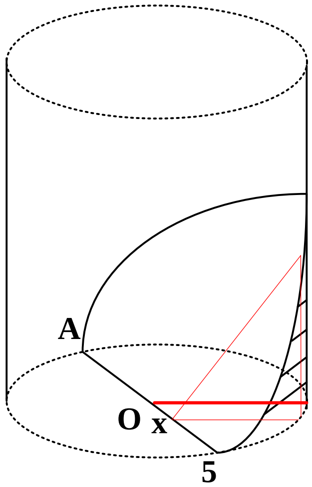
(8) (1→i+j-2)位置有 $1+2+3+\ldots+(i+j-2)=\frac{(i+j-2)(1+i+j-2)}{2}=\frac{(i+j-2)(i+j-1)}{2}$ 個

(9) (i+j-1)位置有 $(i+j-1)-j+1=i$ 個

(10) Total： $\frac{(i+j-2)(i+j-1)}{2}+i$ 個

(11) $a_{ij}=2[\frac{(i+j-2)(i+j-1)}{2}+i]$ = $2(i+j-2)(i+j-1)+2i$ ###

3. 計算



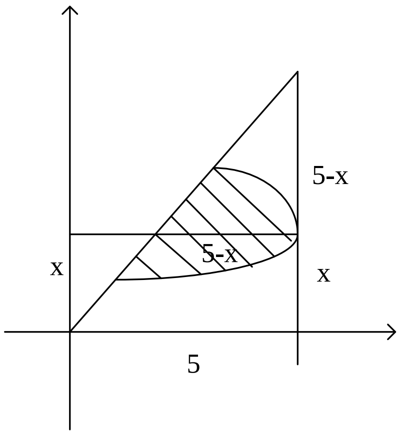
⇨

解：

(12) $x^2+y^2=25$ ⇨ $y=\sqrt{25-x^2}$ ⇨直角三角形面積= $\frac{1}{2}\sqrt{25-x^2}\times\sqrt{25-x^2}=\frac{1}{2}(25-x^2)$

(13) 所求體積 = $2\int_0^5\frac{1}{2}(25-x^2)dx=\int_0^5(25-x^2)dx=\frac{250}{3}$ ###

3. 計算



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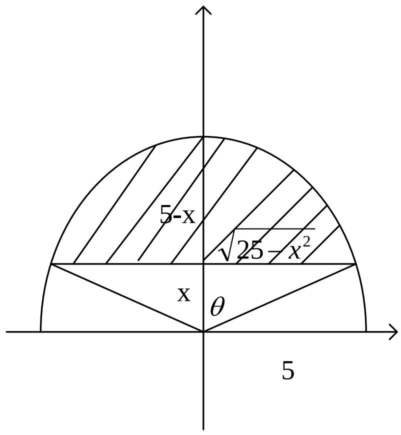

⇨

解：

(14) 弓形面積 ＝ 扇形 － 三角形 ＝ $2(\frac{1}{2}r^2\theta-\frac{1}{2}x\sqrt{25-x^2})$

(15) 所求體積 ＝ $\int_0^5 2(\frac{1}{2}r^2\theta-\frac{1}{2}x\sqrt{25-x^2})dx=\int_0^5 (25\theta-x\sqrt{25-x^2})dx$ ＝ **(2.1) － (2.2)**

(2.1) $\int_0^5 25\theta\, dx = \int 25\theta(-5\sin\theta)d\theta = \int(-125\theta\sin\theta)d\theta = -125\int(\theta\sin\theta)d\theta$ ### ;

$[\cos\theta=\frac{x}{5}\Rightarrow x=5\cos\theta\Rightarrow dx=-5\sin\theta d\theta]$

(2.1.1) $\int(\theta\sin\theta)d\theta = -\theta\cos\theta+\int\cos\theta d\theta = -\theta\cos\theta+\sin\theta$ ### ;

$\left[\begin{matrix} v=\theta, & du=\sin\theta d\theta\Rightarrow \\ dv=d\theta, & u=-\cos\theta \end{matrix}\right]$

(2.1.2) $-\theta\cos\theta+\sin\theta = -\cos^{-1}\frac{x}{5}\cdot\frac{x}{5}+\frac{\sqrt{25-x^2}}{5}\Big|_0^5 = -1$ ###

$[\cos\theta=\frac{x}{5}]$

(2.2) $\int_0^5 x\sqrt{25-x^2}dx = \int 5\cos\theta(5\sin\theta)(-5\sin\theta)d\theta = \int(-125\cos\theta\sin^2\theta)d\theta$

$= -125\int(\cos\theta\sin^2\theta)d\theta$ ### ;

(2.2.1) $\int(\cos\theta\sin^2\theta)d\theta = \frac{1}{3}\sin^3\theta = \frac{1}{3}(\frac{\sqrt{25-x^2}}{5})^3\Big|_0^5 = -\frac{1}{3}$ ###;

(2.3) 所求體積 ＝ **(2.1) － (2.2)** ＝ $(-125)(-1)-(-125)(-\frac{1}{3})]$ ＝ $\frac{250}{3}$ ###