1.開口向下之二次函數且對稱軸$x=3$有最大值，故選(3)

2.實係數方程，共軛虛根知x^2-4x+5爲其因式，所以

$$x^3-ax^2-bx-10=(x^2-4x+5)(x-2)=x^3-6x^2+13x-10=0$$

3.(1) $\det(cA)=c^n\cdot\det(A)$ (2) $\det(\begin{vmatrix}1&0\\0&1\end{vmatrix})=1\neq\det(\begin{vmatrix}1&0\\0&0\end{vmatrix})+\det(\begin{vmatrix}0&0\\0&1\end{vmatrix})=0$ (3) $\det(A^T)=\det(A)$

故選(4)

4. $\pi\int_1^4(\sqrt{x}-1)^2dx=\pi\left[\frac{1}{2}x^2-\frac{4}{3}x^{\frac{3}{2}}+x\right]_1^4=\frac{7}{6}\pi$

5.乙：三場內獲勝+四場內獲勝+5 場內獲勝

$$P=(\frac{2}{5})^3+C_1^3\cdot\frac{3}{5}\cdot(\frac{2}{5})^3+C_2^4\cdot(\frac{3}{5})^2\cdot(\frac{2}{5})^3=(\frac{2}{5})^3\cdot(1+\frac{9}{5}+\frac{54}{25})=\frac{8}{125}\cdot\frac{124}{25}=\frac{992}{3125}$$

6. $\overrightarrow{AH}=\overrightarrow{AO}+\overrightarrow{OH}=-\overrightarrow{OA}+\overrightarrow{OA}+\overrightarrow{OB}+\overrightarrow{OC}=\overrightarrow{OB}+\overrightarrow{OC}$

$\because \overline{OB}=\overline{OC}\therefore\overrightarrow{AH}\perp\overrightarrow{BC}$，同理$\overrightarrow{BH}\perp\overrightarrow{AC}$，$\overrightarrow{CH}\perp\overrightarrow{AB}$，$H$爲三高交點，故選(4)

7. $\angle ACB=90^\circ$ $\overline{CD}=2\sqrt{25^2-(\frac{40\times30}{50})^2}=14$，故選(4)

8. $\frac{\Delta_x}{\Delta}=\frac{\begin{vmatrix}c_1&b_1\\c_2&b_2\end{vmatrix}}{\begin{vmatrix}a_1&b_1\\a_2&b_2\end{vmatrix}}=2$, $\frac{\Delta_y}{\Delta}=\frac{\begin{vmatrix}a_1&c_1\\a_2&c_2\end{vmatrix}}{\begin{vmatrix}a_1&b_1\\a_2&b_2\end{vmatrix}}=-3$

$$\Rightarrow(x,y)=(\frac{\Delta_x}{\Delta},\frac{\Delta_y}{\Delta})=(\frac{\begin{vmatrix}6c_1&-2a_1\\6c_2&-2a_2\end{vmatrix}}{\begin{vmatrix}3b_1&-2a_1\\3b_2&-2a_2\end{vmatrix}},\frac{\begin{vmatrix}3b_1&6c_1\\3b_2&6c_2\end{vmatrix}}{\begin{vmatrix}3b_1&-2a_1\\3b_2&-2a_2\end{vmatrix}})=(\frac{12}{6}\cdot\frac{\Delta_y}{\Delta},\frac{-18}{6}\cdot\frac{\Delta_x}{\Delta})=(-6,-6)$$

9. $\Delta=\begin{vmatrix}1&2&-3\\1&3&-2\\1&a&b\end{vmatrix}=-a+b+5$, $\Delta_x=\begin{vmatrix}1&2&-3\\-1&3&-2\\1&a&b\end{vmatrix}=5a+5b+5$, $\Delta_y=\begin{vmatrix}1&1&-3\\1&-1&-2\\1&1&b\end{vmatrix}=-2b-6$

$\Delta_z=\begin{vmatrix}1&2&1\\1&3&-1\\1&a&1\end{vmatrix}=2a-4$

(1) $\Delta\neq0$,恰有一組解 (2) $\Delta=0$, $\Delta_x\neq0$,無解 (3) $\Delta=0$, $\Delta_x\neq0$,無解

(4) $\Delta=0$, $\Delta_x=\Delta_y=\Delta_z=0$,無限多解，故選(3)

10.$\because$ $\overline{PF_1}$ 中點在短軸上，$\therefore$由相似性質 $\overline{PF_2}$ 爲正焦弦的一半，所求$\frac{1}{2}\cdot\frac{2b^2}{a}=\frac{2(\sqrt{5})^2}{2\times 5}=1$

11. $a=\sqrt{2^2+3^2}=\sqrt{13}$ ，故選(2)

