

## 計算第 2 題

$$f(x) = \lim_{n \rightarrow \infty} \frac{x^{2n-1} + ax^2 + bx}{x^{2n} + 1} = \lim_{n \rightarrow \infty} \frac{1 + \frac{ax^2 + bx}{x^{2n-1}}}{x + \frac{1}{x^{2n-1}}}$$

$$(1) |x| < 1, \quad f(x) = ax^2 + bx$$

$$(2) |x| > 1, \quad f(x) = \frac{1}{x}$$

$$(3) \lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} \frac{1}{x} = 1, \quad \lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} (ax^2 + bx) = a + b$$

$$f(1) = \frac{1 + a + b}{2}$$

$$\frac{1 + a + b}{2} = 1 = a + b$$

$$a + b = 1$$

$$(4) \lim_{x \rightarrow -1^+} f(x) = \lim_{x \rightarrow -1^+} (ax^2 + bx) = a - b, \quad \lim_{x \rightarrow -1^-} f(x) = \lim_{x \rightarrow -1^-} \frac{1}{x} = -1$$

$$f(-1) = \frac{-1 + a - b}{2}$$

$$\frac{-1 + a - b}{2} = a - b = -1$$

$$a - b = -1$$

$$a = 0, b = 1$$