

臺北市 109 學年度市立國民中學正式教師聯合甄選

數學（雙語）科題本

請不要翻到次頁！

讀完本頁的說明，聽從監試委員的指示才開始作答

※請先確認你的答案卡、准考證與座位號碼是否一致無誤。

請閱讀以下測驗作答說明

測驗說明：

這是臺北市 109 學年度市立國民中學正式教師聯合甄選數學（雙語）科題本，題本採雙面印刷，共 90 題，每題只有一個正確或最佳的答案。測驗時間共 100 分鐘，作答開始與結束請聽從監試委員的指示。

注意事項：

1. 所有試題均為四選一的選擇題，答錯不倒扣。
2. 依試場規則規定，答案卡上不得書寫姓名及任何標記。故意污損答案卡、損壞試題本，或在答案卡上顯示自己身份者，該科測驗不予計分。

作答方式：

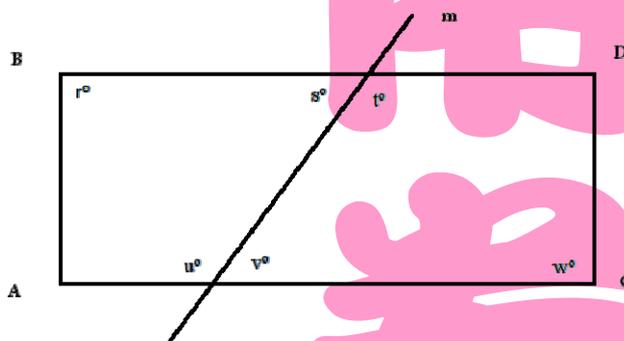
請依照題意從四個選項中選出一個正確或最佳的答案，並用 2B 鉛筆在答案卡上相應的位置劃記，請務必將選項塗黑、塗滿。如果需要修改答案，請使用橡皮擦擦拭乾淨，重新塗黑答案。

請聽到鈴（鐘）聲響後再翻頁作答

貳、數學(雙語)

選擇題 (共 40 題, 每題 1.5 分, 共 60 分)

51. Suppose $x^2 + 2$ is a factor of $x^4 + 2x^3 + \alpha x^2 + \beta x + 2$. What is the value of $\alpha + \beta$?
- (A) 5
(B) 6
(C) 7
(D) 8
52. If x is a positive even integer and y is a negative odd integer, then which of the following must be a positive odd integer?
- (A) $x^3 y^2$
(B) $(xy + 2)^2$
(C) $xy^2 - 1$
(D) $x + y$
53. Over a year, Brendan sold an average of 12 umbrellas per day on rainy days, and an average of 3 umbrellas per day on clear days. If the weather was rainy one day in five, and this was not a leap year, what was Brendan's average daily umbrella sales for the year?
- (A) 4.8
(B) 5.2
(C) 6.75
(D) 7.3
54. If line m intersects rectangle $ABCD$ as shown, then which of the following is equal to t ?



- (A) w
(B) $r + s$
(C) $w - v$
(D) $r + w - s$

55. In an experiment, it is found that the probability that a released bee will land on a painted target is $\frac{2}{5}$. It is also found that when a bee lands on the target, the probability that the bee will attempt to sting the target is $\frac{1}{3}$. In this experiment, what is the probability that a released bee will land on the target and attempt to sting it?

(A) $\frac{2}{15}$

(B) $\frac{1}{5}$

(C) $\frac{2}{5}$

(D) $\frac{1}{3}$

56. $\lim_{x \rightarrow 3} \frac{x^2 + x - 12}{2x^2 - 6x} =$

(A) 1.17

(B) 2.25

(C) 3.33

(D) The limit does not exist.

57. The use of a neighborhood car wash costs n dollars for a membership and p cents for each wash. If a membership includes a bonus 4 free washes, which of the following reflects the cost in dollars of getting a membership at the car wash and washing a car q times, if q is greater than 4?