12.(1) $(\begin{vmatrix}1 & 0\\1 & 1\end{vmatrix},\begin{vmatrix}0 & 2\\1 & 0\end{vmatrix},\begin{vmatrix}2 & 1\\0 & 1\end{vmatrix})\cdot(2,3,1)=-2$ (2) $(\begin{vmatrix}-1 & 0\\1 & 1\end{vmatrix},\begin{vmatrix}0 & 1\\1 & 0\end{vmatrix},\begin{vmatrix}1 & -1\\0 & 1\end{vmatrix})\cdot(2,3,1)=-4$

(3) $(-1,1,-1)\cdot(2,3,1)=0$ (4) $(2,1,1)\cdot(2,3,1)=8$

故選(3)

13. $50\times\frac{1}{6}=8.\bar{3}$，由大數法則選(1)

14. $\frac{1}{\sqrt{k+1}}<\frac{1}{\sqrt{k}}<\frac{1}{\sqrt{k-1}}\Rightarrow\frac{1}{\sqrt{k}+\sqrt{k+1}}<\frac{1}{2\sqrt{k}}<\frac{1}{\sqrt{k}+\sqrt{k-1}}$

$\Rightarrow 2(\sqrt{k+1}-\sqrt{k})<\frac{1}{\sqrt{k}}<2(\sqrt{k}-\sqrt{k-1})\Rightarrow\sum 2(\sqrt{k+1}-\sqrt{k})<\sum\frac{1}{\sqrt{k}}<\sum 2(\sqrt{k}-\sqrt{k-1})$

$\Rightarrow 198<2(\sqrt{n+1}-1)=2(\sqrt{10001}-1)<\sum\frac{1}{\sqrt{k}}<2\sqrt{n}=2\sqrt{10000}=200$

故選(2)

15. $3^{4\times 3}=3^{12}$

16.相鄰兩數間差值固定(1)10 (2)10 (3)9 (4)11，可知(3)標準差最小，故選(3)

17.(1) $2\sin(60°+74°)=2\sin 134°$ (2) $2\sin(60°+64°)=2\sin 124°$

(3) $2\sin(60°+54°)=2\sin 114°$ (4) $2\sin(60°+44°)=2\sin 104°$

故選(1)

18. $\lim\limits_{x\to\frac{\pi}{4}}\frac{\sec x-\sqrt{2}}{x-\frac{\pi}{4}}\overset{*}{=}\lim\limits_{x\to\frac{\pi}{4}}\sec x\tan x=\sqrt{2}\cdot 1=\sqrt{2}$

19. $\lim\limits_{\substack{x\to\infty\\y\to 2}}(1+\frac{1}{x})^{\frac{x^2}{x+y}}=\lim\limits_{x\to\infty}(1+\frac{1}{x})^{\frac{x^2}{x}}=\lim\limits_{x\to\infty}(1+\frac{1}{x})^{x}=e$

20. $\varepsilon^3=1$ 且 $1+\varepsilon+\varepsilon^2=0$

$$\begin{vmatrix}1 & 1 & \varepsilon\\1 & 1 & \varepsilon^2\\\varepsilon^2 & \varepsilon & 1\end{vmatrix}=1+\varepsilon^4+\varepsilon^2-\varepsilon^3-\varepsilon^3-1=1+\varepsilon+\varepsilon^2-1-1-1=-3$$

21. $\log 875^{16}=16(3\log 5+\log 7)=16(3\times 0.699+0.8451)=47.0736$ ，有 47+1=48 位數

22.(1)Ratio test $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|=\lim_{n\to\infty}\left|\frac{100^{n+1}}{(n+1)!}\cdot\frac{n!}{100^n}\right|=\lim_{n\to\infty}\frac{100}{n+1}=0$ ，收斂

(2)Ratio test $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|=\lim_{n\to\infty}\left|\frac{[(n+1)!]^2}{[2(n+1)]!}\cdot\frac{(2n)!}{(n!)^2}\right|=\lim_{n\to\infty}\frac{(n+1)^2}{(2n+1)(2n+2)}=\lim_{n\to\infty}\frac{n+1}{2(2n+1)}=\frac{1}{4}$ ，收斂

(3)Ratio test $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|=\lim_{n\to\infty}\left|\frac{(n+1)!}{(n+1)^{n+1}}\cdot\frac{n^n}{n!}\right|=\lim_{n\to\infty}\frac{n^n}{(n+1)^n}=\lim_{n\to\infty}\left(\frac{n}{n+1}\right)^n=\lim_{n\to\infty}(\frac{1}{1+\frac{1}{n}}^n)=e^{-1}$ ，收斂

(4) $\sum_{n=1}^{\infty}\frac{1}{3n}=\sum_{n=1}^{\infty}(\frac{1}{3n}+\frac{1}{3n}-\frac{1}{3n})<\sum_{n=1}^{\infty}(\frac{1}{3n-2}+\frac{1}{3n-1}-\frac{1}{3n})$ 又 $\sum_{n=1}^{\infty}\frac{1}{3n}=\frac{1}{3}\sum_{n=1}^{\infty}\frac{1}{n}$ 發散，

