1220 Number Theory Problems (The J29 Project)

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Abstract

This problem set is my main source for writing a book. It is nothing but a set of problems posted by active users of AoPS/MathLinks, and it will be a really good source for preparing for mathematical olympiads.

I just like to thank all people who posted the problems: **Andrew** (WakeUp), **Goutham**, **Orlando** (orl), **Valentin** (Valentin Vornicu), **Darij** (darij grinberg), **Vesselin** (vess), **Gabriel** (harazi), **April**, **Arne**, and **Kunihiko** (kunny). And a special thanks goes to **Ben** (bluecarneal) who helped me a *lot* to edit and compile this problem set.

But How To Use It? You probably ask this question when you see the problems have been categorized by their posters, not by thir level/subject. First, let me tell you why are the problems are categorized like that. It's simple: the large number of problems. It would take me a very long time to categorize all these problems and put them altogether.

The Solution. Problems posted by me, Andrew, Goutham, Orlando, Valentin, and April are almost all problems of known competitions like USAMO, Iran national olympiad, etc. So, they are all olympiad level problems and you can solve them when you think you're prepared enough in most subjects of number theory. Problems posted by Vesselin, Gabriel, and in some cases, Darij, are **really hard!** Obviously, they need more skills to solve. And finally problems posted by Arne and Kunihiko are random and you can find problems of different levels in their parts.

Enjoy Problem Solving!

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Chapter 1

Problems

1.1 Amir Hossein

1.1.1 Amir Hossein - Part 1

1. Show that there exist infinitely many non similar triangles such that the side-lengths are positive integers and the areas of squares constructed on their sides are in arithmetic progression.

2. Let n be a positive integer. Find the number of those numbers of $2n$ digits in the binary system for which the sum of digits in the odd places is equal to the sum of digits in the even places.

3. Find the necessary and sufficient condition for numbers $a \in \mathbb{Z} \setminus \{-1, 0, 1\}$, $b, c \in \mathbb{Z} \setminus \{0\}$, and $d \in \mathbb{N} \setminus \{0, 1\}$ for which $a^n + bn + c$ is divisible by d for each natural number n.

4. Find the 73th digit from the end of the number $\underbrace{111\ldots1}_{2012 \text{ digits}}{}^2$.

5. Find all numbers $x, y \in \mathbb{N}$ for which the relation $x + 2y + \frac{3x}{y} = 2012$ holds.

6. Let p be a prime number. Given that the equation

$$p^k + p^l + p^m = n^2$$

has an integer solution, prove that $p + 1$ is divisible by 8.

7. Find all integer solutions of the equation the equation $2x^2 - y^{14} = 1$.

8. Do there exist integers m, n and a function $f : \mathbb{R} \to \mathbb{R}$ satisfying simultaneously the following two conditions $f(f(x)) = 2f(x) - x - 2$ for any $x \in \mathbb{R}$, $m \leq n$ and $f(m) = n$?

9. Show that there are infinitely many positive integer numbers n such that $n^2 + 1$ has two positive divisors whose difference is n.

10. Consider the triangular numbers $T_n = \frac{n(n+1)}{2}, n \in \mathbb{N}$.

- **(a)**If a_n is the last digit of T_n, show that the sequence (a_n) is periodic and find its basic period.
- **(b)** If s_n is the sum of the first n terms of the sequence (T_n), prove that for every $n \geq 3$ there is at least one perfect square between s_{n-1} and s_n.

11. Find all integers x and prime numbers p satisfying $x^8 + 2^{2^x+2} = p$.

12. We say that the set of step lengths $D \subset \mathbb{Z}_+ = \{1, 2, \ldots\}$ is excellent if it has the following property: If we split the set of integers into two subsets A and $\mathbb{Z} \setminus A$, at least other set contains element $a-d, a, a+d$ (i.e. $\{a-d, a, a+d\} \subset A$ or $\{a-d, a, a+d\} \in \mathbb{Z} \setminus A$ from some integer $a \in \mathbb{Z}, d \in D$.) For example the set of one element $\{1\}$ is not excellent as the set of integer can be split into even and odd numbers, and neither of these contains three consecutive integer. Show that the set $\{1, 2, 3, 4\}$ is excellent but it has no proper subset which is excellent.

13. Let n be a positive integer and let α_n be the number of 1's within binary representation of n.

Show that for all positive integers r,

$$2^{2n-\alpha_n} \mid \sum_{k=-n}^{n} {C_{2n}}^{n+k} k^{2r}.$$

14. The function $f : \mathbb{N} \to \mathbb{R}$ satisfies $f(1) = 1, f(2) = 2$ and

$$f(n+2) = f(n+2-f(n+1)) + f(n+1-f(n)).$$

Show that $0 \leq f(n+1) - f(n) \leq 1$. Find all n for which $f(n) = 1025$.

15. Let $x_{n+1} = 4x_n - x_{n-1}$, $x_0 = 0$, $x_1 = 1$, and $y_{n+1} = 4y_n - y_{n-1}$, $y_0 = 1$, $y_1 = 2$. Show that for all $n \geq 0$ that $y_n^2 = 3x_n^2 + 1$.

16. Find all solutions of $a^2 + b^2 = n!$ for positive integers a, b, n with $a \leq b$ and $n < 14$.

17. Let a, b, c, d, e be integers such that $1 \leq a < b < c < d < e$. Prove that

$$\frac{1}{[a,b]} + \frac{1}{[b,c]} + \frac{1}{[c,d]} + \frac{1}{[d,e]} \leq \frac{15}{16},$$

where $[m, n]$ denotes the least common multiple of m and n (e.g. $[4, 6] = 12$).

18. N is an integer whose representation in base b is 777. Find the smallest integer b for which N is the fourth power of an integer.

19. Let a, b, c some positive integers and x, y, z some integer numbers such that we have

- **a)** $ax^2 + by^2 + cz^2 = abc + 2xyz - 1$, and
- **b)** $ab + bc + ca \geq x^2 + y^2 + z^2$.

Prove that a, b, c are all sums of three squares of integer numbers.

20. Suppose the set of prime factors dividing at least one of the numbers $[a], [a^2], [a^3], \dots$ is finite. Does it follow that a is integer?

1.1.2 Amir Hossein - Part 2

21. Determine all pairs (x, y) of positive integers such that $\frac{x^2y+x+y}{xy^2+y+11}$ is an integer.

22. We call a positive integer n amazing if there exist positive integers a, b, c such that the equality

$$n = (b, c)(a, bc) + (c, a)(b, ca) + (a, b)(c, ab)$$

holds. Prove that there exist 2011 consecutive positive integers which are amazing.
Note. By (m, n) we denote the greatest common divisor of positive integers m and n.

23. Let A and B be disjoint nonempty sets with $A \cup B = \{1, 2, 3, \dots, 10\}$. Show that there exist elements $a \in A$ and $b \in B$ such that the number $a^3 + ab^2 + b^3$ is divisible by 11.

24. Let k and m, with $k > m$, be positive integers such that the number $km(k^2 - m^2)$ is divisible by $k^3 - m^3$. Prove that $(k - m)^3 > 3km$.

25. Initially, only the integer 44 is written on a board. An integer a on the board can be re- placed with four pairwise different integers a_1, a_2, a_3, a_4 such that the arithmetic mean $\frac{1}{4}(a_1+a_2+a_3+a_4)$ of the four new integers is equal to the number a. In a step we simultaneously replace all the integers on the board in the above way. After 30 steps we end up with $n = 4^{30}$ integers $b_1, b2, \dots, b_n$ on the board. Prove that

$$\frac{b_1^2 + b_2^2 + b_3^2 + \cdots + b_n^2}{n} \geq 2011.$$

26. Determine all finite increasing arithmetic progressions in which each term is the reciprocal of a positive integer and the sum of all the terms is 1.

