

# Functional Equations Problems

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Dedicated to pco.

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## 1 Definitions

- $\mathbb{N}$  is the set of positive integers.
- $\mathbb{N} \cup \{0\} = \mathbb{N}^*$  is the set of non-negative integers.
- $\mathbb{Z}$  is the set of integers.
- $\mathbb{Q}$  is the set of rational numbers.
- $\mathbb{R}^+$  is the set of positive real numbers.
- $\mathbb{R}$  is the set of real numbers.
- If a function  $f$  is defined on the set  $A$  to the set  $B$ , we write  $f : A \rightarrow B$  and read “ $f$  is a function from the set  $A$  to the set  $B$ .”

## 2 Problems

1. Find all surjective functions  $f : \mathbb{N} \rightarrow \mathbb{N}$  such that  $f(n) \geq n + (-1)^n, \forall n \in \mathbb{N}$ .

2. Find all functions  $g : \mathbb{R} \rightarrow \mathbb{R}$  such that for any real numbers  $x$  and  $y$

$$g(x+y) + g(x)g(y) = g(xy) + g(x) + g(y).$$

3. Find all real valued functions defined on the reals such that for every real  $x, y$

$$f(x^2 - y^2) = xf(x) - yf(y).$$

4. Find all real valued functions defined on the reals such that for every real  $x, y$ :

$$f(xf(x) + f(y)) = f(x)^2 + y.$$

5. Find all functions  $f : \mathbb{N} \rightarrow \mathbb{N}$  such that  $f(f(n)) + (f(n))^2 = n^2 + 3n + 3$  for all positive integers  $n$ .

6. Let  $n$  be a positive integer. Find all strictly increasing functions  $f : \mathbb{N}^* \rightarrow \mathbb{N}^*$  such that the equation

$$\frac{f(x)}{k^n} = k - x$$

has an integral solution  $x$  for all  $k \in \mathbb{N}$ .

7. Find all functions  $f : \mathbb{R}^+ \rightarrow \mathbb{R}^+$  such that

$$f\left(\frac{x+y}{2}\right) = \frac{2f(x)f(y)}{f(x)+f(y)} \quad \forall x, y \in \mathbb{R}^+.$$

8. Find all functions  $f : \mathbb{R} \rightarrow \mathbb{R}$  such that

$$f(1-x) = 1 - f(f(x)) \quad \forall x \in \mathbb{R}.$$

9. Find all functions  $f : \mathbb{R}^+ \rightarrow \mathbb{R}^+$  such that

$$f(1 + xf(y)) = yf(x+y) \quad \forall x, y \in \mathbb{R}^+.$$

10. Find all functions  $f : \mathbb{R}^+ \rightarrow \mathbb{R}^+$  such that

$$f(xf(y)) = f(x+y) \quad \forall x, y \in \mathbb{R}^+.$$

11. Find all functions  $f : \mathbb{R} \rightarrow \mathbb{R}$  such that

$$f(f(x) + y) = f(x^2 - y) + 4yf(x) \quad \forall x, y \in \mathbb{R}.$$

12. Find all functions  $f, g, h : \mathbb{R} \rightarrow \mathbb{R}$  such that

$$f(x+y) + g(x-y) = 2h(x) + 2h(y) \quad \forall x, y \in \mathbb{R}.$$

13. Find all functions  $f : \mathbb{R} \rightarrow \mathbb{R}$  such that

$$f(x + y + z) = f(x) \cdot f(1 - y) + f(y) \cdot f(1 - z) + f(z) \cdot f(1 - x) \quad \forall x, y, z \in \mathbb{R}.$$

14. Find all functions  $f : \mathbb{R} \rightarrow \mathbb{R}$  such that

$$f(f(x) - f(y)) = (x - y)^2 f(x + y) \quad \forall x, y \in \mathbb{R}.$$

15. Find all functions  $f, g : \mathbb{R} \rightarrow \mathbb{R}$  such that

- If  $x < y$ , then  $f(x) < f(y)$ ;
- for all  $x, y \in \mathbb{R}$ , we have  $f(xy) = g(y)f(x) + f(y)$ .

