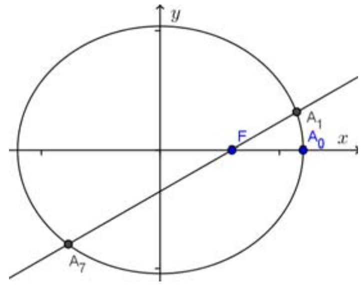
9.設橢圓曲線$\Gamma:\dfrac{x^2}{36}+\dfrac{y^2}{27}=1$與直線$L: x=12$，若$A_0$，$F$的坐標分別為$(6,0),(3,0)$，在曲線$\Gamma$上另有 11 個點$A_k$，$k=1,2,3,\cdots,11$使得$\angle A_0FA_1=\angle A_1FA_2=\cdots=\angle A_{11}FA_0$，令$d_k$為$A_k$到$L$的距離，試求$\sum_{k=0}^{11}\dfrac{1}{d_k}=$______。【100.中壢高中二招　◆◇◇圓錐曲線方程式】



【解】：如右圖所示，過F的任意一直線（斜率為m）交橢圓與兩點$A_1(x_1,y_1)$、$A_7(x_2,y_2)$【12 個點中的兩點】

$$\begin{cases} y=m(x-3) \\ \dfrac{x^2}{36}+\dfrac{y^2}{27}=1 \end{cases} \Rightarrow (3+4m^2)x^2-24m^2x+36m^2-108=0$$

(1)將根平移$x'=x-12$：$(3+4m^2)x^2+(72m^2+72)x+(324m^2+324)=0$

(2)取負根$x''=-x'=12-x$：$(3+4m^2)x^2-(72m^2+72)x+(324m^2+324)=0$

(3)取倒根$x'''=\dfrac{1}{x''}=\dfrac{1}{12-x}$：$(324m^2+324)x^2-(72m^2+72)x+(3+4m^2)=0$

故可得知$\dfrac{1}{12-x_1}+\dfrac{1}{12-x_7}=\dfrac{72m^2+72}{324m^2+324}=\dfrac{2}{9}$（為一定值）

$$\sum_{k=0}^{11}\frac{1}{d_k}=\frac{1}{12-d_0}+\frac{1}{12-d_1}+\cdots+\frac{1}{12-d_{11}}=(\frac{1}{12-d_1}+\frac{1}{12-d_7})+\cdots+(\frac{1}{12-d_6}+\frac{1}{12-d_0})$$

$$=\frac{2}{9}\times 6=\frac{4}{3}$$