27. A binary sequence is constructed as follows. If the sum of the digits of the positive integer k is even, the k-th term of the sequence is 0. Otherwise, it is 1. Prove that this sequence is not periodic.

28. Find all (finite) increasing arithmetic progressions, consisting only of prime numbers, such that the number of terms is larger than the common difference.

29. Let p and q be integers greater than 1. Assume that $p \mid q^3 - 1$ and $q \mid p-1$. Prove that $p = q^{3/2} + 1$ or $p = q^2 + q + 1$.

30. Find all functions $f : \mathbb{N} \cup \{0\} \to \mathbb{N} \cup \{0\}$ such that $f(1) > 0$ and

$$f(m^2 + 3n^2) = (f(m))^2 + 3(f(n))^2 \quad \forall m, n \in \mathbb{N} \cup \{0\}.$$

31. Prove that there exists a subset S of positive integers such that we can represent each positive integer as difference of two elements of S in exactly one way.

32. Prove that there exist infinitely many positive integers which can't be represented as sum of less than 10 odd positive integers' perfect squares.

33. The rows and columns of a $2^n \times 2^n$ table are numbered from 0 to $2^n - 1$. The cells of the table have been coloured with the following property being satisfied: for each $0 \le i, j \le 2^n - 1$, the j-th cell in the i-th row and the $(i+j)$-th cell in the j-th row have the same colour. (The indices of the cells in a row are considered modulo 2^n.) Prove that the maximal possible number of colours is 2^n.

34. Let a, b be integers, and let $P(x) = ax^3 + bx$. For any positive integer n we say that the pair (a, b) is n-good if $n | P(m) - P(k)$ implies $n | m - k$ for all integers m, k. We say that (a, b) is *very good* if (a, b) is n-good for infinitely many positive integers n.

- **(a)** Find a pair (a, b) which is 51-good, but not very good.
- **(b)** Show that all 2010-good pairs are very good.

35. Find the smallest number n such that there exist polynomials $f_1, f_2, \ldots, f_n$ with rational coefficients satisfying

$$x^2 + 7 = f_1(x)^2 + f_2(x)^2 + \ldots + f_n(x)^2 .$$

36. Find all pairs (m, n) of nonnegative integers for which

$$m^2 + 2 \cdot 3^n = m\left(2^{n+1} - 1\right).$$

37. Find the least positive integer n for which there exists a set $\{s_1, s_2, \ldots, s_n\}$ consisting of n distinct positive integers such that

$$\left(1 - \frac{1}{s_1}\right)\left(1 - \frac{1}{s_2}\right) \cdots \left(1 - \frac{1}{s_n}\right) = \frac{51}{2010}.$$

38. For integers x, y, and z, we have $(x-y)(y-z)(z-x) = x+y+z$. Prove that $27 | x+y+z$.

39. For a positive integer n, numbers $2n+1$ and $3n+1$ are both perfect squares. Is it possible for $5n+3$ to be prime?

40. A positive integer K is given. Define the sequence (a_n) by $a_1 = 1$ and a_n is the n-th positive integer greater than a_{n-1} which is congruent to n modulo K.

- **(a)** Find an explicit formula for a_n.
- **(b)** What is the result if $K = 2$?

1.1.3 Amir Hossein - Part 3

41. Let a be a fixed integer. Find all integer solutions x, y, z of the system

$$5x + (a+2)y + (a+2)z = a,$$

$$(2a+4)x + (a^2+3)y + (2a+2)z = 3a-1,$$

$$(2a+4)x + (2a+2)y + (a^2+3)z = a+1.$$

42. Let $F(n) = 13^{6n+1} + 30^{6n+1} + 100^{6n+1} + 200^{6n+1}$ and let

$$G(n) = 2F(n) + 2n(n-2)F(1) - n(n-1)F(2).$$

Prove by induction that for all integers $n \geq 0$, $G(n)$ is divisible by 7^3.

43. Let $P(x) = x^3 - px^2 + qx - r$ be a cubic polynomial with integer roots a, b, c.

- **(a)** Show that the greatest common divisor of p, q, r is equal to 1 if the greatest common divisor of a, b, c is equal to 1.
- **(b)** What are the roots of polynomial $Q(x) = x^3 - 98x^2 + 98sx - 98t$ with s, t positive integers.

44. Let Q_n be the product of the squares of even numbers less than or equal to n and K_n equal to the product of cubes of odd numbers less than or equal to n. What is the highest power of 98, that **a)**Q_n, **b)** K_n or **c)** Q_nK_n divides? If one divides $Q_{98}K_{98}$ by the highest power of 98, then one get a number N. By which power-of-two number is N still divisible?

45. Prove that for each positive integer n, the sum of the numbers of digits of 4^n and of 25^n (in the decimal system) is odd.

46. Find all pairs of integers (m, n) such that

$$\left|(m^2 + 2000m + 999999) - (3n^3 + 9n^2 + 27n)\right| = 1.$$

47. Let b be a positive integer. Find all 2002-tuples $(a_1, a_2, \ldots, a_{2002})$, of natural numbers such that

$$\sum_{j=1}^{2002} a_j^{a_j} = 2002b^b.$$

48. Determine all integers m for which all solutions of the equation $3x^3 - 3x^2 + m = 0$ are rational.

49. Prove that, for any integer $g > 2$, there is a unique three-digit number $\overline{abc}_g$ in base g whose representation in some base $h = g \pm 1$ is $\overline{cba}_h$.

50. For every lattice point (x, y) with x, y non-negative integers, a square of side $\frac{0.9}{2^x 5^y}$ with center at the point (x, y) is constructed. Compute the area of the union of all these squares.

51. Consider the polynomial $P(n) = n^3 - n^2 - 5n + 2$. Determine all integers n for which $P(n)^2$ is a square of a prime.

52. Find all triples of prime numbers (p, q, r) such that $p^q + p^r$ is a perfect square.

53. Find all functions $f : \mathbb{N} \to \mathbb{N}$ such that

$$f(n) = 2 \cdot \lfloor \sqrt{f(n-1)} \rfloor + f(n-1) + 12n + 3, \qquad \forall n \in \mathbb{N}$$

where $\lfloor x \rfloor$ is the greatest integer not exceeding x, for all real numbers x.

54. Find all quadruple $(m, n, p, q) \in \mathbb{Z}^4$ such that

$$p^m q^n = (p+q)^2 + 1.$$

55. Let p be the product of two consecutive integers greater than 2. Show that there are no integers $x_1, x_2, \ldots, x_p$ satisfying the equation

$$\sum_{i=1}^{p} x_i^2 - \frac{4}{4 \cdot p + 1} \left(\sum_{i=1}^{p} x_i \right)^2 = 1.$$

56. Consider the equation

$$x^2 + y^2 + z^2 + t^2 - N \cdot x \cdot y \cdot z \cdot t - N = 0$$

where N is a given positive integer.

- **a)** Prove that for an infinite number of values of N, this equation has positive integral solutions (each such solution consists of four positive integers x, y, z, t),
- **b)** Let $N = 4 \cdot k \cdot (8 \cdot m + 7)$ where k, m are no-negative integers. Prove that the considered equation has no positive integral solutions.

57. In the sequence $00, 01, 02, 03, \ldots, 99$ the terms are rearranged so that each term is obtained from the previous one by increasing or decreasing one of its digits by 1 (for example, 29 can be followed by $19, 39$, or 28, but not by 30 or 20). What is the maximal number of terms that could remain on their places?

