

$$\Leftrightarrow \cos \theta + \cos 2\theta + \cos 4\theta + \cos 8\theta = x$$

$$\begin{aligned} & \cos^2 \theta + \cos^2 2\theta + \cos^2 4\theta + \cos^2 8\theta \\ &= \frac{1 + \cos 2\theta}{2} + \frac{1 + \cos 4\theta}{2} + \frac{1 + \cos 8\theta}{2} + \frac{1 + \cos 16\theta}{2} \\ &= 2 + \frac{1}{2}(\cos 2\theta + \cos 4\theta + \cos 8\theta + \cos 16\theta) \\ &= 2 + \frac{1}{2}(\cos 2\theta + \cos 4\theta + \cos 8\theta + \cos \theta) \end{aligned}$$

$$\begin{aligned} & 2 \cos \theta \cos 2\theta + 2 \cos \theta \cos 4\theta + 2 \cos \theta \cos 8\theta + 2 \cos 2\theta \cos 4\theta + 2 \cos 2\theta \cos 8\theta + 2 \cos 4\theta \cos 8\theta \\ &= (\cos \theta + \cos 3\theta) + (\cos 3\theta + \cos 5\theta) + (\cos 7\theta + \cos 9\theta) + (\cos 2\theta + \cos 6\theta) + (\cos 6\theta + \cos 10\theta) + (\cos 4\theta + \cos 12\theta) \\ &= (\cos \theta + \cos 3\theta) + (\cos 14\theta + \cos 5\theta) + (\cos 7\theta + \cos 9\theta) + (\cos 2\theta + \cos 6\theta) + (\cos 11\theta + \cos 10\theta) + (\cos 4\theta + \cos 12\theta) \\ &= -(\cos 0 + \cos 8\theta + \cos 13\theta + \cos 15\theta + \cos 16\theta) \\ &= -(1 + \cos 8\theta + \cos 4\theta + \cos 2\theta + \cos \theta) \end{aligned}$$

$$x^2 = 2 + \frac{1}{2}x - (1 + x)$$

$$2x^2 + x - 2 = 0$$

$$x = \frac{-1 \pm \sqrt{17}}{4} \text{ (負不合)}$$