16. Find all functions  $f : \mathbb{R} \rightarrow \mathbb{R}$  such that

$$f((x + z)(y + z)) = (f(x) + f(z))(f(y) + f(z)) \quad \forall x, y, z \in \mathbb{R}.$$

17. Find all functions  $f : \mathbb{R} \rightarrow \mathbb{R}$  that satisfy

$$f(x^3 + y^3) = x^2 f(x) + y f(y^2)$$

for all  $x, y \in \mathbb{R}$ .

18. Find all functions  $f : \mathbb{R} \rightarrow \mathbb{R}$  that satisfy

$$f(m + n f(m)) = f(m) + m f(n)$$

for all  $m$  and  $n$ .

19. Find all functions  $f : \mathbb{R} \rightarrow \mathbb{R}$  such that  $f(x)f(y) = f(x + y) + xy$  for all  $x, y \in \mathbb{R}$ .

20. Find all functions  $f : \mathbb{N} \cup \{0\} \rightarrow \mathbb{N} \cup \{0\}$  such that  $x \cdot 3^{f(y)}$  divides  $f(x) \cdot 3^y$  for all  $x, y \in \mathbb{N} \cup \{0\}$ .

21. Find all continuous functions  $f : \mathbb{R} \rightarrow \mathbb{R}$  such that

$$f(x + y)f(x - y) = (f(x)f(y))^2 \quad \forall x, y \in \mathbb{R}.$$

22. Find all functions  $f : \mathbb{R} \rightarrow \mathbb{R}$  such that

$$(x + y)(f(x) - f(y)) = (x - y)f(x + y) \quad \forall x, y \in \mathbb{R}.$$

23. Find all functions  $f : \mathbb{R} \rightarrow \mathbb{R}$  such that

$$f((f(x) + y)) = f(x^2 - y) + 4f(x)y \quad \forall x, y \in \mathbb{R}.$$

24. Find all the functions  $f : \mathbb{Z} \rightarrow \mathbb{R}$  such that

$$f(m + n - mn) = f(m) + f(n) - f(mn) \quad \forall m, n \in \mathbb{Z}$$

25. Find all functions  $f : (0, 1) \rightarrow (0, 1)$  such that  $f(\frac{1}{2}) = \frac{1}{2}$  and

$$(f(ab))^2 = (af(b) + f(a))(bf(a) + f(b)) \quad \forall a, b \in (0, 1).$$

26. Find all functions  $f : \mathbb{Q} \rightarrow \mathbb{Q}$  such that

$$f(x + y + f(x)) = x + f(x) + f(y) \quad \forall x, y \in \mathbb{Q}.$$

27. Find all functions  $f : \mathbb{R} \rightarrow \mathbb{R}$  such that

$$f(x^2 + f(y)) = (x - y)^2 f(x + y) \quad \forall x, y \in \mathbb{R}.$$

28. Find all functions  $f : \mathbb{R} \rightarrow \mathbb{R}$  such that

- $f(x + y) = f(x) + f(y) \quad \forall x, y \in \mathbb{R},$
- $f(x) = x^2 f(\frac{1}{x}) \quad \forall x \in \mathbb{R} \setminus \{0\}.$

