

第 2 題

$$\begin{aligned}\lim_{n \rightarrow \infty} P_n &= \lim_{n \rightarrow \infty} \sum_{k=1}^n \left[\frac{1}{n} \times \left(\frac{n+k}{2n} \right)^3 \right] \\ &= \lim_{n \rightarrow \infty} \sum_{k=1}^n \left[\frac{1}{n} \times \left(\frac{1}{2} + \frac{1}{2} \times \frac{k}{n} \right)^3 \right] \\ &= \int_0^1 \left(\frac{1}{2} + \frac{1}{2} x \right)^3 dx \\ &= \frac{15}{32}\end{aligned}$$