107-全國高中教師聯招 詳解整理

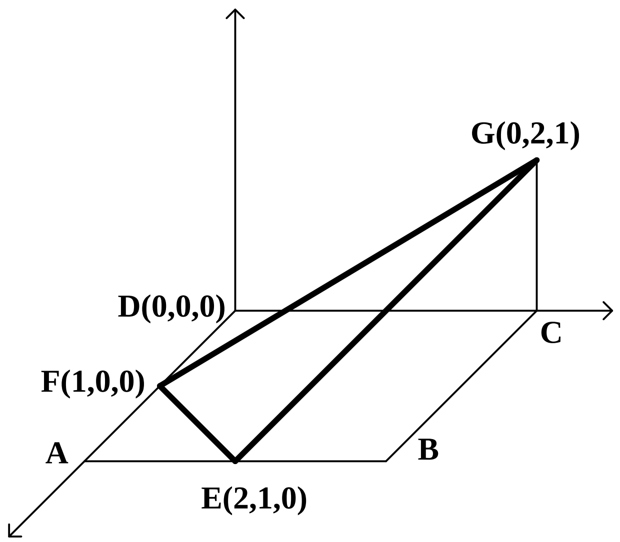
1. 單選

解：

(1) 向量 $\overrightarrow{GE}=(2,-1,-1)$，$\overrightarrow{GF}=(1,-2,-1)$

(2) 法向量 $\vec{n}=(1,-1,3)$ ⇨ 平面 EFG：$x-y+3z=1$

(3) $d(D,EFG)=\dfrac{|0+0++0-1|}{\sqrt{1^2+1^2+3^2}}=\dfrac{1}{\sqrt{11}}$ ###



2. 單選

解：

(1) $\begin{cases}\log_8 a+\log_4 b=3\\ \log_8 b+\log_4 a=7\end{cases}$ ⇨ $\begin{cases}\dfrac{1}{3}\log_2 a+\dfrac{1}{2}\log_2 b=3\\ \dfrac{1}{2}\log_2 a+\dfrac{1}{3}\log_2 b=7\end{cases}$

⇨ $\log_2 a=18,\ \log_2 b=-6$ ⇨ $\log_2 a+\log_2 b=12$ ⇨ $ab=2^{12}=4096$ ###

3. 單選

解：

(1) $\begin{vmatrix}a & b\\ b & c\end{vmatrix}>0$ ⇨ $ac-b^2>0$ ⇨ $ac>b^2$ (b 是偶數)

b	2 ($b^2=4$)	4 ($b^2=16$)	6 ($b^2=36$)
(ac)	(15)(16), (23)~(26), (32)~(36), (42)~(46), (51)~(56), (61)~(66)	(36), (45)(46), (54)~(56), (63)~(66)	無

(2) $p=\dfrac{38}{6\times3\times6}=0.35$ ###

4. 單選

解：

(*) $a+b=2$

(1) $\sqrt{2a+1}+\sqrt{3b+2}=\sqrt{2}\sqrt{a+\frac{1}{2}}+\sqrt{3}\sqrt{b+\frac{2}{3}}$

(2) $\left(\left(\sqrt{a+\frac{1}{2}}\right)^2+\left(\sqrt{b+\frac{2}{3}}\right)^2\right)\left((\sqrt{2})^2+(\sqrt{3})^2\right)\geq\left(\sqrt{2}\sqrt{a+\frac{1}{2}}+\sqrt{3}\sqrt{b+\frac{2}{3}}\right)^2$

$\Rightarrow\ (a+b+\frac{7}{6})(2+3)\geq(\sqrt{2a+1}+\sqrt{3b+2})^2$

$\Rightarrow\ \frac{95}{6}\geq(\sqrt{2a+1}+\sqrt{3b+2})^2\ \Rightarrow\ (\sqrt{2a+1}+\sqrt{3b+2})\leq\sqrt{\frac{95}{6}}=\frac{\sqrt{570}}{6}$ ###

