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四面體 $D-ABC$ ，作 $\overline{BE} \perp \overline{AC}$ 於 E ， $\overline{DF} \perp \overline{AC}$ 於 F

$$\triangle ABC = \sqrt{11(11-9)(11-8)(11-5)} = 6\sqrt{11}$$

$$\triangle ACD = \sqrt{15(15-11)(15-10)(15-9)} = 30\sqrt{2}$$

$$\overline{BE} = \frac{\triangle ABC \times 2}{\overline{AC}} = \frac{12\sqrt{11}}{9} = \frac{4\sqrt{11}}{3}$$

$$\overline{DF} = \frac{\triangle ACD \times 2}{\overline{AC}} = \frac{60\sqrt{2}}{9} = \frac{20\sqrt{2}}{3}$$

$$\overline{AE} = \sqrt{\overline{AB}^2 - \overline{BE}^2} = \sqrt{25 - \frac{176}{9}} = \frac{7}{3}$$

$$\overline{AF} = \sqrt{\overline{AD}^2 - \overline{DF}^2} = \sqrt{100 - \frac{800}{9}} = \frac{10}{3}$$

$$A(0,0,0), C(9,0,0), B\left(\frac{7}{3}, \frac{4\sqrt{11}}{3}, 0\right), D\left(\frac{10}{3}, y, z\right)$$

$$\overline{AD}^2 = \left(\frac{10}{3}\right)^2 + y^2 + z^2 = 100$$

$$\overline{BD}^2 = \left(\frac{10}{3} - \frac{7}{3}\right)^2 + \left(y - \frac{4\sqrt{11}}{3}\right)^2 + z^2 = 144$$

$$y = -\frac{311\sqrt{11}}{264}$$

$$\cos \theta = \frac{y}{\overline{DF}} = \frac{-\frac{311\sqrt{11}}{264}}{\frac{20\sqrt{2}}{3}} = -\frac{311\sqrt{22}}{3520}$$