

令

$$PA = PB = x, PO = \sqrt{x^2 + 1}$$

$$\cos \angle APB = \cos 2\angle APO = 2\cos^2 \angle APO - 1 = \frac{x^2 - 1}{x^2 + 1}$$

$$\overline{PA} \cdot \overline{PB} = x \times x \times \frac{x^2 - 1}{x^2 + 1} = \frac{x^4 - x^2}{x^2 + 1} = x^2 + 1 + \frac{2}{x^2 + 1} - 3 \geq 2\sqrt{2} - 3$$

等號成立於 $x = \sqrt{\sqrt{2} - 1}$