29. Let  $a > \frac{3}{4}$  be a real number. Find all functions  $f : \mathbb{R} \rightarrow \mathbb{R}$  such that

$$f(f(x)) + a = x^2 \quad \forall x \in \mathbb{R}.$$

30. Find all injective functions  $f : \mathbb{N} \rightarrow \mathbb{N}$  which satisfy

$$f(f(n)) \leq \frac{n + f(n)}{2} \quad \forall n \in \mathbb{N}.$$

31. Find all continuous functions  $f(x), g(x), q(x) : \mathbb{R} \rightarrow \mathbb{R}$  such that

$$f(x^2) + f(y^2) = [q(x) - q(y)]g(x + y) \quad \forall x, y \in \mathbb{R}.$$

32. Find all functions  $f : \mathbb{R} \rightarrow \mathbb{R}$  so that

$$f(x + y) + f(x - y) = 2f(x) \cos y \quad \forall x, y \in \mathbb{R}.$$

33. Find all functions  $f : \mathbb{R} \rightarrow \mathbb{R}$  such that

$$f(x - f(y)) = f(x) + x \cdot f(y) + f(f(y)) \quad \forall x, y \in \mathbb{R}.$$

34. Find all functions  $f : \mathbb{R}^+ \rightarrow \mathbb{R}^+$  such that

$$f(f(x)) = 6x - f(x) \quad \forall x \in \mathbb{R}^+.$$

35. Find all functions  $f : \mathbb{R} \rightarrow \mathbb{R}$  such that

$$f(x + y) + f(xy) + 1 = f(x) + f(y) + f(xy + 1) \quad \forall x, y \in \mathbb{R}.$$

36. Find all functions  $f : \mathbb{R} \rightarrow \mathbb{R}$  such that

$$f(x)f(y \cdot f(x) - 1) = x^2 f(y) - f(x) \quad \forall x, y \in \mathbb{R}.$$

37. Find all functions  $f : \mathbb{R} \rightarrow \mathbb{R}$  such that

$$f(x^3 + y^3) = x \cdot f(x^2) + y \cdot f(y^2) \quad \forall x, y \in \mathbb{R}.$$

38. Find all functions  $f : \mathbb{Q} \rightarrow \mathbb{R}$  such that

$$|f(x) - f(y)| \leq (x - y)^2 \quad \forall x, y \in \mathbb{Q}.$$

39. Find all functions  $f : \mathbb{R} \rightarrow \mathbb{R}^+$  such that

$$f(x + y) = f(x^2 + y^2) \quad \forall x \in \mathbb{R}^+.$$

40. Find all functions  $f : \mathbb{R} \rightarrow \mathbb{R}$  such that

$$x^2 y^2 (f(x + y) - f(x) - f(y)) = 3(x + y)f(x)f(y) \quad \forall x, y \in \mathbb{R}.$$

41. Find all functions  $f : \mathbb{R} \rightarrow \mathbb{R}$  such that

$$f(f(x) + f(y) + f(z)) = f(f(x) - f(y)) + f(2xy + f(z)) + 2f(xz - yz)$$

for all reals  $x, y$ .

42. Find all functions  $f : \mathbb{N} \rightarrow \mathbb{N}$  such that  $m^2 + f(n)|(f(m))^2 + n$  for all positive integers  $m, n$ .

43. Let  $n$  be a positive integer. Find all functions  $f : \mathbb{R} \rightarrow \mathbb{R}$  such that

$$f(x + f(y)) = f(x) + y^n \quad \forall x, y \in \mathbb{R}.$$

44. Find all the functions  $f : \mathbb{N} \rightarrow \mathbb{N}$  such that

$$3f(f(f(n))) + 2f(f(n)) + f(n) = 6n \quad \forall n \in \mathbb{N}.$$

45. Find all functions  $f : \mathbb{N}^* \rightarrow \mathbb{N}^*$  satisfying

$$(f^2(m) + f(n)) \mid (m^2 + n)^2$$

for any two positive integers  $m$  and  $n$ .

