

12. 設  $a_1, a_2, a_3, \dots, a_{104}$  為一等差數列， $b_1, b_2, b_3, \dots, b_{104}$  為一等比數列，若級數

$$a_1 + a_2 + a_3 + \dots + a_{104} = 2015, \quad b_1 + b_2 + b_3 + \dots + b_{104} = 520, \quad \text{且兩數列滿足}$$

$$a_1 b_1 + a_2 b_2 + a_3 b_3 + \dots + a_{104} b_{104} = 20000, \quad \text{求 } a_1 b_{104} + a_2 b_{103} + a_3 b_{102} + \dots + a_{104} b_1 = \underline{\hspace{2cm}} \circ$$

【104.文華高中】

【解】：

$$a_1 + a_{104} = a_2 + a_{103} = a_3 + a_{102} = \dots = a_{52} + a_{53} = \frac{2015}{52}$$

$$\begin{aligned} & (a_1 b_1 + a_2 b_2 + a_3 b_3 + \dots + a_{104} b_{104}) + (a_1 b_{104} + a_2 b_{103} + a_3 b_{102} + \dots + a_{104} b_1) \\ &= (a_1 + a_{104})b_1 + (a_2 + a_{103})b_2 + (a_3 + a_{102})b_3 + \dots + (a_{104} + a_1)b_{104} \\ &= \frac{2015}{52}(b_1 + b_2 + b_3 + \dots + b_{104}) = \frac{2015}{52} \times 520 = 20150 \end{aligned}$$

$$\text{故所求 } a_1 b_{104} + a_2 b_{103} + a_3 b_{102} + \dots + a_{104} b_1 = 150$$