

9. 出處:信欣茗數學園地

$$\begin{cases} \sqrt{x} = \sqrt{y-1323} - \sqrt{z-1323} \\ \sqrt{y} = \sqrt{x-675} + \sqrt{z-675} \\ \sqrt{z} = \sqrt{y-3675} - \sqrt{x-3675} \end{cases} \Rightarrow \begin{cases} \sqrt{y-1323} = \sqrt{x} + \sqrt{z-1323} \\ \sqrt{y} = \sqrt{x-675} + \sqrt{z-675} \\ \sqrt{y-3675} = \sqrt{z} + \sqrt{x-3675} \end{cases} \Rightarrow$$

$$\begin{cases} y = x + z + 2\sqrt{x}\sqrt{z-1323} \\ y = x + z - 1350 + 2\sqrt{x-675}\sqrt{z-675} \\ y = z + x + 2\sqrt{z}\sqrt{x-3675} \end{cases} \Rightarrow \begin{cases} \sqrt{x}\sqrt{z-1323} = \sqrt{z}\sqrt{x-3675} \\ xz - 1323x = zx - z3675 \\ x = z3675/1323 = \frac{25}{9}z \end{cases}$$

$$\begin{cases} \sqrt{x} = \sqrt{y-1323} - \sqrt{z-1323} \\ \sqrt{x-675} = \sqrt{y} - \sqrt{z-675} \\ \sqrt{x-3675} = \sqrt{y-3675} - \sqrt{z} \end{cases} \Rightarrow \begin{cases} x = y + z - 2646 - 2\sqrt{y-1323}\sqrt{z-1323} \\ x = y + z - 2\sqrt{y}\sqrt{z-675} \\ x = y + z - 2\sqrt{y-3675}\sqrt{z} \end{cases}$$

$$\begin{cases} x = y + z - 2\sqrt{y}\sqrt{z-675} \\ x = y + z - 2\sqrt{y-3675}\sqrt{z} \end{cases} \Rightarrow \begin{cases} \sqrt{y}\sqrt{z-675} = \sqrt{y-3675}\sqrt{z} \\ yz - 675y = yz - 3675z \Rightarrow y = \frac{49}{9}z \end{cases}$$

$x : y : z = 25 : 49 : 9 \Rightarrow$  令  $x = 25t, y = 49t, z = 9t$  代回原式

$$y = z + x + 2\sqrt{z}\sqrt{x-3675}, 49t = 34t + 2\sqrt{9t(25t-3675)}$$

$$225t^2 = 4 \times 9t(25t - 3675), 675t = 3675 \times 36, t = 196$$

$$x = 196 \times 25 = 4900$$

Z=1764

計算 3

p 是質數,  $p > 2$ , 若  $\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{p-1} = \frac{b}{a}$ , 則  $p \mid b$

$$\left(\frac{1}{1} + \frac{1}{p-1}\right) + \left(\frac{1}{2} + \frac{1}{p-2}\right) + \left(\frac{1}{3} + \frac{1}{p-3}\right) + \dots = p \left[ \frac{1}{p-1} + \frac{1}{2(p-2)} + \dots \right]$$

顯然,  $p \mid p \left[ \frac{1}{p-1} + \frac{1}{2(p-2)} + \dots \right]$  的分子部分, 也就是  $p \mid b$