臺北市立陽明高級中學 111 學年度第 1 次正式教師甄選高中數學科試題

Taipei Municipal Yang Ming High School Teacher Recruitment Exam

[Calculator is allowed]

1. Let $f(x) = |2x+3| - 5$ write a function g whose graph is horizontal shrink of the graph of f by a factor of $\frac{1}{3}$.

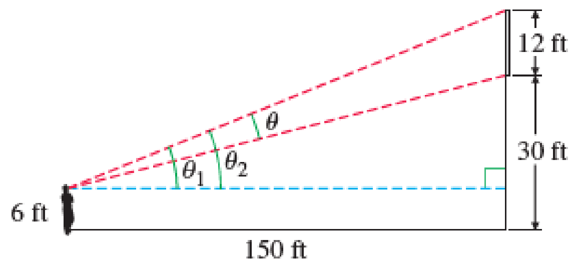
2. If $f(6) = 30$, and $f'(x) = \frac{x^2}{x+3}$. Estimate $f(6.02)$ using the line tangent to f at $x = 6$.

3. Find $\left[\left(\frac{-\sqrt{2}}{2}\right) + i\left(\frac{\sqrt{2}}{2}\right)\right]^8$ using De Moivre's theorem.

4. At a yearly rate of 5% compounded continuously, how long does it take (to the nearest year) for an investment to triple?

5. Let R be the region in the first quadrant bounded by the graph of $y = 2\sqrt{x}$, the horizontal line $y = 6$, and the y-axis.
(1) Find the area of R.
(2) Find the volume of the solid generated when R is rotated about the horizontal line $y = 7$.

6. An advertising agency is designing a 40 feet long by 12 feet high billboard. The billboard is mounted on a wall with the bottom of the billboard 30 feet above the ground. A man, whose eyes are 6 feet above the ground, stands 150 feet from the wall. Find the angle θ (to the nearest degree) between the man's line of sight to the top of the billboard and his line of sight to the bottom of the billboard. Refer to the figure.



7. Find the equilibrium price and then find the consumers' and producers' surplus at the equilibrium price level, if $p = D(x) = 20 - 0.05x$ and $p = S(x) = 2 + 0.0002x^2$

8. A blood test indicates the presence of Amyotrophic lateral sclerosis (ALS) 95% of the time when ALS is actually present. The same test indicates the presence of ALS 0.5% of the time when the disease is not actually present. One percent of the population actually has ALS. Calculate the probability that a person actually has ALS given that the test indicates the presence of ALS. Round your answer to the nearest thousandth.

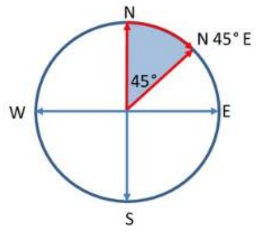
9. Hot water is dripping through a coffeemaker, filling a large cup with coffee. The amount of coffee in the cup at time, $0 \le t \le 6$, is given by a differentiable function C, where t is measured in minutes. Selected values of $C(t)$, measured in ounces, are given in the table below.

t (minuets)	0	1	2	3	4	5	6
$C(t)$ (ounces)	0	5.3	8.8	11.2	12.8	13.8	14.5

(1) Use the data in the table to approximate $C'(4.5)$, and indicate units of measure.
(2) Use a midpoint sum with <u>three</u> subintervals of equal length indicated by the data in the table to approximate the value of $\frac{1}{6}\int_0^6 C(t)dt$. Using correct units, explain the meaning of $\frac{1}{6}\int_0^6 C(t)dt$ in the context of the problem.

10. The notation for compass direction is shown below as an example.

The course for a boat race starts at point A and proceeds in the direction S $52°$W to point B, then in the direction S $40°$E to point C, and finally back to A. Point C lies 8 km directly south of point A. Approximate the total distance of the race course. Round the answer to the nearest hundredth.



11. The concentration of a medication injected into the bloodstream drops at a rate proportional to the existing concentration. If the factor of proportionality is 30% per hour, in how many hours will the concentration be one-tenth of the initial concentration? Round the answer to the nearest hundredth.

12. Your student claims that it is possible for a rational equation of the form

$\frac{x-a}{b} = \frac{x-c}{d}$, where b $\neq 0$ and d $\neq 0$, to have extraneous solutions. Is your student correct? How would you explain to the student?

13. Consider the infinite geometric series $\frac{1}{5} + \frac{1}{10} + \frac{1}{20} + \frac{1}{40} + \frac{1}{80} + \cdots$. How do you graphically and algebraically explain to students what happens to S_n as n increases?