9.

ACA+ BCB = ACB+ BCA+ I

ACA+ BCB−ACB− BCA =I

(A−B)CA−(A−B)CB= I (左分配)

(A−B)C(A−B)= I (右分配)

$C=(A-B)^{-1}I(A-B)^{-1}$

$\quad =((A-B)^{-1})^2$

$$A-B=\begin{pmatrix}2 & 0 & 0\\ 1 & 1 & 0\\ 0 & 3 & 2\end{pmatrix}-\begin{pmatrix}1 & 1 & 1\\ 1 & 0 & 1\\ 0 & 3 & 1\end{pmatrix}=\begin{pmatrix}1 & -1 & -1\\ 0 & 1 & -1\\ 0 & 0 & 1\end{pmatrix}$$

(法一)三階反矩陣公式

$\det(A-B)=1$

$$(A-B)^{-1}=\frac{1}{\det(A-B)}\begin{pmatrix}\begin{vmatrix}1 & -1\\ 0 & 1\end{vmatrix} & -\begin{vmatrix}0 & -1\\ 0 & 1\end{vmatrix} & \begin{vmatrix}0 & 1\\ 0 & 0\end{vmatrix}\\ -\begin{vmatrix}-1 & 1\\ 0 & 1\end{vmatrix} & \begin{vmatrix}1 & -1\\ 0 & 1\end{vmatrix} & -\begin{vmatrix}1 & -1\\ 0 & 0\end{vmatrix}\\ \begin{vmatrix}-1 & -1\\ 1 & -1\end{vmatrix} & -\begin{vmatrix}1 & -1\\ 0 & -1\end{vmatrix} & \begin{vmatrix}1 & -1\\ 0 & 1\end{vmatrix}\end{pmatrix}^t$$

$$=\begin{pmatrix}1 & 0 & 0\\ 1 & 1 & 0\\ 2 & 1 & 1\end{pmatrix}^t=\begin{pmatrix}1 & 1 & 2\\ 0 & 1 & 1\\ 0 & 0 & 1\end{pmatrix}$$

(法二)利用 Gauss-Jordan Elimination 求$(A-B)^{-1}$

$$\left(\begin{array}{ccc|ccc}1 & -1 & -1 & 1 & 0 & 0\\ 0 & 1 & -1 & 0 & 1 & 0\\ 0 & 0 & 1 & 0 & 0 & 1\end{array}\right)\to(A-B)^{-1}=\begin{pmatrix}1 & 1 & 2\\ 0 & 1 & 1\\ 0 & 0 & 1\end{pmatrix}$$

$$((A-B)^{-1})^2=\begin{pmatrix}1 & 2 & 5\\ 0 & 1 & 2\\ 0 & 0 & 1\end{pmatrix}$$