(A) $100n + pq - 4p$

(B) $n + 100pq - 25p$

(C) $n + pq - \frac{p}{25}$

(D) $n + \frac{pq}{100} - \frac{p}{25}$

58. Jules can make m muffins in s minutes. Alice can make n muffins in t minutes. Which of the following gives the number of muffins that Jules and Alice can make together in 30 minutes?

(A) $\frac{m+n}{30st}$

(B) $\frac{30(m+n)}{st}$

(C) $30(mt + ns)$

(D) $\frac{30(mt + ns)}{st}$

59. Which of the following expresses the possible values of s , if s is the perimeter of an equilateral triangle with an area of 12?

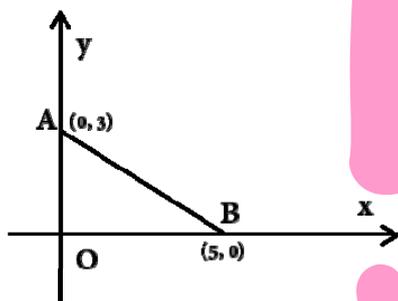
(A) $14 < s < 15$

(B) $15 < s < 16$

(C) $16 < s < 17$

(D) $17 < s < 18$

60. If the triangle created by OAB is rotated around the x-axis, what is the volume of the generated solid?



(A) 15.70

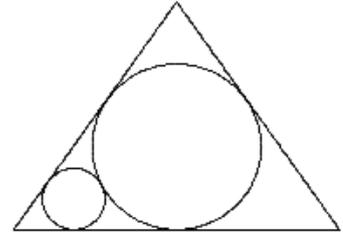
(B) 33.33

(C) 40.00

(D) 47.12

61. If $\sin x = 0.707$, then what is the value of $(\sin x) \cdot (\cos x) \cdot (\tan x)$?
- (A) 1.0
(B) 0.707
(C) 0.5
(D) 0.4
62. Possible ingredients for the pizza include: pepperoni, meatballs, green peppers, mushrooms, tomato, sausage, anchovies, onion, garlic, and broccoli. If a pizza must have 3 toppings chosen from the ingredient list and no ingredient may be used more than once on a given pizza, how many different kinds of pizza can be made?
- (A) 720
(B) 360
(C) 120
(D) 90
63. Circle O is the inscribed circle of $\triangle ABC$, intersecting the line segments \overline{AB} , \overline{BC} , \overline{AC} at three points of P, Q and R, respectively.
- Given $\overline{AB} = 5$, $\overline{BC} = 6$, $\overline{AC} = 7$, which of the following is correct?
- (A) $\overline{AP} = 2$
(B) $\overline{BQ} = 3$
(C) The radius of circle O is $\frac{2\sqrt{6}}{3}$.
(D) The area of $\triangle ABC$ is 83.
64. Suppose the sequence of numbers goes as 3, 6, 11, 18, 27, 38, ... The first term is 3, the second term is 6, according to this rule, find the value of the 100th term.
- (A) 9801
(B) 9803
(C) 10001
(D) 10002

65. The radius of the inscribed circle of a regular triangle is 1. There is a small circle of which two tangents to this circle and the two sides of the triangle take place at the same time, as shown in the figure. The radius of the small circle is



- (A) $\frac{1}{3}$
 (B) $\frac{1}{2}$
 (C) $\frac{\sqrt{3}}{6}$
 (D) $\frac{\sqrt{3}-1}{2}$
66. Let a and b be real numbers. Given that the parabola equation on the coordinate plane $y = x^2 + ax + b$ intersects the x-axis intersect at P, Q , and $\overline{PQ} = 7$. If the intersection of the parabola $y = x^2 + ax + (b + 2)$ and the x-axis is R, S , then $\overline{RS} =$
- (A) $\sqrt{35}$
 (B) $\sqrt{41}$
 (C) $\sqrt{47}$
 (D) $\sqrt{57}$
67. For any real number, $\frac{x^2 + 2ax + 1}{3x^2 - 2x + 3} \leq 1$ is always true, then the range of real numbers a is
- (A) $-3 \leq a \leq 1$
 (B) $-1 \leq a \leq 3$
 (C) $-2 \leq a \leq 1$
 (D) $-1 \leq a \leq 2$
68. Let x be a real number.
 Given $f(x) = 2(x^2 + 2x - 1)^2 - 4x^2 - 8x - 3, -2 \leq x \leq 1$. Let the maximum of $f(x)$ is M and the minimum is m . Evaluate $M + m =$
- (A) 0
 (B) 3
 (C) -3
 (D) 6