58. Each term of a sequence of positive integers is obtained from the previous term by adding to it its largest digit. What is the maximal number of successive odd terms in such a sequence?

59. Determine all integers a and b such that

$$(19a + b)^{18} + (a + b)^{18} + (a + 19b)^{18}$$

is a perfect square.

60. Let a be a non-zero real number. For each integer n, we define $S_n = a^n + a^{-n}$. Prove that if for some integer k, the sums S_k and S_{k+1} are integers, then the sums S_n are integers for all integers n.

61. Find all pairs (a, b) of rational numbers such that $|a - b| = |ab(a + b)|$.

1.1.4 Amir Hossein - Part 4

62. Find all positive integers x, y such that

$$y^3 - 3^x = 100.$$

63. Notice that in the fraction $\frac{16}{64}$ we can perform a simplification as $\frac{1\cancel{6}}{\cancel{6}4} = \frac{1}{4}$ obtaining a correct equality. Find all fractions whose numerators and denominators are two-digit positive integers for which such a simplification is correct.

64. Let a and b be coprime integers, greater than or equal to 1. Prove that all integers n greater than or equal to $(a-1)(b-1)$ can be written in the form:

$$n = ua + vb, \qquad \text{with}(u, v) \in \mathbb{N} \times \mathbb{N}.$$

65. Consider the set E consisting of pairs of integers (a, b), with $a \geq 1$ and $b \geq 1$, that satisfy in the decimal system the following properties:

- **(i)** b is written with three digits, as $\overline{\alpha_2\alpha_1\alpha_0}$, $\alpha_2 \neq 0$;
- **(ii)** a is written as $\overline{\beta_p \dots \beta_1\beta_0}$ for some p;
- **(iii)** $(a+b)^2$ is written as $\overline{\beta_p \dots \beta_1\beta_0\alpha_2\alpha_1\alpha_0}$.

Find the elements of E.

66. For $k = 1, 2, \dots$ consider the k-tuples $(a_1, a_2, \dots, a_k)$ of positive integers such that

$$a_1 + 2a_2 + \cdots + ka_k = 1979.$$

Show that there are as many such k-tuples with odd k as there are with even k.

67. Show that for no integers $a \geq 1, n \geq 1$ is the sum

$$1 + \frac{1}{1+a} + \frac{1}{1+2a} + \cdots + \frac{1}{1+na}$$

an integer.

68. Describe which positive integers do not belong to the set

$$E = \left\{ \lfloor n + \sqrt{n} + \frac{1}{2} \rfloor | n \in \mathbb{N} \right\}.$$

69. Find all non-negative integers a for which $49a^3 + 42a^2 + 11a + 1$ is a perfect cube.

70. For $n \in \mathbb{N}$, let $f(n)$ be the number of positive integers $k \leq n$ that do not contain the digit 9. Does there exist a positive real number p such that $\frac{f(n)}{n} \geq p$ for all positive integers n?

71. Let m, n, and d be positive integers. We know that the numbers m^2n+1 and mn^2+1 are both divisible by d. Show that the numbers m^3+1 and n^3+1 are also divisible by d.

72. Find all pairs (a, b) of positive rational numbers such that

$$\sqrt{a}+\sqrt{b}=\sqrt{4+\sqrt{7}}.$$

73. Let $a_1, a_2, \ldots, a_n, \ldots$ be any permutation of all positive integers. Prove that there exist infinitely many positive integers i such that $\gcd(a_i, a_{i+1}) \leq \frac{3}{4}i$.

74. Let $n > 1$ be an integer, and let k be the number of distinct prime divisors of n. Prove that there exists an integer a, $1 < a < \frac{n}{k}+1$, such that $n \mid a^2 - a$.

75. Let $\{b_n\}_{n\geq 1}^{\infty}$ be a sequence of positive integers. The sequence $\{a_n\}_{n\geq 1}^{\infty}$ is defined as follows: a_1 is a fixed positive integer and

$$a_{n+1} = a_n^{b_n} + 1, \qquad \forall n \geq 1.$$

Find all positive integers $m \geq 3$ with the following property: If the sequence $\{a_n \mod m\}_{n\geq 1}^{\infty}$ is eventually periodic, then there exist positive integers q, u, v with $2 \leq q \leq m-1$, such that the sequence $\{b_{v+ut} \mod q\}_{t\geq 1}^{\infty}$ is purely periodic.

76. Simplify

$$\sum_{k=0}^{n} \frac{(2n)!}{(k!)^2((n-k)!)^2}.$$

77. Find all functions $f : \mathbb{Z} \to \mathbb{Z}$ such that $f(a^3 - b^2) = f(a)^3 - f(b)^2$ holds for all integers $a, b \in \mathbb{Z}$.

78. Find all increasing sequences $\{a_i\}_{i=1}^{\infty}$ such that

$$d(x_1 + x_2 + \cdots + x_k) = d(a_{x_1} + a_{x_2} + \cdots + a_{x_k}),$$

holds for all k-tuples $(x_1, x_2, \cdots, x_k)$ of positive integers, where $d(n)$ is number of integer divisors of a positive integer n, and $k \geq 3$ is a fixed integer.

79. Let y be a prime number and let x, z be positive integers such that z is not divisible by neither y nor 3, and the equation

$$x^3 - y^3 = z^2$$

holds. Find all such triples (x, y, z).

80. Does there exist a positive integer m such that the equation

$$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{abc} = \frac{m}{a+b+c}$$

has infinitely many solutions in positive integers?

1.1.5 Amir Hossein - Part 5

81. Find all distinct positive integers $a_1, a_2, a_3, \ldots, a_n$ such that

$$\frac{1}{a_1} + \frac{2}{a_2} + \frac{3}{a_3} + \cdots + \frac{n}{a_n} = \frac{a_1 + a_2 + a_3 + \cdots + a_n}{2}.$$

82. Show that if $1 + 2^n + 4^n$ is a prime, then $n = 3^k$ for some positive integer k.

83. Show that there exist no natural numbers x, y such that $x^3 + xy^3 + y^2 + 3$ divides $x^2 + y^3 + 3y - 1$.

84. Find all positive integer triples of (a, b, c) so that $2a = b+c$ and $2a^3 = b^3+c^3$.

85. Find all integers $0 \le a_1, a_2, a_3, a_4 \le 9$ such that

$$\overline{a_1a_2a_3a_4} = (\overline{a_1a_4} + 1)(\overline{a_2a_3} + 1).$$

86. Find all integers $a_1, a_2, \ldots a_n$ satisfying $0 \le a_i \le 9$ for all $1 \le i \le n$, and

$$\overline{a_1a_2a_3\ldots a_n} = (\overline{a_1a_2} + 1)(\overline{a_2a_3} + 1)\cdots(\overline{a_{n-1}a_n} + 1).$$

87. Find all integers m, n satisfying the equation

$$n^2 + n + 1 = m^3.$$

88. Solve the equation $x^3 + 48 = y^4$ over positive integers.

89. Find all positive integers a, b, c, d, e, f such that the numbers $\overline{ab}, \overline{cd}, \overline{ef}$, and $\overline{abcdef}$ are all perfect squares.

90. Let $f : \mathbb{N} \to \mathbb{N}$ be an injective function such that there exists a positive integer k for which $f(n) \le n^k$. Prove that there exist infinitely many primes q such that the equation $f(x) \equiv 0 \pmod{q}$ has a solution in prime numbers.

91. Let n and k be two positive integers. Prove that there exist infinitely many perfect squares of the form $n \cdot 2^k - 7$.