5. 單選

解：

3R
2B
1Y

甲 乙 丙

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RR BB RY $3!=6$

RR RB BY $3!=6$

RB RB RY $\frac{3!}{2!}=3$

###

6. 單選

解：

(*) $\begin{cases} a+b+c+d+e=10 \\ a^2+b^2+c^2+d^2+e^2=\frac{205}{4} \end{cases}\ \Rightarrow\ \begin{cases} a+b+d+e=10-c \\ a^2+b^2+d^2+e^2=\frac{205}{4}-c^2 \end{cases}$

(2) $(a^2+b^2+d^2+e^2)(1^2+1^2+1^2+1^2)\geq(a+b+d+e)^2$

$\Rightarrow\ (a^2+b^2+d^2+e^2)(1^2+1^2+1^2+1^2)\geq(a+b+d+e)^2$

$\Rightarrow\ (\frac{205}{4}-c^2)\times4\geq(10-c)^2\ \Rightarrow\ 205-4c^2\geq100-2c+c^2$

$\Rightarrow\ c^2-4c-21\leq0\ \Rightarrow\ -3\leq c\leq7$ ###

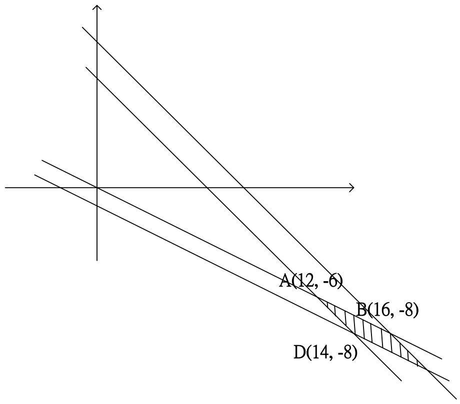
7. 單選

解：

(1) $\vec{A}\cdot\vec{B}=|\vec{A}||\vec{B}|\cos 60^{0}=1$

(2) $\vec{U}\cdot\vec{V}=(\vec{A}+\vec{B})\cdot(x\vec{A}+y\vec{B})=2x+5y$

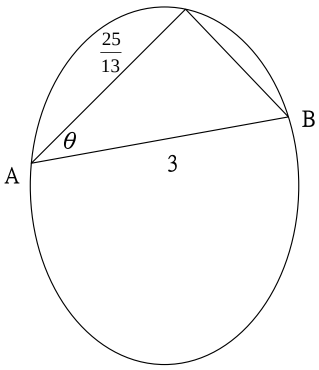
(*) $\begin{cases} 6\le x+y\le 8 \\ -2\le x+2y\le 0 \end{cases}$ ⇨



(3) $f(A)=-6$ ， $f(B)=-18$ ， $f(C)=-14$ ， $f(D)=-12$ ###

8. 單選

解：



(*)

(1) $\dfrac{a}{\sin A}=\dfrac{b}{\sin B}=\dfrac{c}{\sin C}=2R$ ⇨ $\dfrac{a}{\sin A}=\dfrac{25/13}{\sin B}=\dfrac{3}{\sin C}=5$ ⇨ $\begin{cases} \sin B=5/13 \\ \sin C=3/5 \end{cases}$

(2) $\cos A=\cos(\pi-(B+C))=-\cos(B+C)=-(\cos B\cos C-\sin B\sin C)$

$=-(\dfrac{12}{13}\times\dfrac{-4}{5}-\dfrac{5}{13}\times\dfrac{3}{5})=\dfrac{63}{65}$ ###

9. **複選題**

解：

$$\frac{b+c}{2a}=\frac{a-c}{2b}=\frac{a-b}{2c}=\frac{2a}{2a+2b+2c}=\frac{a}{a+b+c}\text{令 } t$$

$$\therefore \begin{cases} b+c=2at \\ a-c=2bt \\ a-b=2ct \end{cases} \Rightarrow \begin{cases} (-2t)a+b+c=0 \\ a+(-2t)b-c=0 \\ a-b-(2t)c=0 \end{cases}$$

