國立竹東高級中學 104 學年度第 1 次教師甄選數學科部分試題 (共 11 題)

1. Let $\{a_i\}_{i=1}^n \subset \mathbb{R}$, where $m \le a_i \le M, \forall i \in \{1, 2, \cdots, n\}$ and $\sum\limits_{i=1}^n a_i = 0$. Prove

$$\frac{1}{n}\sum_{i=1}^n a_i^2 \le |mM|$$

2. Let $S = \{(x, y, z) \in \mathbb{R}^3 | x^2 + y^2 + z^2 = 1\}$, $\{P_i\}_{i=1}^n \subset S$, $O = (0, 0, 0)$, such that $\sum\limits_{i=1}^n \overrightarrow{OP_i} = \overrightarrow{0}$. Prove

$$\sum_{i=1}^n \overline{AP_i} \ge n$$

3. Let $a, b \in \mathbb{R}$ such that $\sin(a\cos x) \ne \cos(b\sin x), \forall x \in \mathbb{R}$. Prove

$$a^2 + b^2 < \frac{\pi^2}{4}$$

4. Let $\{x_i\}_{i=1}^5 \subset \mathbb{R}$, satisfy $\sum\limits_{i=1}^5 x_i = 8$ and $\sum\limits_{i=1}^5 x_i^2 = 16$. Find the maximum extreme value of $2x_1 + x_2$

5. 已知 $\begin{cases} x + y = 7 \\ x^2 + xy + y^2 = 25 \\ y^2 + yz + z^2 = 36 \end{cases}$, 求出 $xy + yz$ 的值.