

計算 2 :  $a_1 = 1, a_{n+1} = 2a_n + n^2$

$$\text{令 } [a_{n+1} + H(n+1)] = 2[a_n + H(n)] \Rightarrow a_{n+1} = 2a_n + 2H(n) - H(n+1)$$

$$\text{令 } H(n) = an^2 + bn + c \Rightarrow 2H(n) - H(n+1) = n^2 \Rightarrow H(n) = n^2 + 2n + 3$$

$$[a_n + H(n)] = 2[a_{n-1} + H(n-1)]$$

$$[a_{n-1} + H(n-1)] = 2[a_{n-2} + H(n-2)]$$

$$[a_{n-2} + H(n-2)] = 2[a_{n-3} + H(n-3)]$$

...

$$[a_2 + H(2)] = 2[a_1 + H(1)]$$

相乘

$$\Rightarrow [a_n + H(n)] = 2^{n-1}[a_1 + H(1)] = 2^{n-1} \cdot 7$$

$$\Rightarrow a_n = 2^{n-1} \cdot 7 - (n^2 + 2n + 3)$$