

$$\begin{aligned}
& 1. [(x-1) + 1]^{2009} \\
& = C_0^{2009}(x-1)^0 1^{2009} + C_1^{2009}(x-1)^1 1^{2008} + C_2^{2009}(x-1)^2 1^{2007} + \dots + \\
& C_{2009}^{2009}(x-1)^{2009} 1^0 \\
& \text{除以}(x-1)^2 \text{餘式 } C_0^{2009}(x-1)^0 1^{2009} + C_1^{2009}(x-1)^1 1^{2008} = 2009(x-1) + 1 \\
& \text{令原式為 } x^{2009} = (x-1)^2(x^2+1)Q(x) + (x-1)^2(ax+b) + 2009(x-1) + 1 \\
& \text{以 } x=i \text{ 帶入上式整理後} \\
& i = (2a-2008) + (-2b+2009)i \\
& a = 1004, b = 1004 \\
& \text{因此餘式} = 1004(x-1)^2(x+1) + 2009(x-1) + 1
\end{aligned}$$