92. Let n be a positive integer and suppose that $\phi(n) = \frac{n}{k}$, where k is the greatest perfect square such that $k \mid n$. Let $a_1, a_2, \ldots, a_n$ be n positive integers such that $a_i = p_1^{a_{1i}} \cdot p_2^{a_{2i}} \cdots p_n^{a_{ni}}$, where p_i are prime numbers and a_{ji} are non-negative integers, $1 \le i \le n, 1 \le j \le n$. We know that $p_i \mid \phi(a_i)$, and if $p_i \mid \phi(a_j)$, then $p_j \mid \phi(a_i)$. Prove that there exist integers $k_1, k_2, \ldots, k_m$ with $1 \le k_1 \le k_2 \le \cdots \le k_m \le n$ such that

$$\phi(a_{k_1} \cdot a_{k_2} \cdots a_{k_m}) = p_1 \cdot p_2 \cdots p_n.$$

93. Find all integer solutions to the equation

$$3a^2 - 4b^3 = 7^c.$$

94. Find all non-negative integer solutions of the equation

$$2^x + 3^y = z^2.$$

95. Find all pairs (p, q) of prime numbers such that

$$m^{3pq} \equiv m \pmod{3pq} \qquad \forall m \in \mathbb{Z}.$$

96. Find all functions $f : \mathbb{N} \to \mathbb{N}$ such that

$$f(m)^{2k} + f(n) \mid (m^{2k} + n)^{2k} \qquad \forall m, n, k \in \mathbb{N}.$$

97. Numbers $u_{n,k}$ $(1 \leq k \leq n)$ are defined as follows

$$u_{1,1} = 1, \quad u_{n,k} = \binom{n}{k} - \sum_{d|n, d|k, d>1} u_{n/d,k/d}.$$

(the empty sum is defined to be equal to zero). Prove that $n \mid u_{n,k}$ for every natural number n and for every k $(1 \leq k \leq n)$.

98. Let $(a_n)_{n\geq 0}$ and $(b_n)_{n\geq 0}$ be two sequences of natural numbers. Determine whether there exists a pair (p, q) of natural numbers that satisfy

$$p < q \quad \text{and} \quad a_p \leq a_q, b_p \leq b_q.$$

99. Determine the sum of all positive integers whose digits (in base ten) form either a strictly increasing or a strictly decreasing sequence.

100. Show that the set S of natural numbers n for which $\frac{3}{n}$ cannot be written as the sum of two reciprocals of natural numbers ($S = \left\{n | \frac{3}{n} \neq \frac{1}{p} + \frac{1}{q} \text{ for any } p, q \in \mathbb{N}\right\}$) is not the union of finitely many arithmetic progressions.

1.1.6 Amir Hossein - Part 6

101. Given any two real numbers α and $\beta, 0 \leq \alpha < \beta \leq 1$, prove that there exists a natural number m such that

$$\alpha < \frac{\phi(m)}{m} < \beta.$$

102.
- **(a)** Prove that $\frac{1}{n+1} \cdot \binom{2n}{n}$ is an integer for $n \geq 0$.
- **(b)** Given a positive integer k, determine the smallest integer C_k with the property that $\frac{C_k}{n+k+1} \cdot \binom{2n}{n}$ is an integer for all $n \geq k$.

103. Find all prime numbers p for which the number of ordered pairs of integers (x, y) with $0 \leq x, y < p$ satisfying the condition

$$y^2 \equiv x^3 - x \pmod{p}$$

is exactly p.

104. Let m and n be positive integers. Prove that for each odd positive integer b there are infinitely many primes p such that $p^n \equiv 1 \pmod{b^m}$ implies $b^{m-1}|n$.

105. Let c be a positive integer, and a number sequence $x_1, x_2, \ldots$ satisfy $x_1 = c$ and

$$x_n = x_{n-1} + \left\lfloor \frac{2x_{n-1} - (n+2)}{n} \right\rfloor, \quad n = 2, 3, \ldots$$

Determine the expression of x_n in terms of n and c.

106. Find all positive integers a such that the number

$$A = a^{a+1^{a+2}} + (a+1)^{a+2^{a+3}}$$

is a perfect power of a prime.

107. Find all triples (n, a, b) of positive integers such that the numbers

$$\frac{a^n + b^{n-1}}{a^n - b^{n-1}} \quad \text{and} \quad \frac{b^n + a^{n-1}}{b^n - a^{n-1}}$$

are both integers.

108. Find all positive integers a and b for which

$$\frac{a^2 + b}{b^2 - a^3} \quad \text{and} \quad \frac{b^2 + a}{a^2 - b^3}$$

are both integers.

109. Prove that for every integer $n \geq 2$ there exist n different positive integers such that for any two of these integers a and b their sum $a + b$ is divisible by their difference $a - b$.

110. Find the largest integer N satisfying the following two conditions:

- **(i)** $\left[\frac{N}{3}\right]$ consists of three equal digits;
- **(ii)** $\left[\frac{N}{3}\right] = 1 + 2 + 3 + \cdots + n$ for some positive integer n.

111. Determine a positive constant c such that the equation

$$xy^2 - y^2 - x + y = c$$

has precisely three solutions (x, y) in positive integers.

112. Find all prime numbers p and positive integers m such that $2p^2 + p + 9 = m^2$.

113.
- **a)** Prove that for any positive integer n there exist a pair of positive integers (m, k) such that

$$k + m^k + n^{m^k} = 2009^n.$$

- **b)** Prove that there are infinitely many positive integers n for which there is only one such pair.

114. Let p be a prime. Find number of non-congruent numbers modulo p which are congruent to infinitely many terms of the sequence

$$1, 11, 111, \dots$$

115. Let m, n be two positive integers such that $\gcd(m, n) = 1$. Prove that the equation

$$x^m t^n + y^m s^n = v^m k^n$$

has infinitely many solutions in $\mathbb{N}$.

116. Determine all pairs (n, m) of positive integers for which there exists an infinite sequence $\{x_k\}$ of 0's and 1's with the properties that if $x_i = 0$ then $x_{i+m} = 1$ and if $x_i = 1$ then $x_{i+n} = 0$.

117. The sequence $a_{n,k}$, $k = 1, 2, 3, \dots, 2^n$, $n = 0, 1, 2, \dots$, is defined by the following recurrence formula:

$$a_1 = 2, \qquad a_{n,k} = 2a_{n-1,k}^3, \qquad , a_{n,k+2^{n-1}} = \frac{1}{2}a_{n-1,k}^3$$

$$\text{for} \quad k = 1, 2, 3, \dots, 2^{n-1} \ , n = 0, 1, 2, \dots$$

Prove that the numbers $a_{n,k}$ are all different.

118. Let p be a prime number greater than 5. Let V be the collection of all positive integers n that can be written in the form $n = kp+1$ or $n = kp-1$ ($k = 1, 2, \dots$). A number $n \in V$ is called *indecomposable* in V if it is impossible to find $k, l \in V$ such that $n = kl$. Prove that there exists a number $N \in V$ that can be factorized into indecomposable factors in V in more than one way.

119. Let z be an integer > 1 and let M be the set of all numbers of the form $z_k = 1 + z + \cdots + z^k$, $k = 0, 1, \dots$. Determine the set T of divisors of at least one of the numbers z_k from M.

120. If p and q are distinct prime numbers, then there are integers x_0 and y_0 such that $1 = px_0 + qy_0$. Determine the maximum value of $b - a$, where a and b are positive integers with the following property: If $a \le t \le b$, and t is an integer, then there are integers x and y with $0 \le x \le q - 1$ and $0 \le y \le p - 1$ such that $t = px + qy$.

1.1.7 Amir Hossein - Part 7

121. Let p be a prime number and n a positive integer. Prove that the product

$$N = \frac{1}{p^{n^2}} \prod_{i=1;2\nmid i}^{2n-1} \left[((p-1)!)\binom{p^2 i}{pi}\right]$$

Is a positive integer that is not divisible by p.

