

12.

$$x, y \in \square, \text{ 試解聯立方程式 } \begin{cases} x + y = 5 \\ x^4 + y^4 = 97 \end{cases} \cdot \text{ (12 分)}$$

$$\text{Sol : } \because x^2 + 2xy + y^2 = 25$$

$$\begin{aligned} \Rightarrow (x^2 + y^2)^2 &= (25 - 2xy)^2 \Rightarrow x^4 + 2x^2y^2 + y^4 = 625 - 100xy + 4x^2y^2 \Rightarrow 97 + 2x^2y^2 = 625 - 100xy \\ \Rightarrow x^2y^2 - 50xy + 264 &= 0 \Rightarrow xy = 6 \text{ or } 44 \end{aligned}$$

$$(1) \begin{cases} x + y = 5 \\ xy = 6 \end{cases} \Rightarrow \begin{cases} x = 2 \\ y = 3 \end{cases} \text{ or } \begin{cases} x = 3 \\ y = 2 \end{cases}$$

$$(2) \begin{cases} x + y = 5 \\ xy = 44 \end{cases} \Rightarrow \begin{cases} x = \frac{5 + \sqrt{151}i}{2} \\ y = \frac{5 - \sqrt{151}i}{2} \end{cases} \text{ or } \begin{cases} x = \frac{5 - \sqrt{151}i}{2} \\ y = \frac{5 + \sqrt{151}i}{2} \end{cases}$$

-----共四組解.