除了(0,0,0)以外還有其他解 (∵ 分母 $a,b,c\neq 0$)

$$\therefore 0=\begin{vmatrix} -2t & 1 & 1 \\ 1 & -2t & -1 \\ 1 & -1 & -2t \end{vmatrix}=0$$

$$\therefore (t+1)(8t^2-8t+2)=0 \Rightarrow t=-1 \text{ or } \frac{1}{2} \ \#$$

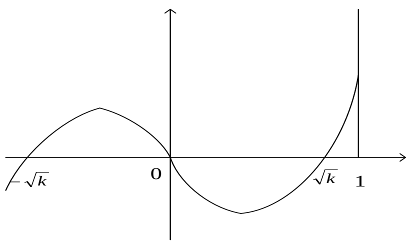
（借）###

10. **複選題**

解：

(1) $\int_0^1 |x^3-kx|\,dx=\int_0^k |x^3-kx|\,dx+\int_k^1 |x^3-kx|\,dx=-\int_0^k (x^3-kx)\,dx+\int_k^1 (x^3-kx)\,dx$

$$=\frac{1}{2}k^2-\frac{1}{2}k+\frac{1}{4}=\frac{1}{2}\left(k-\frac{1}{2}\right)^2+\frac{1}{8}$$ ###



11. **複選題**

解：

(*) 令 $a_n=a+(n-1)d$ ⇨ $b_n=f(a_n)=2^{a+(n-1)d}$

(A) $b_1=2^a$，$b_2=2^{a+d}=2^a\times 2^d$，$b_3=2^{a+2d}=b_2\times 2^d$，… G.P. （ √ ）

(B) $d>0$ ⇨ $r=2^d>1$

(C) $b_n = 2^{a+(n-1)d} = 2^{1+(n-1)\times\frac{1}{2}} > 4096$ ⇨ $1+(n-1)\times\dfrac{1}{2} > 12$ ⇨ $n > 23$ (√)

(D) $\sum_{i=1}^{n} a_i = 22$ ⇨ $\dfrac{n[1+1+(n-1)\times\frac{1}{2}]}{2} = 22$ ⇨ $n = 8$

⇨ $\sum_{i=1}^{8} b_i = 2^1 + 2^{3/2} + 2^2 + \dots + 2^{9/2} = 30(\sqrt{2}+1)$ ###

12. 複選題

解：

(*) $\begin{cases} \mu_x = 50,\ \sigma_x = 8 \\ \mu_y = M,\ \sigma_y = 12 \end{cases}$

(A) $y = \dfrac{3}{4}x + 20$ ⇨ $M = \dfrac{3}{4}\times 50 + 20 = 57.5$ (√)

(B) (i) $\sigma_x^{\,2} = \dfrac{\sum_{i=1}^{40}(x_i-\mu_x)^2}{n} = 8^2$ ⇨ $\sum_{i=1}^{40}(x_i-\mu_x)^2 = 64\times 40 = 2560$ #

(ii) $y = ax + b$ ⇨ $y = \dfrac{S_{xy}}{S_{xx}}x + b$ ⇨ $\dfrac{S_{xy}}{S_{xx}} = \dfrac{\sum_{i=1}^{40}(x_i-\mu_x)(y_i-\mu_y)}{\sum_{i=1}^{40}(x_i-\mu_x)^2} = \dfrac{3}{4}$

⇨ $\sum_{i=1}^{40}(x_i-\mu_x)(y_i-\mu_y) = \dfrac{3}{4}\times 2560 = 1920$ #

(iii) $r = \dfrac{\sum_{i=1}^{40}(\frac{x_i-\mu_x}{\sigma_x})(\frac{y_i-\mu_x}{\sigma_y})}{n} = \dfrac{1920 / 8\times 12}{40} = 0.5$ (√)

(D) $\sigma_x = 8,\ \sigma_y = 12$ (√)