122. Find all integer solutions of the equation

$$x^2 + y^2 = (x - y)^3.$$

123. Note that $8^3 - 7^3 = 169 = 13^2$ and $13 = 2^2 + 3^2$. Prove that if the difference between two consecutive cubes is a square, then it is the square of the sum of two consecutive squares.

124. Let $x_n = 2^{2^n} + 1$ and let m be the least common multiple of $x_2, x_3, \ldots, x_{1971}$. Find the last digit of m.

125. Let us denote by $s(n) = \sum_{d|n} d$ the sum of divisors of a positive integer n (1 and n included). If n has at most 5 distinct prime divisors, prove that $s(n) < \frac{77}{16}n$. Also prove that there exists a natural number n for which $s(n) < \frac{76}{16}n$ holds.

126. Let x and y be two real numbers. Prove that the equations

$$\lfloor x \rfloor + \lfloor y \rfloor = \lfloor x + y \rfloor, \quad \lfloor -x \rfloor + \lfloor -y \rfloor = \lfloor -x - y \rfloor$$

Holds if and only if at least one of x or y be integer.

127. Does there exist a number $n = \overline{a_1a_2a_3a_4a_5a_6}$ such that $\overline{a_1a_2a_3} + 4 = \overline{a_4a_5a_6}$ (all bases are 10) and $n = a^k$ for some positive integers a, k with $k \geq 3$?

128. Find the smallest positive integer for which when we move the last right digit of the number to the left, the remaining number be $\frac{3}{2}$ times of the original number.

129.

- **(a)** Solve the equation $m! + 2 = n^2$ in positive integers.
- **(b)** Solve the equation $m! + 1 = n^2$ in positive integers.
- **(c)** Solve the equation $m! + k = n^2$ in positive integers.

130. Solve the following system of equations in positive integers

$$\begin{cases} a^3 - b^3 - c^3 = 3abc \\ \\ a^2 = 2(b + c) \end{cases}$$

131. Let n be a positive integer. 1369^n positive rational numbers are given with this property: if we remove one of the numbers, then we can divide remain numbers into 1368 sets with equal number of elements such that the product of the numbers of the sets be equal. Prove that all of the numbers are equal.

132. Let $\{a_n\}_{n=1}^{\infty}$ be a sequence such that $a_1 = \frac{1}{2}$ and

$$a_n = \left(\frac{2n - 3}{2n}\right) a_{n-1} \qquad \forall n \geq 2.$$

Prove that for every positive integer n, we have $\sum_{k=1}^{n} a_k < 1$.

133. Let $f : \mathbb{N} \to \mathbb{N}$ be a function satisfying

$$f(f(m) + f(n)) = m + n \qquad \forall m, n \in \mathbb{N}.$$

Prove that $f(x) = x \quad \forall x \in \mathbb{N}$.

134. Solve the equation $x^2y^2 + y^2z^2 + z^2x^2 = z^4$ in integers.

135.

- **(a)** For every positive integer n prove that

$$1 + \frac{1}{2^2} + \frac{1}{3^2} + \cdots + \frac{1}{n^2} < 2$$

- **(b)** Let $X = \{1, 2, 3, \ldots, n\}$ $(n \geq 1)$ and let A_k be non-empty subsets of X $(k = 1, 2, 3, \ldots, 2^n - 1)$. If a_k be the product of all elements of the set A_k, prove that

$$\sum_{i=1}^{m}\sum_{j=1}^{m} \frac{1}{a_i \cdot j^2} < 2n + 1$$

136. Find all integer solutions to the equation

$$(x^2 - x)(x^2 - 2x + 2) = y^2 - 1.$$

137. Prove that the equation $x + x^2 = y + y^2 + y^3$ do not have any solutions in positive integers.

138. Let $X \neq \varnothing$ be a finite set and let $f : X \to X$ be a function such that for every $x \in X$ and a fixed prime p we have $f^p(x) = x$. Let $Y = \{x \in X | f(x) \neq x\}$. Prove that the number of the members of the set Y is divisible by p.

139. Prove that for any positive integer t,

$$1 + 2^t + 3^t + \cdots + 9^t - 3(1 + 6^t + 8^t)$$

is divisible by 18.

140. Let $n > 3$ be an odd positive integer and $n = \prod_{i=1}^{k} p_i^{\alpha_i}$ where p_i are primes and α_i are positive integers. We know that

$$m = n(1 - \frac{1}{p_1})(1 - \frac{1}{p_2})(1 - \frac{1}{p_3}) \cdots (1 - \frac{1}{p_n}).$$

Prove that there exists a prime P such that $P | 2^m - 1$ but $P \nmid n$.

1.1.8 Amir Hossein - Part 8

141. Let $\overline{a_1a_2a_3\ldots a_n}$ be the representation of a $n-$digits number in base 10. Prove that there exists a one-to-one function like $f : \{0, 1, 2, 3, \ldots, 9\} \to \{0, 1, 2, 3, \ldots, 9\}$ such that $f(a_1) \neq 0$ and the number $\overline{f(a_1)f(a_2)f(a_3)\ldots f(a_n)}$ is divisible by 3.

142. Let n, r be positive integers. Find the smallest positive integer m satisfying the following condition. For each partition of the set $\{1, 2, \dots, m\}$ into r subsets $A_1, A_2, \dots, A_r$, there exist two numbers a and b in some $A_i, 1 \le i \le r$, such that

$$1 < \frac{a}{b} < 1 + \frac{1}{n}.$$

143. Let $n \ge 0$ be an integer. Prove that

$$\lceil \sqrt{n} + \sqrt{n+1} + \sqrt{n+2} \rceil = \lceil \sqrt{9n+8} \rceil$$

Where $\lceil x \rceil$ is the smallest integer which is greater or equal to x.

144. Prove that for every positive integer $n \ge 3$ there exist two sets $A = \{x_1, x_2, \dots, x_n\}$ and $B = \{y_1, y_2, \dots, y_n\}$ for which
i) $A \cap B = \varnothing$.
ii) $x_1 + x_2 + \cdots + x_n = y_1 + y_2 + \cdots + y_n$.
iii) $x_1^2 + x_2^2 + \cdots + x_n^2 = y_1^2 + y_2^2 + \cdots + y_n^2$.

145. Let $x \ge 1$ be a real number. Prove or disprove that there exists a positive integer n such that $\gcd([x], [nx]) = 1$.

146. Find all pairs of positive integers a, b such that

$$ab = 160 + 90 \gcd(a, b)$$

147. Find all prime numbers p, q and r such that $p > q > r$ and the numbers $p - q, p - r$ and $q - r$ are also prime.

148. Let a, b, c be positive integers. Prove that $a^2 + b^2 + c^2$ is divisible by 4, if and only if a, b, c are even.

149. Let a, b and c be nonzero digits. Let p be a prime number which divides the three digit numbers $\overline{abc}$ and $\overline{cba}$. Show that p divides at least one of the numbers $a + b + c, a - b + c$ and $a - c$.

150. Find the smallest three-digit number such that the following holds: If the order of digits of this number is reversed and the number obtained by this is added to the original number, the resulting number consists of only odd digits.

151. Find all prime numbers p, q, r such that

$$15p + 7pq + qr = pqr.$$

152. Let p be a prime number. A rational number x, with $0 < x < 1$, is written in lowest terms. The rational number obtained from x by adding p to both the numerator and the denominator differs from x by $\frac{1}{p^2}$. Determine all rational numbers x with this property.

