

第 12 題

還沒擲滿 n 次前要結束，一定是後兩次擲出相同點數

故

$$\begin{aligned} E_n &= \left[2 \times 1 \times \frac{1}{6} + 3 \times 1 \times \frac{5}{6} \times \frac{1}{6} + 4 \times 1 \times \left(\frac{5}{6}\right)^2 \times \frac{1}{6} + \cdots + (n-1) \times 1 \times \left(\frac{5}{6}\right)^{n-3} \times \frac{1}{6} \right] + n \times 1 \times \left(\frac{5}{6}\right)^{n-2} \times 1 \\ &= \left[7 - 5 \left(\frac{5}{6}\right)^{n-3} - (n-1) \left(\frac{5}{6}\right)^{n-2} \right] + n \left(\frac{5}{6}\right)^{n-2} \\ &= 7 - 5 \left(\frac{5}{6}\right)^{n-3} + \left(\frac{5}{6}\right)^{n-2} \\ &= 7 - 6 \left(\frac{5}{6}\right)^{n-1} \end{aligned}$$