69. Let x be a real number. Suppose the maximum value of $\frac{2ax+b}{x^2+1}$ is 4, and the minimum value is -1 , then $a^2+b=$

- (A) 3
- (B) 5
- (C) 7
- (D) 9

70. Let $f(x) = x^2 + 10x + 3$, $g(x) = \frac{1}{\sqrt{x+22}}$. Which of the following is $g(f(x))$?

- (A) $\frac{1}{x-5}$
- (B) $\frac{1}{x+5}$
- (C) $\sqrt{x^2+10x+3}$
- (D) $\frac{1}{x^2+10x+3}$

71. On the Cartesian plane a point is a lattice point if both of its coordinates are integers. If there are 80 lattice points in the region (boundary included) enclosed by two axis and the line $y = -2x + k$. What is the value of k ?

- (A) 9
- (B) 11
- (C) 18
- (D) 22

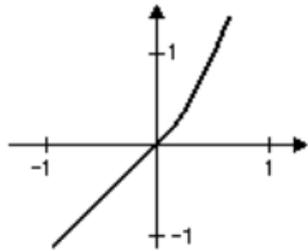
72. a is a positive number. The unit digit of $a + a^2 + \dots + a^9$ cannot be?

- (A) 6
- (B) 5
- (C) 4
- (D) 3

73. In the space there are two vectors \vec{a}, \vec{b} . If $|\vec{a}|=2$, $|\vec{b}|=3$, $\vec{a} \cdot \vec{b} = 4$, find $|\vec{a} \times \vec{b}|$?
- (A) $\sqrt{5}$
(B) $2\sqrt{5}$
(C) $3\sqrt{5}$
(D) $4\sqrt{5}$
74. If $\log 2 = 0.3010$, $\log 3 = 0.4771$. How many digits are there in the decimal expansion of the number $2^{50} \cdot 3^{35}$?
- (A) 31
(B) 32
(C) 33
(D) 34
75. How many integers are there for the equation $(n^2 - n - 1)^{n+2} = 1$?
- (A) 4
(B) 3
(C) 2
(D) 1
76. There are 5 ways to write 4 as a sum of positive numbers, namely 5, 4+1, 3+2, 2+1+1, 1+1+1+1. Note that the order of the summands is irrelevant. How many ways are there to write 8 into a sum of positive numbers, with order or the summands irrelevant?
- (A) 20
(B) 21
(C) 22
(D) 23
77. How many integers n among $[0,10]$ are there such that the equation $|x-1| - |x-2| + 2|x-3| = n$ has exactly two solutions?
- (A) 8
(B) 9
(C) 10
(D) 11

78. Find the sum of the coefficients when we write the vector $(-1,2,0)$ as the linear combination of three vectors $(2,1,5)$, $(-1,1,3)$ and $(3,1,9)$.
- (A) 2
(B) 3
(C) 5
(D) 7
79. Initially in the first box there are 1 black ball and 1 white ball and in the second box there is a white ball. Tom choose a ball randomly from the first box, put it into the second box, then choose a ball randomly from the second box and put it back to the first box. The above process is called a “round”. After the third round, the probability that the two balls in the first box are of the different colors is $\frac{n}{m}$ (in the reduced fraction) . Find the value of $m+n$.
- (A) 107
(B) 109
(C) 111
(D) 113
80. According to Anna Sfard, a mathematical concept can be conceived in two different ways: structurally and operationally. Which of the following is to view $\sqrt{2}$ operationally?
- (A) $\sqrt{2}$ as an irrational number.
(B) $\sqrt{2}$ as a symbol.
(C) $\sqrt{2}$ as an infinite decimal.
(D) $\sqrt{2}$ as taking a square root of 2.