1. 填充

解：

(1) $(\sin^2 63^0 - 3\sin^2 27^0)(\sin^2 9^0 - 3\cos^2 171^0)$

$= (\cos^2 27^0 - 3\sin^2 27^0)((1-\cos^2 9^0) - 3\cos^2 9^0)$

$= (1 - 4\sin^2 27^0)(1 - 4\cos^2 9^0)$

$= (1 - 4\times\dfrac{1-\cos 54^0}{2})(1 - 4\times\dfrac{1+\cos 18^0}{2})$

$= (2\cos 54^0 - 1)(-2\cos 18^0 - 1)$

$= (2\cos 54^0 - 1)\cdot\dfrac{\sin 54^0}{\sin 54^0}(-2\cos 18^0 - 1)\cdot\dfrac{\sin 18^0}{\sin 18^0}$

$= (\dfrac{2\sin 54^0\cos 54^0 - \sin 54^0}{\sin 54^0})(\dfrac{-2\sin 18^0\cos 18^0 - \sin 18^0}{\sin 18^0})$

$= (\dfrac{\sin 108^0 - \sin 54^0}{\sin 54^0})(\dfrac{-\sin 36^0 - \sin 18^0}{\sin 18^0})$

$= (\dfrac{\sin 54^0 - \sin 72^0}{\sin 54^0})(\dfrac{\sin 36^0 + \sin 18^0}{\sin 18^0})$

$= (\dfrac{-2\cos 63^0\sin 9^0}{\sin 54^0})(\dfrac{2\sin 27^0\cos 9^0}{\sin 18^0}) = (\dfrac{-2\cos 63^0\sin 27^0}{\sin 54^0})(\dfrac{2\sin 9^0\cos 9^0}{\sin 18^0})$

$= \dfrac{-2\sin 27^0\sin 27^0}{\sin 54^0} = \dfrac{-2\sin 27^0\sin 27^0}{2\sin 27^0\cos 27^0} = -\tan 27^0 = \tan 333^0$ ###

2. 填充

解：

(*) $a_{n+1} - 1 = a_n + 2\sqrt{1+a_n}$

(1) 令 $A_n = a_n + 1$ ⇨ $A_{n+1} = a_{n+1} + 1$

(2) 原式 ⇨ $A_{n+1} = A_n + 2\sqrt{A_n} + 1$ ⇨ $A_{n+1} = (\sqrt{A_n} + 1)^2$ ⇨ $\sqrt{A_{n+1}} = \sqrt{A_n} + 1$

(3) $\sqrt{A_2} = \sqrt{A_1} + 1$

$\sqrt{A_3} = \sqrt{A_2} + 1$

… …

$\sqrt{A_n} = \sqrt{A_{n-1}} + 1$

⇨ $\sqrt{A_n} = \sqrt{A_1} + n - 1 = n$ ⇨ $A_n = n^2$ ⇨ $a_n + 1 = n^2$ ⇨ $a_{30} = 30^2 - 1 = 899$ ###

3. 填充

解：

(1) $f(t)=\dfrac{1}{2t}$ ⇨ $f(x)=\dfrac{1}{2x}$ ⇨ $2xf(x)=1$ ⇨ $2xf(x)-1=0$

(2) 令 $2xf(x)-1=A(x-1)(x-2)(x-3)\ldots(x-2019)$ ⇨ $A=\dfrac{1}{2019!}$

⇨ $2xf(x)-1=\dfrac{1}{2019!}(x-1)(x-2)(x-3)\ldots(x-2019)$

(3) $2\times 2020\times f(2020)-1=\dfrac{1}{2019!}(2020-1)(2020-2)(2020-3)\ldots(2020-2019)$

⇨ $f(2020)=\dfrac{1}{2020}$ ###

4. 填充

解：

(1) $\dfrac{1}{2\times3\times4\times5}+\dfrac{1}{3\times4\times5\times6}+\dfrac{1}{4\times5\times6\times7}+\ldots$

$=\dfrac{1}{3}(\dfrac{1}{2\times3\times4}-\dfrac{1}{3\times4\times5})+\dfrac{1}{3}(\dfrac{1}{3\times4\times5}-\dfrac{1}{4\times5\times6})+\ldots=\dfrac{1}{72}$ ###