153. Prove that the two last digits of 9^{9^9} and $9^{9^{9^9}}$ in decimal representation are equal.

154. Let n be an even positive integer. Show that there exists a permutation $(x_1, x_2, \ldots, x_n)$ of the set $\{1, 2, \ldots, n\}$, such that for each $i \in \{1, 2, \ldots, n\}, x_{i+1}$ is one of the numbers $2x_i, 2x_i - 1, 2x_i - n, 2x_i - n - 1$, where $x_{n+1} = x_1$.

155. A prime number p and integers x, y, z with $0 < x < y < z < p$ are given. Show that if the numbers x^3, y^3, z^3 give the same remainder when divided by p, then $x^2 + y^2 + z^2$ is divisible by $x + y + z$.

156. Let x be a positive integer and also let it be a perfect cube. Let n be number of the digits of x. Can we find a general form for n ?

157. Define the sequence (x_n) by $x_0 = 0$ and for all $n \in \mathbb{N}$,

$$x_n = \begin{cases} x_{n-1} + (3^r - 1)/2, & \text{if } n = 3^{r-1}(3k+1); \\ x_{n-1} - (3^r + 1)/2, & \text{if } n = 3^{r-1}(3k+2). \end{cases}$$

where $k \in \mathbb{N}_0, r \in \mathbb{N}$. Prove that every integer occurs in this sequence exactly once.

158. Show that there exists a positive integer N such that the decimal representation of 2000^N starts with the digits 200120012001.

159. Find all natural numbers m such that

$$1! \cdot 3! \cdot 5! \cdots (2m-1)! = \left(\frac{m(m+1)}{2}\right)!.$$

160. Find all functions $f : \mathbb{N} \to \mathbb{N}$ such that for all positive integers m, n,

- **(i)** $mf(f(m)) = (f(m))^2$,
- **(ii)**If $\gcd(m, n) = d$, then $f(mn) \cdot f(d) = d \cdot f(m) \cdot f(n)$,
- **(iii)** $f(m) = m$ if and only if $m = 1$.

1.1.9 Amir Hossein - Part 9

161. Find all solutions $(x, y) \in \mathbb{Z}^2$ of the equation

$$x^3 - y^3 = 2xy + 8.$$

162. We are given $2n$ natural numbers

$$1, 1, 2, 2, 3, 3, \ldots, n-1, n-1, n, n.$$

Find all n for which these numbers can be arranged in a row such that for each $k \leq n$, there are exactly k numbers between the two numbers k.

163. Let n be a positive integer and let $x_1, x_2, \ldots, x_n$ be positive and distinct integers such that for every positive integer k,

$$x_1 x_2 x_3 \cdots x_n | (x_1 + k)(x_2 + k) \cdots (x_n + k).$$

Prove that

$$\{x_1, x_2, \ldots, x_n\} = \{1, 2, \ldots, n\}.$$

164. Let n be a positive integer, prove that

$$\operatorname{lcm}(1, 2, 3, \dots, n) \geq 2^{n-1}.$$

165. Find all positive integers $n > 1$ such that for every integer a we have

$$n | a^{25} - a.$$

166. Let p, q be two consecutive odd primes. Prove that $p+q$ has at least three prime divisors (not necessary distinct).

167. Find all positive integers x, y such that

$$x^2 + 3^x = y^2.$$

168. Find all positive integers n such that we can divide the set $\{1, 2, 3, \dots, n\}$ into three sets with the same sum of members.

169. Let a and b be integers. Is it possible to find integers p and q such that the integers $p+na$ and $q+nb$ have no common prime factor no matter how the integer n is chosen?

170. In the system of base $n^2 + 1$ find a number N with n different digits such that:

- **(i)** N is a multiple of n. Let $N = nN'$.
- **(ii)** The number N and N' have the same number n of different digits in base $n^2 + 1$, none of them being zero.
- **(iii)** If $s(C)$ denotes the number in base $n^2 + 1$ obtained by applying the permutation s to the n digits of the number C, then for each permutation s, $s(N) = ns(N')$.

171. Consider the expansion

$$(1 + x + x^2 + x^3 + x^4)^{496} = a_0 + a_1 x + \cdots + a_{1984} x^{1984}.$$

- **(a)** Determine the greatest common divisor of the coefficients $a_3, a_8, a_{13}, \dots, a_{1983}$.
- **(b)** Prove that $10^{340} < a_{992} < 10^{347}$.

172. For every $a \in \mathbb{N}$ denote by $M(a)$ the number of elements of the set

$$\{b \in \mathbb{N} | a + b \text{ is a divisor of } ab\}.$$

Find $\max_{a \leq 1983} M(a)$.

173. Find all positive integers k, m such that

$$k! + 48 = 48(k+1)^m.$$

174. Solve the equation

$$5^x \times 7^y + 4 = 3^z$$

in integers.

175. Let a, b, c be positive integers satisfying $\gcd(a, b) = \gcd(b, c) = \gcd(c, a) = 1$. Show that $2abc - ab - bc - ca$ cannot be represented as $bcx + cay + abz$ with nonnegative integers x, y, z.

176. Does there exist an infinite number of sets C consisting of 1983 consecutive natural numbers such that each of the numbers is divisible by some number of the form a^{1983}, with $a \in \mathbb{N}, a \neq 1$?

177. Let $b \geq 2$ be a positive integer.

- **(a)** Show that for an integer N, written in base b, to be equal to the sum of the squares of its digits, it is necessary either that $N = 1$ or that N have only two digits.
- **(b)** Give a complete list of all integers not exceeding 50 that, relative to some base b, are equal to the sum of the squares of their digits.
- **(c)** Show that for any base b the number of two-digit integers that are equal to the sum of the squares of their digits is even.
- **(d)** Show that for any odd base b there is an integer other than 1 that is equal to the sum of the squares of its digits.

178. Let p be a prime number and $a_1, a_2, \dots, a_{(p+1)/2}$ different natural numbers less than or equal to p. Prove that for each natural number r less than or equal to p, there exist two numbers (perhaps equal) a_i and a_j such that

$$p \equiv a_i a_j \pmod{r}.$$

179. Which of the numbers $1, 2, \dots, 1983$ has the largest number of divisors?

180. Find all numbers $x \in \mathbb{Z}$ for which the number

$$x^4 + x^3 + x^2 + x + 1$$

is a perfect square.

1.1.10 Amir Hossein - Part 10

181. Find the last two digits of a sum of eighth powers of 100 consecutive integers.

182. Find all positive numbers p for which the equation $x^2 + px + 3p = 0$ has integral roots.

183. Let $a_1, a_2, \ldots, a_n$ $(n \geq 2)$ be a sequence of integers. Show that there is a subsequence $a_{k_1}, a_{k_2}, \ldots, a_{k_m}$, where $1 \leq k_1 < k_2 < \cdots < k_m \leq n$, such that $a_{k_1}^2 + a_{k_2}^2 + \cdots + a_{k_m}^2$ is divisible by n.

184.

- **(a)** Find the number of ways 500 can be represented as a sum of consecutive integers.
- **(b)** Find the number of such representations for $N = 2^\alpha 3^\beta 5^\gamma, \alpha, \beta, \gamma \in \mathbb{N}$. Which of these representations consist only of natural numbers ?
- **(c)** Determine the number of such representations for an arbitrary natural number N.

185. Find digits x, y, z such that the equality

$$\sqrt{\underbrace{\overline{xx\cdots x}}_{n \text{ times}} - \underbrace{\overline{yy\cdots y}}_{n \text{ times}}} = \underbrace{\overline{zz\cdots z}}_{n \text{ times}}$$

holds for at least two values of $n \in \mathbb{N}$, and in that case find all n for which this equality is true.

