$y - f(t) = f'(t)(x - t)$

$y_{(x=0)} \Rightarrow y = f(t) - t f'(t) = g(t)$

$\Rightarrow f(t) = g(t) + t f'(t)$

$\int_{-4}^{4} f(t)dt = \int_{-4}^{4} g(t)dt + \underbrace{\int_{-4}^{4} t f'(t)dt}$ int. by parts

$\left(\int_{-4}^{4} t f'(t)dt = \left[t f(t)\right]_{-4}^{4} - \int_{-4}^{4} f(t)dt \right)$

$\Rightarrow \dfrac{4\{f(4)+f(4)\}}{2} = \dfrac{\int_{-4}^{4} g(t)dt}{2} + \dfrac{2\int_{-4}^{4} t f'(t)dt}{2}$

$2\{f(4)+f(-4)\} = \frac{1}{2}\int_{-4}^{4} g(t)dt + \boxed{\int_{-4}^{4} t f'(t)dt}$

$\Rightarrow 2\{f(4)+f(-4)\} - \int_{-4}^{4} f(t)dt = -\frac{1}{2}\underbrace{\int_{-4}^{4} g(t)dt}_{=?}$

$\int_{-4}^{-3} + \int_{-3}^{-2} + \int_{-2}^{-1} + \int_{-1}^{0} + \int_{0}^{1} + \int_{1}^{2} + \int_{2}^{3} + \int_{3}^{4}$

另-方面 $g(t) = f(t) - t f'(t)$

$\int_0^1 g(t)dt = \underbrace{\int_0^1 f(t)dt}_{-\frac{\ln 10}{4}} - \underbrace{\int_0^1 t f'(t)dt}$

$\left(\int_0^1 t f'(t)dt = \left[t f(t)\right]_0^1 - \int_0^1 f(t)dt \right.$

$= f(1) - \left(-\frac{\ln 10}{4}\right)$

$\left. = 4 + \frac{\ln 17}{8} + \frac{\ln 10}{4} \right)$

$\int_0^1 g(t)dt = -4 - \frac{\ln 10}{2} - \frac{\ln 17}{8}$

又 $(1+t^2)\{g(1+t) - g(t)\} = 2t$

$g(1+t) - g(t) = \frac{2t}{1+t^2}$

$\Rightarrow g(1+t) = \frac{2t}{1+t^2} + g(t)$

$\left(\int_n^{n+1} g(t)dt = \int_{n-1}^{n} g(t+1)dt \right)$

$\Rightarrow \int_{n-1}^{n} g(1+t)dt = \int_{n-1}^{n} \left(\frac{2t}{1+t^2} + g(t)\right)dt$

$= \underbrace{\int_{n-1}^{n} \frac{2t}{1+t^2}dt} + \int_{n-1}^{n} g(t)dt$

$\int_{n-1}^{n} \frac{2t}{t^2+1}dt = \left[\ln(t^2+1)\right]_{n-1}^{n}$

$\boxed{= \ln(n^2+1) - \ln\{(n-1)^2+1\}}$

$\int_0^1 g(t)dt = -4 - \frac{\ln 10}{2} - \frac{\ln 17}{8}$

$\int_1^2 g(t)dt = \int_0^1 g(t)dt + \boxed{\ln(n^2+1) - \ln\{(n-1)^2+1\}}$

$= -4 - \frac{\ln 10}{2} - \frac{\ln 17}{8} + \ln 2$

又 $\int_2^3 g(t)dt = -4 - \frac{\ln 10}{2} - \frac{\ln 17}{4} + \cancel{\ln 2} + \ln 5 - \cancel{\ln 2}$

$= -4 - \frac{\ln 10}{2} - \frac{\ln 17}{4} + \ln 5$

同理

$\int_3^4 g(t)dt = -4 - \frac{\ln 10}{2} - \frac{\ln 17}{4} + \ln 10$

$\int_0^1 g(t)dt = \int_{-1}^{0} g(t)dt + \boxed{\ln 1 - \ln 2}$

$\Rightarrow \int_{-1}^{0} g(t)dt = -4 - \frac{\ln 10}{2} - \frac{\ln 17}{4} + \ln 2$

同理

$\int_{-2}^{-1} g(t)dt = -4 - \frac{\ln 10}{2} - \frac{\ln 17}{4} + \ln 5$

$\int_{-3}^{-2} g(t)dt = -4 - \frac{\ln 10}{2} - \frac{\ln 17}{4} + \ln 10$

$\int_{-4}^{-3} g(t)dt = \underbrace{-4 - \frac{\ln 10}{2} - \frac{\ln 17}{4}}_{= A} + \ln 17$

$\therefore \int_{-4}^{4} g(t)dt = A + \ln 17 + A + \ln 10 + A + \ln 5 + A + \ln 2 + A + A + \ln 2 + A + \ln 5 + A + \ln 10$

$= 8A + \ln(2^2 \times 5^2 \times 10^2 \times 17)$

$= 8\left(-4 - \frac{\ln 10}{2} - \frac{\ln 17}{8}\right) + \cancel{\ln 17} + \cancel{\ln 10^4}$

$= -32$

原式 $= -\frac{1}{2}\int_{-4}^{4} g(t)dt = -\frac{1}{2} \times (-32) = 16$