5. 填充

解：

(*) $A=\begin{bmatrix}1&0\\-1&2\end{bmatrix}$，$B=\begin{bmatrix}0&0\\-1&1\end{bmatrix}$，$I=\begin{bmatrix}1&0\\0&1\end{bmatrix}$ ⇨ $B^2=B$

(1) ⇨ $A=I+B$ ⇨ $A^8=(I+B)^8=C_0^8I^8+C_1^8B+C_2^8B^2+\ldots+C_8^8B^8$

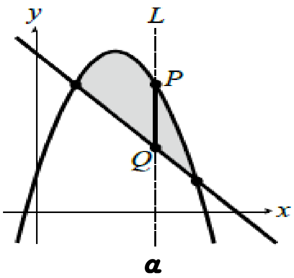
$=I+(C_1^8+C_2^8+\ldots+C_8^8)B=I+255B$ ###

6. 填充

解：

$$(*)\ \begin{cases} f(x) = -x^2 + 5x + 1 \\ g(x) = -x + 6 \end{cases} \Rightarrow \begin{cases} f(a) = -a^2 + 5a + 1 \\ g(a) = -a + 6 \end{cases}$$

(1) $\overline{PQ} = -a^2 + 6a - 5 = -(a-3)^2 + 4$ ###



7. 填充

解：

(1) $\sum_{n-1}^{100} [\frac{1}{2}(\log_2 n) - 1] = -1 \times 3 + 0 \times 12 + 1 \times 48 + 2 \times 37 = 119$ ###

n	1, 2, 3	4～15	16～63	64～100
$\frac{1}{2}(\log_2 n) - 1$	−1	0	1	2

8. 填充

解：

(1) $\sqrt{m + \sqrt{m^2 - n}} + \sqrt{m - \sqrt{m^2 - n}} = 6$ （平方）

$\Rightarrow m + \sqrt{m^2 - n} + 2\sqrt{m + \sqrt{m^2 - n}} \times \sqrt{m - \sqrt{m^2 - n}} + m - \sqrt{m^2 - n} = 36$

$\Rightarrow 2m + 2\sqrt{n} = 36 \Rightarrow m + \sqrt{n} = 18$

n	1^2	2^2	3^2	4^2	5^2	6^2	7^2	8^2	9^2
m	17	16	15	14	13	12	11	10	9

(2) $1^2 + 2^2 + 3^2 + ... + 9^2 = \frac{9(9+1)(2 \times 9 + 1)}{6} = 285$ ###

9. 填充

解：

$$(*)\begin{cases} a_n = a_{n-1} - a_{n-2} \\ \sum_{n=1}^{40} a_n = 30 \\ \sum_{n=1}^{80} a_n = 78 \end{cases}$$

(1) 令 $\begin{cases} a_1 = a \\ a_2 = b \end{cases}$ ⇨ $a_3 = a_2 - a_1 = b - a$ ⇨ $a_4 = a_3 - a_2 = -a$

⇨ $a_5 = a_4 - a_3 = -b$ ⇨ $a_6 = a_5 - a_4 = -b + a$ ⇨ $a_7 = a_6 - a_5 = a$

⇨ $a_8 = a_7 - a_6 = b$ （循環了）且 $a_1 + a_2 + \ldots + a_6 = 0$

(2) $\begin{cases} \sum_{n=1}^{40} a_n = -a + 2b = 30 \\ \sum_{n=1}^{80} a_n = a + b = 78 \end{cases}$ ⇨ $a = 42\,,\ b = 36$

⇨ $\sum_{n=1}^{123} a_n = a_1 + a_2 + a_3 = a + b + b - a = 2b = 72$ ###

1. 計算

解：

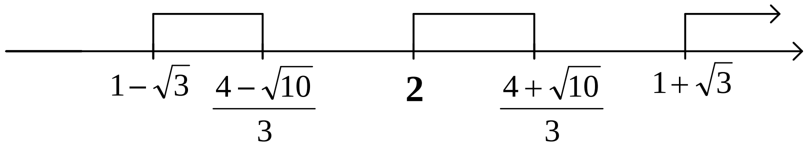
(1) $x^3 - 4x^2 + 2x + 4 = 0$ ⇨ $(x-2)(x^2 - 2x - 2) = 0$ ⇨ $x = 2,\ x = 1 \pm \sqrt{3}$