186. Does there exist an integer z that can be written in two different ways as $z = x! + y!$, where x, y are natural numbers with $x \leq y$?

187. Let p be a prime. Prove that the sequence

$$1^1, 2^2, 3^3, \ldots, n^n, \ldots$$

is periodic modulo p, i.e. the sequence obtained from remainders of this sequence when dividing by p is periodic.

188. Let $a_0, a_1, \ldots, a_k$ $(k \geq 1)$ be positive integers. Find all positive integers y such that

$$a_0|y, (a_0 + a_1)|(y + a1), \ldots, (a_0 + a_n)|(y + a_n).$$

189. For which digits a do exist integers $n \geq 4$ such that each digit of $\frac{n(n+1)}{2}$ equals a ?

190. Show that for any $n \not\equiv 0 \pmod{10}$ there exists a multiple of n not containing the digit 0 in its decimal expansion.

191. Let a_i, b_i be coprime positive integers for $i = 1, 2, \ldots, k$, and m the least common multiple of $b_1, \ldots, b_k$. Prove that the greatest common divisor of $a_1 \frac{m}{b_1}, \ldots, a_k \frac{m}{b_k}$ equals the greatest common divisor of $a_1, \ldots, a_k$.

192. Find the integer represented by $\left[\sum_{n=1}^{10^9} n^{-2/3}\right]$. Here $[x]$ denotes the greatest integer less than or equal to x.

193. Prove that for any positive integers x, y, z with $xy - z^2 = 1$ one can find non-negative integers a, b, c, d such that $x = a^2 + b^2, y = c^2 + d^2, z = ac + bd$. Set $z = (2q)!$ to deduce that for any prime number $p = 4q + 1$, p can be represented as the sum of squares of two integers.

194. Let p be a prime and $A = \{a_1, \ldots, a_{p-1}\}$ an arbitrary subset of the set of natural numbers such that none of its elements is divisible by p. Let us define a mapping f from $\mathcal{P}(A)$ (the set of all subsets of A) to the set $P = \{0, 1, \ldots, p-1\}$ in the following way:

- **(i)** if $B = \{a_{i_1}, \ldots, a_{i_k}\} \subset A$ and $\sum_{j=1}^{k} a_{i_j} \equiv n \pmod p$, then $f(B) = n$,
- **(ii)** $f(\emptyset) = 0$, $\emptyset$ being the empty set.

Prove that for each $n \in P$ there exists $B \subset A$ such that $f(B) = n$.

195. Let S be the set of all the odd positive integers that are not multiples of 5 and that are less than $30m$, m being an arbitrary positive integer. What is the smallest integer k such that in any subset of k integers from S there must be two different integers, one of which divides the other?

196. Let m be an positive odd integer not divisible by 3. Prove that $\left[4^m - (2+\sqrt{2})^m\right]$ is divisible by 112.

197. Let $n \geq 4$ be an integer. $a_1, a_2, \ldots, a_n \in (0, 2n)$ are n distinct integers. Prove that there exists a subset of the set $\{a_1, a_2, \ldots, a_n\}$ such that the sum of its elements is divisible by $2n$.

198. The sequence $\{u_n\}$ is defined by $u_1 = 1, u_2 = 1, u_n = u_{n-1} + 2u_{n-2} forn \geq 3$. Prove that for any positive integers n, p $(p > 1), u_{n+p} = u_{n+1}u_p + 2u_nu_{p-1}$. Also find the greatest common divisor of u_n and u_{n+3}.

199. Let a, b, c be integers. Prove that there exist integers p_1, q_1, r_1, p_2, q_2 and r_2, satisfying $a = q_1r_2 - q_2r_1, b = r_1p_2 - r_2p_1$ and $c = p_1q_2 - p_2q_1$.

200. Let α be the positive root of the quadratic equation $x^2 = 1990x + 1$. For any $m, n \in \mathbb{N}$, define the operation $m * n = mn + [\alpha m][\alpha n]$, where $[x]$ is the largest integer no larger than x. Prove that $(p * q) * r = p * (q * r)$ holds for all $p, q, r \in \mathbb{N}$.

1.1.11 Amir Hossein - Part 11

201. Prove that there exist infinitely many positive integers n such that the number $\frac{1^2+2^2+\cdots+n^2}{n}$ is a perfect square. Obviously, 1 is the least integer having this property. Find the next two least integers having this property.

202. Find, with proof, the least positive integer n having the following property: in the binary representation of $\frac{1}{n}$, all the binary representations of $1, 2, \ldots, 1990$ (each consist of consecutive digits) are appeared after the decimal point.

203. We call an integer $k \geq 1$ having property P, if there exists at least one integer $m \geq 1$ which cannot be expressed in the form $m = \varepsilon_1 z_1^k + \varepsilon_2 z_2^k + \cdots + \varepsilon_{2k} z_{2k}^k$, where z_i are nonnegative integer and $\varepsilon_i = 1$ or -1, $i = 1, 2, \ldots, 2k$. Prove that there are infinitely many integers k having the property P.

204. Let N be the number of integral solutions of the equation

$$x^2 - y^2 = z^3 - t^3$$

satisfying the condition $0 \leq x, y, z, t \leq 10^6$, and let M be the number of integral solutions of the equation

$$x^2 - y^2 = z^3 - t^3 + 1$$

satisfying the condition $0 \leq x, y, z, t \leq 10^6$. Prove that $N > M$.

205. Consider the sequences $(a_n), (b_n)$ defined by

$$a_1 = 3, \quad b_1 = 100, \quad a_{n+1} = 3^{a_n}, \quad b_{n+1} = 100^{b_n}$$

Find the smallest integer m for which $b_m > a_{100}$.

206. Let m positive integers $a_1, \ldots, a_m$ be given. Prove that there exist fewer than 2^m positive integers $b_1, \ldots, b_n$ such that all sums of distinct b_ks are distinct and all a_i $(i \leq m)$ occur among them.

207. Let $n \geq 2$ be an integer. Find the maximal cardinality of a set M of pairs (j, k) of integers, $1 \leq j < k \leq n$, with the following property: If $(j, k) \in M$, then $(k, m) \notin M$ for any m.

208. Determine the smallest natural number n having the following property: For every integer $p, p \geq n$, it is possible to subdivide (partition) a given square into p squares (not necessarily equal).

209. Are there integers m and n such that

$$5m^2 - 6mn + 7n^2 = 1985 \ ?$$

210. Find all triples of positive integers x, y, z satisfying

$$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{4}{5}.$$

211. For every integer $r > 1$ find the smallest integer $h(r) > 1$ having the following property: For any partition of the set $\{1, 2, ..., h(r)\}$ into r classes, there exist integers $a \geq 0, 1 \leq x \leq y$ such that the numbers $a+x, a+y, a+x+y$ are contained in the same class of the partition.

212. Let $k \geq 2$ and $n_1, n_2, ..., n_k \geq 1$ natural numbers having the property $n_2 | 2^{n_1} - 1, n_3 | 2^{n_2} - 1, \cdots, n_k | 2^{n_{k-1}} - 1$, and $n_1 | 2^{n_k} - 1$. Show that $n_1 = n_2 = \cdots = n_k = 1$.

213. Let p be a prime. For which k can the set $\{1, 2, \ldots, k\}$ be partitioned into p subsets with equal sums of elements ?

214. Prove that there are infinitely many pairs (k, N) of positive integers such that $1 + 2 + \cdots + k = (k+1) + (k+2) + \cdots + N$.

