

2.先求 $\sin \angle BAC$, $\cos \angle CAD$, $\sin \angle CAD$

令 $\angle BAC = \angle 1$, $\angle CAD = \angle 2$, $\angle BAD = \angle 3$

$\angle 3 = \angle 1 + \angle 2$

$$\cos \angle 1 = \frac{3}{5}, \sin \angle 1 = \frac{4}{5}$$

$$\cos \angle 2 = \frac{\vec{AC} \cdot \vec{AD}}{|\vec{AC}| \cdot |\vec{AD}|} = \frac{12}{13}, \sin \angle 2 = \frac{5}{13}$$

然後利用和角公式求 $\cos \angle BAD$,

$$\cos \angle 3 = \cos(\angle 1 + \angle 2) = \frac{3}{5} \times \frac{12}{13} - \frac{4}{5} \times \frac{5}{13} = \frac{16}{65}$$

$$\vec{AC} \cdot \vec{AD} = 120, \vec{AC} \cdot \vec{AB} = 30,$$

再求 \vec{AB} 內積 \vec{AD}

$$\vec{AB} \cdot \vec{AD} = 16$$

將 $\vec{AC} = x \vec{AB} + y \vec{AD}$ 等號左右同時內積 \vec{AB} , 得一個方程式

將 $\vec{AC} = x \vec{AB} + y \vec{AD}$ 等號左右同時內積 \vec{AD} , 得另一個方程式

$$\begin{cases} 25x + 16y = 30 \\ 16x + 16y = 120 \end{cases}$$

解聯立方程式, 可得 x, y .

$$x = \frac{50}{63}, y = \frac{40}{63}$$