(2) $\dfrac{1}{x-\alpha} + \dfrac{1}{x-\beta} + \dfrac{1}{x-\gamma} > 0$ ⇨ $\dfrac{(x-\alpha)(x-\beta) + (x-\beta)(x-\gamma) + (x-\gamma)(x-\alpha)}{(x-\alpha)(x-\beta)(x-\gamma)} > 0$

⇨ $f(x)[(x-\alpha)(x-\beta) + (x-\beta)(x-\gamma) + (x-\gamma)(x-\alpha)] > 0$

⇨ $(x^3 - 4x^2 + 2x + 4)(3x^2 - 8x + 2) > 0$

⇨ $(x^3 - 4x^2 + 2x + 4)(3x^2 - 8x + 2) = 0$ ⇨ $x = 2,\ x = 1 \pm \sqrt{3},\ \dfrac{4 \pm \sqrt{10}}{3}$



(3) $1 - \sqrt{3} < x < \dfrac{4 - \sqrt{10}}{3}$ or $2 < x < \dfrac{4 + \sqrt{10}}{3}$ or $x > 1 + \sqrt{3}$ ###

2. 計算

解：

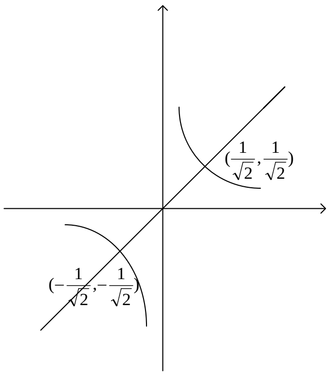
(1) $|z-1-i|-|z+1+i|=2$ ⇨ $\sqrt{(x-1)^2+(y-1)^2}-\sqrt{(x+1)^2+(y+1)^2}=2$

⇨ $\sqrt{(x-1)^2+(y-1)^2}=2+\sqrt{(x+1)^2+(y+1)^2}$

⇨ $(x-1)^2+(y-1)^2=4+(x+1)^2+(y+1)^2+4\sqrt{(x+1)^2+(y+1)^2}$

⇨ $-(x+y+1)=\sqrt{(x+1)^2+(y+1)^2}$ ⇨ $x^2+y^2+1+2xy+2x+2y=(x+1)^2+(y+1)^2$

⇨ $2xy=1$ ⇨ $xy=\frac{1}{2}$ ###



(1*) ⇨ $xy=\frac{1}{2}$ ###

3. 計算

解：

(1) $(1+\frac{1}{n})^n=C_0^n(\frac{1}{n})^0+C_1^n(\frac{1}{n})^1+C_2^n(\frac{1}{n})^2+\ldots+C_k^n(\frac{1}{n})^k+\ldots+C_n^n(\frac{1}{n})^n$

$$=\sum_{k=0}^{n}\frac{n!}{k!(n-k)!}(\frac{1}{n})^k=\sum_{k=0}^{n}\frac{n(n-1)\ldots(n-k+1)}{k!}(\frac{1}{n})^k$$

$$=\sum_{k=0}^{n}\frac{1}{k!}(\frac{n}{n}\times\frac{n-1}{n}\times\ldots\times\frac{n-k+1}{n})$$

(2) $\lim\limits_{n\to\infty}(1+\frac{1}{n})^n=\lim\limits_{n\to\infty}\sum_{k=0}^{n}\frac{1}{k!}(\frac{n}{n}\times\frac{n-1}{n}\times\ldots\times\frac{n-k+1}{n})=\sum_{k=0}^{\infty}\frac{1}{k!}(1\times1\times\ldots\times1)=\sum_{k=0}^{\infty}\frac{1}{k!}$ ###