所以 $\sum_{n=1}^{\infty}(\frac{1}{3n-2}+\frac{1}{3n-1}-\frac{1}{3n})$ 發散，故選(4)

23. $\because 11^{16}\equiv 1(\bmod 17)\ \therefore 11^{104}+1=(11^{16})^6\cdot 11^8+1\equiv(-6)^8+1=(34+2)^4+1\equiv 16+1\equiv 0$

24.9：(09,81,29,61,49,41,69,21,89,01)，則 $9^{9^9}=9^{100k+89}\equiv 9^{89}\equiv 9^9\equiv 89(\bmod 100)$

25.(1)① $x=1$ 代入 $1^3+6\times 1^2+3\times 1+3\neq 0$ ② $x=-1$ 代入 $(-1)^3+6\times(-1)^2+3\times(-1)+3\neq 0$
③ $x=3$ 代入 $3^3+6\times 3^2+3\times 3+3\neq 0$ ④ $x=-3$ 代入 $(-3)^3+6\times(-3)^2+3\times(-3)+3\neq 0$
(2)① $x=1$ 代入 $1^5-5\times 1^3+15\neq 0$ ② $x=-1$ 代入 $(-1)^5-5\times(-1)^3+15\neq 0$
③ $x=3$ 代入 $3^5-5\times 3^3+15\neq 0$ ④ $x=-3$ 代入 $(-3)^5-5\times(-3)^3+15\neq 0$
⑤ $x=5$ 代入 $5^5-5\times 5^3+15\neq 0$ ⑥ $x=-5$ 代入 $(-5)^5-5\times(-5)^3+15\neq 0$
⑦ $x=15$ 代入 $15^5-5\times 15^3+15\neq 0$ ⑧ $x=-15$ 代入 $(-15)^5-5\times(-15)^3+15\neq 0$
(3)① $x=1$ 代入 $1^4+1^3+1^2+1+1\neq 0$ ② $x=-1$ 代入 $(-1)^4+(-1)^3+(-1)^2+(-1)+1\neq 0$
有三個，故選(4)

26.5、6 或 8 的倍數

$\left[\frac{1000}{5}\right]+\left[\frac{1000}{6}\right]+\left[\frac{1000}{8}\right]-\left[\frac{1000}{30}\right]-\left[\frac{1000}{24}\right]-\left[\frac{1000}{40}\right]+\left[\frac{1000}{120}\right]$

$=200+166+125-33-41-25+8=400$

所求 1000－400=600

27. $C_2^4=6$

28. $H_6^3=C_6^8=28$

29. $C_0^5+C_2^5+C_4^5=1+10+5=16$

30. $3^4=81$

31. $x=y+\ln y\Rightarrow 1=y'+\frac{y'}{y}\Rightarrow y'=\frac{y}{1+y}$ ，故選(3)

32. $y^2+2\ln y=x^4\Rightarrow 2ydy+\dfrac{2dy}{y}=4x^3dx\Rightarrow\dfrac{dy}{dx}=4x^3\cdot\dfrac{y}{2y^2+2}=\dfrac{2x^3y}{y^2+1}$

33. $10^{10}=2^{10}\cdot 5^{10}$，所求(10+1)(10+1)=121

34. $n^2-2n=\pm1,\pm2,\pm3,\pm6$有整數解，則$n^2-2n=-1\Rightarrow n=1$或$n^2-2n=3\Rightarrow n=3,-1$
所求$1+3+(-1)=3$

35. $50\times4.2=210$，中位數爲a_{25}、a_{26}的平均值
(4) $26\times7.5=195\Rightarrow 210-195=15<24$(不合)　(3) $26\times7=182\Rightarrow 210-182=28>24$
故選(3)

36. $f(x)=\left|\log_{\frac{1}{2}}x\right|\Rightarrow x\in[\frac{1}{2},4],\ f(x)\in\left|[-2,1]\right|\Rightarrow f(x)\in[0,2]$，所求 2－0=2

37. $\lim\limits_{n\to\infty}\dfrac{1}{\sqrt[n]{|a_n|}}=\lim\limits_{n\to\infty}\dfrac{1}{\sqrt[n]{\left|\frac{1}{2^n}\right|}}=\lim\limits_{n\to\infty}2=2\Rightarrow\left|x^n-0\right|<2\Rightarrow|x|<\sqrt[n]{2}\doteq1$，故選(4)

38. $\int e^x\sin xdx=\sin xe^x-\int e^x\cos xdx=\sin xe^x-\cos xe^x-\int e^x\sin xdx$

