

$$\begin{aligned}
& \int x^2 \sqrt{r^2 - x^2} dx \\
&= -\frac{1}{2} \int x \sqrt{r^2 - x^2} d(r^2 - x^2) \\
&= -\frac{1}{3} \int x d \left((r^2 - x^2)^{\frac{3}{2}} \right) \\
&= -\frac{1}{3} x (r^2 - x^2)^{\frac{3}{2}} + \frac{1}{3} \int (r^2 - x^2)^{\frac{3}{2}} dx \\
&= -\frac{1}{3} x (r^2 - x^2)^{\frac{3}{2}} + \frac{r^2}{3} \int \sqrt{r^2 - x^2} dx - \frac{1}{3} \int x^2 \sqrt{r^2 - x^2} dx
\end{aligned}$$

$$\int x^2 \sqrt{r^2 - x^2} dx = -\frac{1}{4} x (r^2 - x^2)^{\frac{3}{2}} + \frac{r^2}{4} \left(\frac{x}{2} \sqrt{r^2 - x^2} + \frac{r^2}{2} \arcsin \frac{x}{r} \right) + C$$

$$\int_0^r x^2 \sqrt{r^2 - x^2} dx = \frac{\pi}{16} r^4$$