

國立南科國際實驗高級中學 九十八學年度第一次教師甄選

雙語部高中數學科試題卷

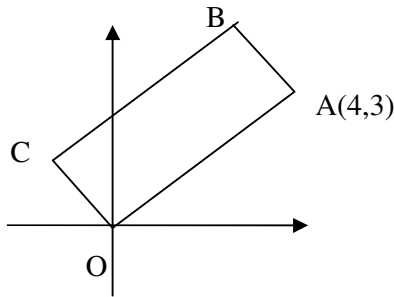
一、填充題：(每題 5 分，共 60 分)

1. Let $A(1,n)$ 、 $B(n,1)$ 、 $C(\frac{1}{n}, \frac{1}{n})$, let A_n represent the area of $\triangle ABC$, calculate

the value of $\lim_{n \rightarrow \infty} \frac{A_n}{n^2} = \underline{\frac{1}{2}}$ 。

2. Calculate $\cos \frac{\pi}{17} \cdot \cos \frac{2\pi}{17} \cdot \cos \frac{4\pi}{17} \cdot \cos \frac{8\pi}{17} = \underline{\frac{1}{16}}$

3. In the coordinates plane, $OABC$ is a rectangle, let $A(4,3)$, $\overline{OC} = 2$, then the coordinates of point B is (2,8,4,6)。



4. Let the function $f(x) = \frac{8}{\sqrt{x-2}}$, the equation of the tangent line at the point (6,4) is $x+2y=14$

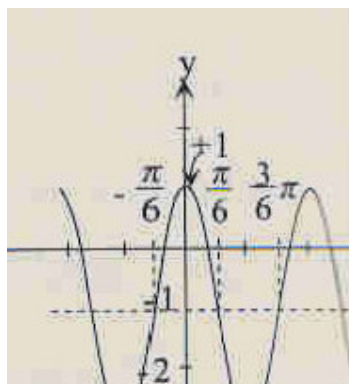
5. To simplify the complex number $\frac{(2+2i)^4}{(1-\sqrt{3}i)^5}$, which is the answer?

(A) $1+\sqrt{3}i$ (B) $-1+\sqrt{3}i$ (C) $1-\sqrt{3}i$ (D) $-1-\sqrt{3}i$ (E) $1+2i$ 。ANS: B

6. Find the function show in the graph?

(A) $y=2\sin(x+\frac{\pi}{2})+1$

(B) $y=2\sin(3x+\frac{\pi}{2})-1$



$$(C)y=3\sin(3x+\pi)-1$$

$$(D)y=2\sin(4x+\frac{\pi}{2})-1$$

$$(E)y=3\sin(4x-\frac{\pi}{2})-1$$

ANS : B

7. Let the matrix $A = \begin{bmatrix} 1 & -2 & -3 \\ 2 & 1 & 3 \\ 3 & -2 & -2 \end{bmatrix}$, if $AX = \begin{bmatrix} -1 & 0 \\ 2 & 1 \\ 0 & 1 \end{bmatrix}$, then $X = \underline{\underline{\begin{bmatrix} 0 & 1 \\ -1 & 2 \\ 1 & -1 \end{bmatrix}}}$

8. Which is the value of the determinant $\begin{vmatrix} 100 & 101 & 102 \\ 26 & 24 & 25 \\ 53 & 51 & 52 \end{vmatrix}$?

(A)0 (B)49 (C)64 (D)81

ANS : (D)

9. Let two lines $L_1 : \frac{x-4}{2} = \frac{y-1}{4} = \frac{z-1}{3}$ and $L_2 : \frac{x-3}{2} = \frac{y+3}{5} = \frac{z-2}{4}$, the shortest distance between two lines is 3 °.

10. Evaluate the integral $\int_0^2 \frac{2x+4}{x^2+4x+3} dx = \underline{\ln 5}$

11. Evaluate the integral $\int_0^{\pi/3} \cos x dx = \underline{\underline{\frac{\sqrt{3}}{2}}}$

12. Let $f(x) = \ln(\sin x)$, $0 < x < \pi$, then $f'(\frac{\pi}{3}) = \underline{\underline{\frac{\sqrt{3}}{3}}}$

二、判斷題：請判斷下列各敘述是否正確，若正確請證明，若錯誤請舉例說明。

(每題 8 分，共 16 分)

1. Let $f(x)$ be defined on an open interval about c . If the limit of the function $f(x)$ exists at $x=c$, then the function $f(x)$ is continuous at $x=c$.

2. Let $f(x)$ be defined on an open interval about c . If $f'(x)$ is

defined at c and $f'(c) = 0$, then the function $f(x)$ has a local maximum or minimum value at $x=c$.

三、計算證明題：(每題 8 分，共 24 分)

1. Let $a > 0$, the region between $f(x) = x^2 - ax$ and the x -axis is called by R . The volume of the solid generated by revolving the region R about the line $y=0$ is A ; the volume of the solid generated by revolving the region R about the line $x=0$ is B . If $A=B$, calculate the value of a ?

ANS : $a=5$

2. Let $f(x) = x^3 + 3x^2 - 9x$, $x \in R$,

(1) Find the local maximum or minimum value of $f(x)$?

(2) If the equation $f(x) - 2k + 1 = 0$ has 3 distinct real roots, find the region of k ?

(1) 在 $x=1$ 時有極小值 -5

在 $x=-3$ 時有極大值 27

(2) $-2 < k < 14$

3. Let the plane equation $E: ax + by + cz + d = 0$, point $A(x_0, y_0, z_0)$ is not at the plane, prove: the distance from point A to E is $\frac{|ax_0 + by_0 + cz_0 + d|}{\sqrt{a^2 + b^2 + c^2}}$