$\Rightarrow\int_0^\pi e^x\sin xdx=\dfrac{1}{2}\left[\sin xe^x-\cos xe^x\right]_0^\pi=\dfrac{1}{2}(e^\pi+1)$

39.橢圓定義，到兩定點距離和爲定値，故選(3)

40.分爲兩對皆分別同組及僅有一對在同組

$$\frac{\dfrac{C_2^{16}\cdot C_2^{14}\cdot C_2^{12}\cdot C_2^{10}\cdot C_2^{8}\cdot C_2^{6}\cdot C_2^{4}\cdot C_2^{2}}{8!}+\dfrac{C_1^{2}\cdot16\cdot15\cdot C_2^{14}\cdot C_2^{12}\cdot C_2^{10}\cdot C_2^{8}\cdot C_2^{6}\cdot C_2^{4}\cdot C_2^{2}}{7!}}{\dfrac{C_2^{20}\cdot C_2^{18}\cdot C_2^{16}\cdot C_2^{14}\cdot C_2^{12}\cdot C_2^{10}\cdot C_2^{8}\cdot C_2^{6}\cdot C_2^{4}\cdot C_2^{2}}{10!}}$$

$$=\frac{\dfrac{C_2^{16}}{8}+\dfrac{2\times16\times15}{1}}{\dfrac{C_2^{20}\cdot C_2^{18}\cdot C_2^{16}}{10\times9\times8}}=\frac{15+2\times16\times15}{19\times17\times15}=\frac{1+2\times16}{19\times17}=\frac{33}{19\times17}$$

41. $\dfrac{1}{2}=\dfrac{2^2+\overline{BC}^2-3^2}{2\cdot2\cdot\overline{BC}}\Rightarrow\overline{BC}=1+\sqrt{6}$，所以$\Delta ABC=\dfrac{1}{2}\cdot2\cdot(1+\sqrt{6})\cdot\sin60^\circ=\dfrac{\sqrt{3}+\sqrt{18}}{2}$

42.令$f(x)=(x^2+1)(ax+b)+2x-1$將$x=\dfrac{1}{2}$代入$\dfrac{5}{4}(\dfrac{a}{2}+b)+1-1=5\Rightarrow\dfrac{a}{2}+b=4\Rightarrow a+2b=8$

43. $A\begin{bmatrix}2 & 0\\ 1 & -1\end{bmatrix}=\begin{bmatrix}-1 & 2\\ 2 & 1\end{bmatrix}\Rightarrow A=\begin{bmatrix}-1 & 2\\ 2 & 1\end{bmatrix}\cdot\frac{1}{-2}\cdot\begin{bmatrix}-1 & 0\\ -1 & 2\end{bmatrix}=\begin{bmatrix}\frac{1}{2} & -2\\ \frac{3}{2} & -1\end{bmatrix}$

所求 $x=A^{-1}b=\frac{2}{5}\begin{bmatrix}-1 & 2\\ -\frac{3}{2} & \frac{1}{2}\end{bmatrix}\begin{bmatrix}1\\ 1\end{bmatrix}=\begin{bmatrix}\frac{2}{5}\\ -\frac{2}{5}\end{bmatrix}$

44. $f(x)$ 為連續函數且在閉區間，所以有最大值及最小值存在→(A)、(B)正確
(C)不一定(D)不一定，故選(4)

45.兩邊平方 $x-5=m^2x_2+4mx+4\Rightarrow(4m-1)^2-4m^2\cdot 9>0\Rightarrow(10m-1)(-2m-1)>0$
$-0.5<m<0.1$繪圖可知$m>0\ \therefore 0<m<0.1$，故選(1)

46. $\cos\theta=\frac{(1,2,0)\cdot(3,2,6)}{\sqrt{5}\cdot\sqrt{49}}=\frac{7}{7\sqrt{5}}=\frac{\sqrt{5}}{5}$

47.① $n=5k+3\Rightarrow -10\sim 9$ 共 20 個 ② $n=3m+1\Rightarrow -17\sim 16$ 共 34 個
③ $n=15t-2\Rightarrow -3\sim 3$ 共 7 個(被 3 除餘 1 且被 5 除餘 3)
所求 $20+34-7=47$

48.(1)反例：折線，故選(1)

49.$(1,-1\ )$、$(3,0)$、$(3,2)$、$(1,1)$為平行四邊形，所求 $(2-0)\times(3-1)=4$

50. $x^2+y^2-2x+4y=0\Rightarrow(x-1)^2+(y+2)^2=5$ 且 $\sqrt{(5-1)^2+(1-(-2))^2}=5$

$\therefore 2\cdot\frac{1}{2}\cdot\sqrt{5}\cdot 2\sqrt{5}=\frac{1}{2}\cdot 5\cdot\overline{AB}\Rightarrow\overline{AB}=4$