46. Find all functions  $f : \mathbb{R}^+ \rightarrow \mathbb{R}^+$  such that

$$f\left(\frac{2xy}{x+y}\right) = \frac{2f(x)f(y)}{f(x) + f(y)} \quad \forall x, y \in \mathbb{R}^+.$$

47. Find all functions  $f : \mathbb{R} \rightarrow \mathbb{R}$  such that

$$f(xy) = \max\{f(x), y\} + \min\{f(y), x\} \quad \forall x, y \in \mathbb{R}.$$

48. Find all functions  $f : \mathbb{R} \rightarrow \mathbb{R}$  such that

- $f(x + f(y)) = y + f(x) \quad \forall x, y \in \mathbb{R}$ , and

- The set  $A = \frac{f(x)}{x}$  is finite.

49. Find all functions  $f : \mathbb{R} \rightarrow \mathbb{R}$  such that

$$f(f(x) + f(y)) + f(f(x)) = 2f(x) + y \quad \forall x, y \in \mathbb{R}.$$

50. Find all functions  $f : \mathbb{R} \rightarrow \mathbb{R}$  such that

$$f(x^2(z^2+1)+f(y)(z+1)) = 1-f(z)(x^2+f(y))-z((1+z)x^2+2f(y)) \quad \forall x, y, z \in \mathbb{R}.$$

51. Prove that there is no bijective function  $f : \{1, 2, 3, \dots\} \rightarrow \{0, 1, 2, 3, \dots\}$  such that

$$f(mn) = f(m) + f(n) + 3f(m)f(n).$$

52. Find all functions  $f : \mathbb{R} \rightarrow \mathbb{R}$  such that

$$f(x - f(y)) = f(f(y)) + xf(y) + f(x) - 1 \quad \forall x, y \in \mathbb{R}.$$

53. Find all functions  $f : \mathbb{R} \rightarrow \mathbb{R}$  such that

$$f(xf(x+y)) = f(yf(x)) + x^2 \quad \forall x, y \in \mathbb{R}.$$

54. Find all functions  $f : \mathbb{R} \rightarrow \mathbb{R}$  such that

$$f(x) = f\left(x^2 + \frac{x}{3} + \frac{1}{9}\right) \quad \forall x \in \mathbb{R}.$$

55. Given  $0 < p < 2$ , find all continuous functions  $f : \mathbb{R} \rightarrow \mathbb{R}$  such that

$$f(f(x)) = f(x) + px \quad \forall x \in \mathbb{R}.$$

56. Find all functions  $f : \mathbb{R} \rightarrow \mathbb{R}$  such that

$$f(x + xy + f(y)) = \left(f(x) + \frac{1}{2}\right) \left(f(y) + \frac{1}{2}\right) \quad \forall x, y \in \mathbb{R}.$$

57. Find all functions  $f : \mathbb{R} \rightarrow \mathbb{R}$  such that

$$f(f(x) + y) = f(x + y) + xf(y) - xy - x + 1 \quad \forall x, y \in \mathbb{R}.$$

58. Find all functions  $f : \mathbb{R} \rightarrow \mathbb{R}$  such that:

$$x(f(x) + f(-x) + 2) + 2f(-x) = 0 \quad \forall x \in \mathbb{R}.$$

59. Find all non-decreasing functions  $f : \mathbb{R}^+ \cup \{0\} \rightarrow \mathbb{R}^+ \cup \{0\}$  such that for each  $x, y \in \mathbb{R}^+ \cup \{0\}$

$$f\left(\frac{x + f(x)}{2} + y\right) = 2x - f(x) + f(f(y)).$$

60. Find all functions  $f : \mathbb{R} \rightarrow \mathbb{R}$  such that:

$$(1 + f(x)f(y))f(x + y) = f(x) + f(y) \quad \forall x, y \in \mathbb{R}.$$

61. For function  $f : \mathbb{R} \rightarrow \mathbb{R}$  given that  $f(x^2 + x + 3) + 2 \cdot f(x^2 - 3x + 5) = 6x^2 - 10x + 17$ . Calculate  $f(2009)$ .

62. Find all the functions  $f : \mathbb{R} \mapsto \mathbb{R}$  such that

$$f(x - f(y)) = f(f(y)) + xf(y) + f(x) - 1$$

for all  $x, y \in \mathbb{R}$ .

