

2007 台大資工

$$\begin{aligned}& \sum_{n=1}^{\infty} \frac{4^{2^{n-1}}}{4^{2^n} - 1} \\&= \frac{4}{(4-1)(4+1)} + \frac{4^2}{(4-1)(4+1)(4^2+1)} + \frac{4^4}{(4-1)(4+1)(4^2+1)(4^4+1)} + \cdots + \frac{4^{2^{n-1}}}{(4-1)(4+1)(4^2+1)(4^4+1)\cdots(4^{2^{n-1}}+1)} + \cdots \\&= \left( \frac{1}{(4-1)} - \frac{1}{(4-1)(4+1)} \right) + \left( \frac{1}{(4-1)(4+1)} - \frac{1}{(4-1)(4+1)(4^2+1)} \right) + \left( \frac{1}{(4-1)(4+1)(4^2+1)} - \frac{1}{(4-1)(4+1)(4^2+1)(4^4+1)} \right) + \cdots \\&+ \left( \frac{1}{(4-1)(4+1)(4^2+1)(4^4+1)\cdots(4^{2^{n-2}}+1)} - \frac{1}{(4-1)(4+1)(4^2+1)(4^4+1)\cdots(4^{2^{n-2}}+1)(4^{2^{n-1}}+1)} \right) + \cdots \\&= \frac{1}{3}\end{aligned}$$

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