第二大題第 4 題

射中 $C_{i+1} - C_i$ 地帶的機率 $= \dfrac{\pi\left(\dfrac{i+1}{n}\right)^2 - \pi\left(\dfrac{i}{n}\right)^2}{\pi\left(\dfrac{n}{n}\right)^2} = \dfrac{2i+1}{n^2}$

所求

$= \displaystyle\sum_{i=0}^{n-1}\left[(n-i)\times\dfrac{2i+1}{n^2}\right]$

$= \dfrac{1}{n^2}\left[-2\times\dfrac{(n-1)n(2n-1)}{6}+(2n-1)\times\dfrac{(n-1)n}{2}+n\times n\right]$

$= \dfrac{(n+1)(2n+1)}{6n}$