2. $\angle A+\angle B+\angle C=\pi$

$\angle\frac{A}{2}+\angle\frac{B}{2}+\angle\frac{c}{2}=\frac{\pi}{2}$

$\angle\frac{A}{2}+\angle\frac{B}{2}=\frac{\pi}{2}-\angle\frac{c}{2}$

$\tan\left(\angle\frac{A}{2}+\angle\frac{B}{2}\right)=\tan\left(\frac{\pi}{2}-\angle\frac{c}{2}\right)=\cot\angle\frac{c}{2}$

$$\frac{\tan\angle\frac{A}{2}+\tan\angle\frac{B}{2}}{1-\tan\angle\frac{A}{2}\tan\angle\frac{B}{2}}=\frac{1}{\tan\angle\frac{C}{2}}$$

$$\tan\angle\frac{A}{2}\tan\angle\frac{C}{2}+\tan\angle\frac{B}{2}\tan\angle\frac{C}{2}=1-\tan\angle\frac{A}{2}\tan\angle\frac{B}{2}$$

$$\tan\angle\frac{A}{2}\tan\angle\frac{C}{2}+\tan\angle\frac{B}{2}\tan\angle\frac{C}{2}+\tan\angle\frac{A}{2}\tan\angle\frac{B}{2}=1$$

$$\frac{1}{3}+\tan\angle\frac{B}{2}\tan\angle\frac{C}{2}+\tan\angle\frac{A}{2}\tan\angle\frac{B}{2}=1$$

$$\tan\angle\frac{B}{2}\tan\angle\frac{C}{2}+\tan\angle\frac{A}{2}\tan\angle\frac{B}{2}=\frac{2}{3}=2\tan\angle\frac{A}{2}\tan\angle\frac{C}{2}$$

同乘$\cot\angle\frac{A}{2}\cot\angle\frac{B}{2}\cot\angle\frac{C}{2}$

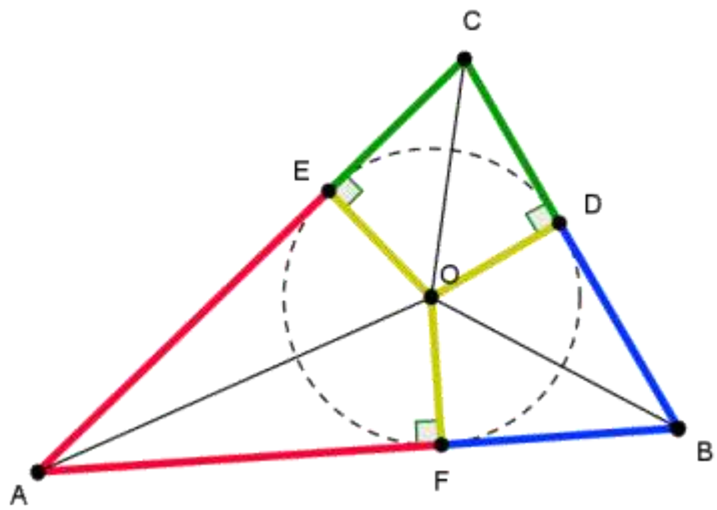
$$\cot\angle\frac{A}{2}+\cot\angle\frac{C}{2}=2\cot\angle\frac{B}{2}$$

令內切圓半徑為 r ,AB=c ,AC=b ,BC=a ,$s=\frac{a+b+c}{2}$

$$\frac{S-a}{r}+\frac{S-c}{r}=2\,\frac{S-b}{r}$$

$2b=a+c$

a,b,c 成等差數列故得證



(PS)AB+BC+AC=a+b+c

AF+FB+CD=$\frac{a+b+c}{2}$=S

AF =S−FB−CD=S−(DB+CD)=S−a

$$\cot\angle\frac{A}{2}=\frac{S-a}{r}$$