國立南科國際實驗高級中學 112 學年度第 1 次正式教師甄選試題卷

科目：雙語部 中等教師數學專長 **數學專業能力**

一、 選擇題：5 題，每題 2 分，共 10 分

1. ( ) How many pairs of positive integers (x, y) are there satisfying $x^2 + y^2 = 6x + 8y$?

   (A) 4 (B) 5 (C) 7 (D) 12

2. ( ) Given $f(x) = x^3 + 3x^2 - 2$. How many distinct real roots of $f(f(x))$?

   (A) 3 (B) 5 (C) 7 (D) 9

3. ( ) Define $t = \frac{2^2}{1\times3} + \frac{4^2}{3\times5} + \cdots + \frac{1314^2}{1313\times1315}$ the notation [t] means "the greatest integer not exceeding t" and {t} means "the fractional part of t". Which of the following is false?

   (A) $t > 1$ (B) $2t > 1314$ (C) $[t] = 657$ (D) $\{t\} > \frac{1}{2}$.

4. ( ) Known a, b, c are side of ΔABC and $\log_a b$ is a double root of equation $x^2 - 2x + sinC + cosC = 0$ then which is correct in the followings?

   (A) acute triangle (B) equilateral triangle

   (C) obtuse angled triangle (D) isosceles right-angled triangle.

5. ( ) Choose the correct relations of the values of following definite integrals.

   $I_1 = \int_{-\infty}^{0} e^{-(x-1)^2}\,dx,\ I_2 = \int_{0}^{\infty} e^{-(x-1)^2}\,dx,\ I_3 = \int_{-\infty}^{\infty} e^{-(2x-1)^2}\,dx,\ I_4 = \int_{-\infty}^{\infty} e^{-2(x-1)^2}\,dx$

   (A) $I_1 < I_2,\ I_3 < I_4$ (B) $I_1 < I_2,\ I_3 > I_4$ (C) $I_1 > I_2,\ I_3 < I_4$ (D) $I_1 > I_2,\ I_3 > I_4$

二、 計算題：8 題，每題 5 分，共 40 分

1. Given a polynomial $f(x)$ whose degree is no less than 3. The remainders of $f(x)$ being divided by $(x-a)(x-b)$, $(x-b)(x-c)$, $(x-a)(x-c)$ are $-x-5$, $4x$, $3x+3$ respectively. Find the remainder of $f(x)$ being divided by $(x-a)(x-b)(x-c)$.

2. How many nonnegative integer solutions are there to the equation $x + 2y + 3z = 17$?

3. Find the largest possible volume of the tetrahedron PQRS, where $P(1,0,0)$, $Q(0,2,0)$, $R(0,0,3)$ and S lies on the unit sphere centered at the origin.

4. The parametric curve $(x, y) = (2\cos\theta, 2\sin\theta + 3\cos\theta)$, $0 \le \theta \le 2\pi$ represents an ellipse. Find the foci that lie in the first quadrant.

5. Let $f(x) = x(x-1)(x-2)\ldots(x-2023)$ and $g(x) = f\big(f(x)\big)$. Calculate $g'(1)$.

6. Find all $\boldsymbol{a}$ such that both of equations $x^2 + \boldsymbol{a}x + 1996 = 0$ and $x^2 + 1996x + \boldsymbol{a} = 0$ has two integer roots.

7. Appling Gram-Schmidt method, it can decomposition vector $\vec{a} = (x, y)$ as $\vec{a} = \alpha\vec{v} + \beta\vec{u}$, where $\vec{v} = (3, -4)$ and $\vec{u} = (4,3)$, α, β are real number. Please find the 2×2 matrix P, such that $P\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$.

8. If the distribution of discrete random variable X is

X	1	2	3	4
P	p	$2p$	p^2	$9p^2$

,where p is positive real number, please find the variance $Var(X)$ of X.

三、 應用題：5 題，每題 8 分，共 40 分

1. There was a class of 41 people divided into three groups A, B, and C, and the average score of subject natural's test was 89.7 points; the average score of group A was 89.8 points; the average score of A, B combination was 88.5 points; the average score of A and C combination was 91 points, and how many people were in each of groups A, B and C?

2. Given an angle θ , the solution of equation $\begin{cases} x\cos\theta - y\sin\theta = 4 \\ x\sin\theta + y\cos\theta = 3 \end{cases}$, is $(1, 2\sqrt{6})$.

   Please find the solution of equation $\begin{cases} x\cos\theta - y\sin\theta = -3 \\ x\sin\theta + y\cos\theta = 4 \end{cases}$.

3. As figure 1, two 3 by 4 rectangles overlap in such a way that their sides are perpendicular. If the area and perimeter of the shaded region are 22 and 20 respectively, compute length AB.

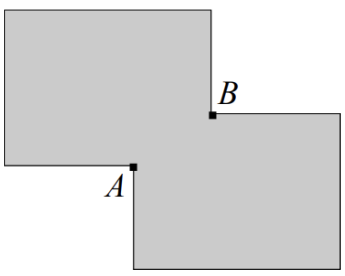

figure 1

4. Compute the greatest real number K for which the graphs of $(|x| - 5)^2 + (|y| - 5)^2 = K$ and $(x - 1)^2 + (y + 1)^2 = 37$ have exactly two intersection points.

5. Suppose the values of α and β are independently chosen uniformly from the interval $(0,2)$. Provided that the numbers $(1, \alpha, \beta)$ can form the sides of a triangle, find the probability that the numbers form an acute triangle.

四、 證明題：1 題，10 分

The sequence $\{a_n\}$：1, 1/2, 1/3,….Is it possible to select a subsequence from $\{a_n\}$ such that each term in it (except the first two) is the difference of the preceding two?

(1) for 5 terms sequence $b_1, b_2, \cdots, b_5$

(2) infinite terms sequence $\{c_n\}$.

Explain your answer!