63. Find all functions  $f : \mathbb{R} \rightarrow \mathbb{R}$  such that  $f(1) = 1$  and

$$f\left(f(x)y + \frac{x}{y}\right) = xyf(x^2 + y^2)$$

for all real numbers  $x$  and  $y$  with  $y \neq 0$ .

64. Find all functions  $f$ , defined on the positive real numbers and taking real numbers such that

$$f(x) + f(y) \leq \frac{f(x+y)}{2}, \quad \frac{f(x)}{x} + \frac{f(y)}{y} \geq \frac{f(x+y)}{x+y}$$

for all  $x, y > 0$ .

65. Find all functions  $f : \mathbb{R} \rightarrow \mathbb{R}$  such that

$$f(xf(y-x)) = f(yf(x)) - x^2 \quad \forall x, y \in \mathbb{R}.$$

66. Find all functions  $f : \mathbb{R} \rightarrow \mathbb{R}$  such that :

$$f(x + f(y + f(z))) = f(x) + f(f(y)) + f(f(f(z))) \quad \forall x, y, z \in \mathbb{R}.$$

67. Find all functions  $f : \mathbb{R}^+ \rightarrow \mathbb{R}$  satisfying the identity

$$f(x)f(y) = y^\alpha f\left(\frac{x}{2}\right) + x^\beta f\left(\frac{y}{2}\right) \quad \forall x, y \in \mathbb{R}^+$$

Where  $\alpha, \beta$  are given real numbers.

68. Find all functions  $f : \mathbb{R} \rightarrow \mathbb{R}$  such that for all  $x, y \in \mathbb{R}$ , we have

$$f(x+y) + f(x)f(y) = f(xy) + (y+1)f(x) + (x+1)f(y).$$

69. Find all functions  $f : \mathbb{R} \rightarrow \mathbb{R}$  such that

$$f(x+y) = 2f(x)f(y) + 3f(x) + 3f(y) + 3 \quad \forall x, y \in \mathbb{R}.$$

70. Find all the continuous bounded functions  $f : \mathbb{R} \rightarrow \mathbb{R}$  such that

$$(f(x))^2 - (f(y))^2 = f(x+y)f(x-y) \text{ for all } x, y \in \mathbb{R}.$$



**71.** Find all functions  $f : \mathbb{R} \rightarrow \mathbb{R}$  such that  $f(x+y) + f(x)f(y) = f(xy) + 2xy + 1$  for all real numbers  $x$  and  $y$ .

**72.** Find all functions  $f : \mathbb{R} \rightarrow \mathbb{R}$  such that

$$f(x^2 + y^2) = f(f(x)) + f(xy) + f(f(y)) \quad \forall x, y \in \mathbb{R}.$$

**73.** Find all functions  $f : \mathbb{R}^+ \rightarrow \mathbb{R}^+$  such that

$$(x+y)f(f(x)y) = x^2f(f(x) + f(y)) \quad \forall x, y \in \mathbb{R}^+.$$

**74.** Find all functions  $f : \mathbb{R} \rightarrow \mathbb{R}$  such that

$$f(x+y^2) \geq (y+1)f(x) \quad \forall x, y \in \mathbb{R}.$$

**75.** Find all functions  $f : \mathbb{R} \rightarrow \mathbb{R}$  such that

$$f(x)f(y) \leq f(xy) \text{ and } f(x) + f(y) \leq f(x+y) \quad \forall x, y \in \mathbb{R}.$$

**76.** Find all functions  $f : \mathbb{Q} \rightarrow \mathbb{R}^+$  such that

- $f(x) \geq 0 \quad \forall x \in \mathbb{Q}, \quad f(x) = 0 \iff x = 0,$
- $f(xy) = f(x) \cdot f(y),$
- $f(x+y) \leq \max\{f(x), f(y)\}$

**77.** Determine all function  $f : \mathbb{R} \rightarrow \mathbb{R}$  satisfying

$$xf(y) - yf(x) = f\left(\frac{y}{x}\right)$$

for all  $x, y \in \mathbb{R}$  with  $x \neq 0$ .