81. David Tall conducted a study to investigate 16 year old students' concept image and concept definition. The students were asked to draw the tangent to the following graph at the origin. Some students did not cope with the task and asserted that "...because the tangent should touch the line at one specific point but this tangent would touch it constantly."



$$y = \begin{cases} x, & x \leq 0 \\ x + x^2, & x \geq 0 \end{cases}$$

Which of the following best describes the cause of the students' answer?

- (A) Almost always the concept definition will be evoked by students when solving problems.
- (B) Almost always the concept image will be evoked by students when solving problems.
- (C) The concept definition includes a form of words used to specify the concept, however, the students were asked to draw a tangent in this case.
- (D) The concept image includes a form of words used to specify the concept, however, the students were asked to draw a tangent in this case.
82. Suppose x is a real number. The maximum of $2x + \sqrt{6-x}$ is?
- (A) $\frac{93}{8}$
- (B) $\frac{95}{8}$
- (C) $\frac{97}{8}$
- (D) $\frac{99}{8}$
83. Five different odd numbers sum up to 85. What is the range of the largest number n among these five?
- (A) $17 \leq n \leq 69$
- (B) $23 \leq n \leq 67$
- (C) $19 \leq n \leq 67$
- (D) $21 \leq n \leq 69$

84. The following is a statement of “algorithm for square-root multiplication” in a secondary school textbook,

“The product of the positive square roots of two positive numbers is equal to the positive square root of the product of the two positive numbers.”

After teaching, Ms. Guo found that most students learned the expressional representation of this statement, $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$, but did not understand the above statement. Please choose the most effective teaching activity from the following to help students understand the statement.

(A) Thoroughly re-teach the concept of the algorithm.

(B) Give students corresponding numerical expressions of the statement, such as $\sqrt{2} \times \sqrt{3} = \sqrt{6}$, and ask students to describe it with words.

(C) Ask students to start with a part of the statement and gradually expand it and then judge the relations among all mathematical objects in the statement.

(D) Ask students to do more exercises pertinent to the algorithm.

85. Let $a = 10 + 6\sqrt{3}$ and $b = 10 - 6\sqrt{3}$. Evaluate $\sqrt[3]{a} + \sqrt[3]{b} =$

(A) 2

(B) $2\sqrt{3}$

(C) 5

(D) $6\sqrt{3}$

86. Matt has 4 more hats than Aaron and half as many hats as Michael. If the three together have 24 hats, how many hats does Michael have?

(A) 7

(B) 9

(C) 12

(D) 14

87. Suppose point O point is the center of an excircle of the acute triangle $\triangle ABC$. If $\angle BAO = 30^\circ$, what is $\angle ACB$?

(A) 40°

(B) 45°

(C) 50°

(D) 60°

88. If $2^n + 2^n = 2^m$, then

- (A) $n + n = m$
- (B) $n + 1 = m$
- (C) $4n = m$
- (D) $m + 1 = n$

89. If the a , b and c satisfy: $ab = c$, $bc = \frac{5}{9}$, and $ac = 15$ then what is one possible value of

abc ?

- (A) 5.0
- (B) 8.33
- (C) 9.28
- (D) 18.54

90. Points S and T lie on the circle with equation $x^2 + y^2 = 16$. If S and T have identical y-coordinates but distinct x-coordinates, then which of the following is **NOT** the possible distance between S and T?

- (A) 5.6
- (B) 8.0
- (C) π
- (D) 11.3

試題結束