

10. 科西不等式

$$(x + y + z) \left( \frac{1}{x} + \frac{1}{y} + \frac{1}{z} \right) \\ = \left( (\sqrt{x})^2 + (\sqrt{y})^2 + (\sqrt{z})^2 \right) \left( \frac{1}{(\sqrt{x})^2} + \frac{1}{(\sqrt{y})^2} + \frac{1}{(\sqrt{z})^2} \right) \geq (1 + 1 + 1)^2 = 9$$

$$\text{即} \left( \frac{1}{x} + \frac{1}{y} + \frac{1}{z} \right) \geq \frac{9}{x+y+z}$$

$\because a, b, c$  為三角形三邊長

$\therefore$  令  $x = b + c - a > 0, y = c + a - b > 0, z = a + b - c > 0, x + y + z = a + b + c$

$$\text{即} \frac{1}{b+c-a} + \frac{1}{c+a-b} + \frac{1}{a+b-c} \geq \frac{9}{a+b+c}$$