**78.** Determine all functions  $f : \mathbb{N} \rightarrow \mathbb{N}$  such that

$$\sum_{k=1}^n \frac{1}{f(k) \cdot f(k+1)} = \frac{f(f(n))}{f(n+1)} \quad \forall n \in \mathbb{N}.$$

**79.** Find all functions  $f : \mathbb{N} \rightarrow \mathbb{N}$  such that for all  $m, n \in \mathbb{N}$ ,

$$(2^m + 1)f(n)f(2^m n) = 2^m f(n)^2 + f(2^m n)^2 + (2^m - 1)^2 n.$$

**80.** Find all functions  $f : \mathbb{R} \rightarrow \mathbb{R}$  such that

$$f(x - f(y)) = f(f(y)) - 2xf(y) + f(x) \quad \forall x, y \in \mathbb{R}.$$

**81.** Find all functions  $f : \mathbb{R} \rightarrow \mathbb{R}$  such that

$$f(f(x) - y^2) = f(x)^2 - 2f(x)y^2 + f(f(y)) \quad \forall x, y \in \mathbb{R}.$$

**82.** Find all functions  $f : [0, +\infty) \rightarrow [0, +\infty)$  such that:

$$f(x + f(x) + 2y) = 2x + f(2f(y)) \quad \forall x, y \in [0, +\infty).$$

**83.** Find all functions  $f : \mathbb{R} \rightarrow \mathbb{R}$  such that

$$f(x^2) + f(xy) = f(x)f(y) + yf(x) + xf(x + y)$$

for all  $x, y \in \mathbb{R}$ .

**84.** Find all functions  $f : \mathbb{Q} \rightarrow \mathbb{Q}$  such that

$$f(x + f(x) + 2y) = 2x + 2f(f(y)) \quad \forall x, y \in \mathbb{Q}.$$

**85.** Find all functions  $f : \mathbb{R} \rightarrow \mathbb{R}$  such that

- $f\left(\frac{x+f(x)}{2} + y + f(2z)\right) = 2x - f(x) + f(f(f(y))) + 2f(f(z)) \quad \forall x, y, z \in \mathbb{R},$
- $f(f(0)) = f(0).$

**86.** Find all functions  $f : \mathbb{R}^+ \rightarrow \mathbb{R}^+$  which satisfy the following conditions:

- $f(x + f(y)) = f(x)f(y)$  for all  $x, y > 0$ ;
- there are at most finitely many  $x$  with  $f(x) = 1$ .

**87.** Find all functions  $f : \mathbb{N} \cup \{0\} \rightarrow \mathbb{N} \cup \{0\}$  such that for all  $m, n \in \mathbb{N} \cup \{0\}$ ,

$$mf(n) + nf(m) = (m + n)f(m^2 + n^2).$$

**88.** Find all functions  $f : (0, 1) \rightarrow \mathbb{R}$  such that

$$f(xyz) = xf(x) + yf(y) + zf(z)$$

for all real numbers  $x, y, z \in (0, 1)$ .

**89.** Find all functions  $f : \mathbb{Z} \mapsto \mathbb{Z}$  satisfying the condition:  $f(x^3 + y^3 + z^3) = f(x)^3 + f(y)^3 + f(z)^3$ .

