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$$A=\frac{1}{2}\begin{bmatrix}\cos\frac{2\pi}{n} & -\sin\frac{2\pi}{n}\\ \sin\frac{2\pi}{n} & \cos\frac{2\pi}{n}\end{bmatrix}$$

$$P_k=A^{k-1}P_1=\frac{1}{2^{k-1}}\begin{bmatrix}\cos\frac{2(k-1)\pi}{n} & -\sin\frac{2(k-1)\pi}{n}\\ \sin\frac{2(k-1)\pi}{n} & \cos\frac{2(k-1)\pi}{n}\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix}=\frac{1}{2^{k-1}}\begin{bmatrix}\cos\frac{2(k-1)\pi}{n}\\ \sin\frac{2(k-1)\pi}{n}\end{bmatrix}$$

$$S_k=\frac{1}{2}\left\|\begin{matrix}\frac{1}{2^{k-1}}\cos\frac{2(k-1)\pi}{n} & \frac{1}{2^{k-1}}\sin\frac{2(k-1)\pi}{n}\\ \frac{1}{2^k}\cos\frac{2k\pi}{n} & \frac{1}{2^k}\sin\frac{2k\pi}{n}\end{matrix}\right\|=\frac{1}{2^{2k}}\sin\frac{2\pi}{n}$$

$$\lim_{n\to\infty}\left(n\sum_{k=1}^{\infty}S_k\right)=\lim_{n\to\infty}\left(n\sum_{k=1}^{\infty}\frac{1}{2^{2k}}\sin\frac{2\pi}{n}\right)=\lim_{\frac{2\pi}{n}\to 0}\left(2\pi\times\frac{\sin\frac{2\pi}{n}}{\frac{2\pi}{n}}\right)\sum_{k=1}^{\infty}\frac{1}{2^{2k}}=2\pi\times\frac{\frac{1}{4}}{1-\frac{1}{4}}=\frac{2}{3}\pi$$