

第 2 題

$$\text{令 } a = \frac{x+y}{2}, b = \sqrt{xy} \text{ , 則 } \frac{2xy}{x+y} = \frac{b^2}{a}$$

$$a + b + \frac{b^2}{a} = 49$$

$$a^2 + (b - 49)a + b^2 = 0$$

$$(b - 49)^2 - 4b^2 \geq 0$$

$$3b^2 + 98b - 49^2 \leq 0$$

$$(b + 49)(3b - 49) \leq 0$$

$$b \leq 16$$

$$b = 1, 2, 3, \dots$$

——帶入 $a^2 + (b - 49)a + b^2 = 0$ 檢驗

易知

$$a = 28, b = 14, x = 28 + 14\sqrt{3}$$

$$a = 25, b = 15, x = 45$$