**90.** Determine all real functions  $f(x)$  that are defined and continuous on the interval  $(-1, 1)$  and that satisfy the functional equation

$$f(x + y) = \frac{f(x) + f(y)}{1 - f(x)f(y)} \quad (x, y, x + y \in (-1, 1)).$$

**91.** Find all functions  $f : \mathbb{R} \rightarrow \mathbb{R}$  such that

$$f(x^n + 2f(y)) = (f(x))^n + y + f(y) \quad \forall x, y \in \mathbb{R}, \quad n \in \mathbb{Z}_{\geq 2}.$$

**92.** Find all functions  $f : \mathbb{R} \rightarrow \mathbb{R}$  such that

$$f(x^2 + y^2) = f(x^2) + f(y^2) + 2f(x)f(y) \quad \forall x, y \in \mathbb{R}.$$

**93.** Find all functions  $f : \mathbb{R} \rightarrow \mathbb{R}$  such that

$$f(x+y)f(x-y) = (f(x) + f(y))^2 - 4x^2f(y) \quad \forall x, y \in \mathbb{R}.$$

**94.** Find all injective functions  $f : \mathbb{N} \rightarrow \mathbb{R}$  such that

$$f(1) = 2, f(2) = 4, \text{ and } f(f(m) + f(n)) = f(f(m)) + f(n) \quad \forall m, n \in \mathbb{N}.$$

**95.** Find all functions  $f : \mathbb{R}^+ \rightarrow \mathbb{R}^+$  such that for any real numbers  $a, b, c, d > 0$  satisfying  $abcd = 1$ , we have

$$(f(a) + f(b))(f(c) + f(d)) = (a + b)(c + d).$$

**96.** Find all functions  $f : \mathbb{R} \rightarrow \mathbb{R}$  such that

$$f(x^2) \left( f(x)^2 + f\left(\frac{1}{y^2}\right) \right) = 1 + f\left(\frac{1}{xy}\right) \quad \forall x, y \in \mathbb{R} \setminus \{0\}.$$

**97.** Find all functions  $f : \mathbb{R} \rightarrow \mathbb{R}$  such that

$$f(f(x) - f(y)) = f(f(x)) - 2x^2f(y) + f(y^2) \quad \forall x, y \in \mathbb{R}.$$

**98.** Find all functions  $f : \mathbb{R}^+ \rightarrow \mathbb{R}^+$  such that

$$f(x+1) = f(x) + 1 \text{ and } f\left(\frac{1}{f(x)}\right) = \frac{1}{x} \quad \forall x, y \in \mathbb{R}^+.$$

**99.** Find all functions  $f : \mathbb{R} \rightarrow \mathbb{R}$  such that

$$f(x + f(x)f(y)) = f(x) + xf(y) \quad \forall x, y \in \mathbb{R}.$$

**100.** Find all continuous functions  $f : \mathbb{R} \rightarrow \mathbb{R}$  such that

$$f(x) + f(y) - f(x+y) = xy \quad \forall x, y \in \mathbb{R}.$$

### 3 Link to Solutions

1. <http://www.artofproblemsolving.com/Forum/viewtopic.php?t=410614>
2. <http://www.artofproblemsolving.com/Forum/viewtopic.php?t=320016>
3. <http://www.artofproblemsolving.com/Forum/viewtopic.php?t=320299>
4. <http://www.artofproblemsolving.com/Forum/viewtopic.php?t=320331>
5. <http://www.artofproblemsolving.com/Forum/viewtopic.php?t=319947>
6. <http://www.artofproblemsolving.com/Forum/viewtopic.php?t=321004>
7. <http://www.artofproblemsolving.com/Forum/viewtopic.php?t=321965>
8. <http://www.artofproblemsolving.com/Forum/viewtopic.php?t=321967>
9. <http://www.artofproblemsolving.com/Forum/viewtopic.php?t=323174>
10. <http://www.artofproblemsolving.com/Forum/viewtopic.php?t=323169>
11. <http://www.artofproblemsolving.com/Forum/viewtopic.php?t=328125>
12. <http://www.artofproblemsolving.com/Forum/viewtopic.php?t=324002>
13. <http://www.artofproblemsolving.com/Forum/viewtopic.php?t=324666>
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