

第6題

$$0 \leq x \leq 2\pi, 0 \leq y \leq 2\pi$$

$$0 \leq x + y \leq 4\pi, -2\pi \leq x - y \leq 2\pi$$

$$\sin x \cos x + \sin y \cos y + \sin x \sin y + \cos x \cos y = 1$$

$$\frac{\sin 2x + \sin 2y}{2} + \cos(x - y) = 1$$

$$\sin(x + y)\cos(x - y) + \cos(x - y) = 1$$

$$\cos(x - y) = \frac{1}{\sin(x + y) + 1} > 0$$

最小值出現在 $\sin(x + y) = 1$ 時，此時 $\cos(x - y) = \frac{1}{2}$

$$\begin{cases} x + y = \frac{\pi}{2} \\ x - y = \frac{5\pi}{3} \end{cases}, \begin{cases} x + y = \frac{\pi}{2} \\ x - y = \frac{\pi}{3} \end{cases}, \begin{cases} x + y = \frac{\pi}{2} \\ x - y = -\frac{\pi}{3} \end{cases}, \begin{cases} x + y = \frac{\pi}{2} \\ x - y = -\frac{5\pi}{3} \end{cases}$$

$$\begin{cases} x + y = \frac{5\pi}{2} \\ x - y = \frac{5\pi}{3} \end{cases}, \begin{cases} x + y = \frac{5\pi}{2} \\ x - y = \frac{\pi}{3} \end{cases}, \begin{cases} x + y = \frac{5\pi}{2} \\ x - y = -\frac{\pi}{3} \end{cases}, \begin{cases} x + y = \frac{5\pi}{2} \\ x - y = -\frac{5\pi}{3} \end{cases}$$

$$(x, y) = \left(\frac{\pi}{12}, \frac{5\pi}{12}\right), \left(\frac{5\pi}{12}, \frac{\pi}{12}\right), \left(\frac{13\pi}{12}, \frac{17\pi}{12}\right), \left(\frac{17\pi}{12}, \frac{13\pi}{12}\right)$$