215. Set $S_n = \sum_{p=1}^{n}(p^5 + p^7)$. Determine the greatest common divisor of S_n and S_{3n}.

216.
- **a)** Call a four-digit number $(xyzt)_B$ in the number system with base B stable if $(xyzt)_B = (dcba)_B - (abcd)_B$, where $a \le b \le c \le d$ are the digits of $(xyzt)_B$ in ascending order. Determine all stable numbers in the number system with base B.
- **b)** With assumptions as in **a** , determine the number of bases $B \le 1985$ such that there is a stable number with base B.

217. Find eight positive integers $n_1, n_2, \dots, n_8$ with the following property:
For every integer k, $-1985 \le k \le 1985$, there are eight integers $a_1, a_2, \dots, a_8$, each belonging to the set $\{-1, 0, 1\}$, such that $k = \sum_{i=1}^{8} a_i n_i$.

218. Solve the equation $2a^3 - b^3 = 4$ in integers.

219. Let p be an odd prime. Find all (x, y) pairs of positive integers such that $p^x - y^p = 1$.

220. Let $\mathbb{N} = 1, 2, 3, \dots.$ For real x, y, set $S(x, y) = \{s | s = [nx + y], n \in \mathbb{N}\}$. Prove that if $r > 1$ is a rational number, there exist real numbers u and v such that

$$S(r, 0) \cap S(u, v) = \emptyset, S(r, 0) \cup S(u, v) = \mathbb{N}.$$

1.1.12 Amir Hossein - Part 12

221. Let A be a set of positive integers such that for any two elements x, y of A, $|x - y| \ge \frac{xy}{25}$. Prove that A contains at most nine elements. Give an example of such a set of nine elements.

222. Let k be a positive integer. Define $u_0 = 0, u_1 = 1$, and $u_n = ku_{n-1} - u_{n-2}, n \ge 2$. Show that for each integer n, the number $u_1^3 + u_2^3 + \cdots + u_n^3$ is a multiple of $u_1 + u_2 + \cdots + u_n$.

223. Find the average of the quantity

$$(a_1 - a_2)^2 + (a_2 - a_3)^2 + \cdots + (a_{n-1} - a_n)^2$$

taken over all permutations $(a_1, a_2, \dots, a_n)$ of $(1, 2, \dots, n)$.

224. Let a and b be integers and n a positive integer. Prove that

$$\frac{b^{n-1}a(a+b)(a+2b)\cdots(a+(n-1)b)}{n!}$$

is an integer.

225. Let d_n be the last nonzero digit of the decimal representation of $n!$. Prove that d_n is aperiodic; that is, there do not exist T and n_0 such that for all $n \ge n_0, d_{n+T} = d_n$.

226. Let n be a positive integer having at least two different prime factors. Show that there exists a permutation $a_1, a_2, \ldots, a_n$ of the integers $1, 2, \ldots, n$ such that
$$\sum_{k=1}^{n} k \cdot \cos \frac{2\pi a_k}{n} = 0.$$

227. Decide whether there exists a set M of positive integers satisfying the following conditions:

- **(i)** For any natural number $m > 1$ there are $a, b \in M$ such that $a+b = m$.
- **(ii)** If $a, b, c, d \in M$, $a, b, c, d > 10$ and $a + b = c + d$, then $a = c$ or $a = d$.

228. Let a be a positive integer and let $\{a_n\}$ be defined by $a_0 = 0$ and
$$a_{n+1} = (a_n + 1)a + (a + 1)a_n + 2\sqrt{a(a + 1)a_n(a_n + 1)} \qquad (n = 1, 2, \ldots).$$
Show that for each positive integer n, a_n is a positive integer.

229. Let n be a positive integer. Let $\sigma(n)$ be the sum of the natural divisors d of n (including 1 and n). We say that an integer $m \geq 1$ is *superabundant* (P.Erdos, 1944) if $\forall k \in \{1, 2, \ldots, m-1\}$, $\frac{\sigma(m)}{m} > \frac{\sigma(k)}{k}$. Prove that there exists an infinity of *superabundant* numbers.

230. In a permutation $(x_1, x_2, \ldots, x_n)$ of the set $1, 2, \ldots, n$ we call a pair (x_i, x_j) discordant if $i < j$ and $x_i > x_j$. Let $d(n, k)$ be the number of such permutations with exactly k discordant pairs. Find $d(n, 2)$ and $d(n, 3)$.

231. Let n be a positive integer and $a_1, a_2, \ldots, a_{2n}$ mutually distinct integers. Find all integers x satisfying
$$(x - a_1) \cdot (x - a_2) \cdots (x - a_{2n}) = (-1)^n (n!)^2.$$

232. Prove that the product of five consecutive positive integers cannot be the square of an integer.

233. Find all positive integers n such that
$$n = d_6^2 + d_7^2 - 1,$$
where $1 = d_1 < d_2 < \cdots < d_k = n$ are all positive divisors of the number n.

234. Prove:
(a) There are infinitely many triples of positive integers m, n, p such that $4mn - m - n = p^2 - 1$.
(b) There are no positive integers m, n, p such that $4mn - m - n = p^2$.

235. It is given that $x = -2272$, $y = 10^3 + 10^2 c + 10b + a$, and $z = 1$ satisfy the equation $ax + by + cz = 1$, where a, b, c are positive integers with $a < b < c$. Find y.

236. Find, with argument, the integer solutions of the equation

$$3z^2 = 2x^3 + 385x^2 + 256x - 58195.$$

237. Let n be a natural number. Solve in integers the equation

$$x^n + y^n = (x-y)^{n+1}.$$

238. Find the integer solutions of the equation

$$\left[\sqrt{2m}\right] = \left[n(2+\sqrt{2})\right].$$

239. Solve the equation $28^x = 19^y + 87^z$, where x, y, z are integers.

240. Numbers $d(n,m)$, with m,n integers, $0 \le m \le n$, are defined by $d(n,0) = d(n,n) = 0$ for all $n \ge 0$ and

$$md(n,m) = md(n-1,m) + (2n-m)d(n-1,m-1) \text{ for all } 0 < m < n.$$

Prove that all the $d(n,m)$ are integers.

1.1.13 Amir Hossein - Part 13

241. Determine the least possible value of the natural number n such that $n!$ ends in exactly 1987 zeros.

242. Let $x_1, x_2, \cdots, x_n$ be n integers. Let $n = p+q$, where p and q are positive integers. For $i = 1, 2, \cdots, n$, put

$$S_i = x_i + x_{i+1} + \cdots + x_{i+p-1} \text{ and } T_i = x_{i+p} + x_{i+p+1} + \cdots + x_{i+n-1}$$

(it is assumed that $x_{i+n} = x_i$ for all i). Next, let $m(a,b)$ be the number of indices i for which S_i leaves the remainder a and T_i leaves the remainder b on division by 3, where $a, b \in \{0,1,2\}$. Show that $m(1,2)$ and $m(2,1)$ leave the same remainder when divided by 3.

243. Five distinct positive integers form an arithmetic progression. Can their product be equal to a^{2008} for some positive integer a ?

244. Given three distinct positive integers such that one of them is the average of the two others. Can the product of these three integers be the perfect 2008th power of a positive integer?

245. Find all positive integers a and b such that $(a+b^2)(b+a^2) = 2^m$ for some integer m.

246. Denote by $[n]!$ the product $1 \cdot 11 \cdot 111 \cdot \ldots \cdot \underbrace{111...1}_{\text{n ones}}$.($n$ factors in total). Prove that $[n+m]!$ is divisible by $[n]! \times [m]!$

247. Are there positive integers $a; b; c$ and d such that $a^3 + b^3 + c^3 + d^3 = 100^{100}$?

248. A sequence $\{an\}$ of positive integers is defined by

$$a_n = \left[n + \sqrt{n} + \frac{1}{2}\right], \qquad \forall n \in \mathbb{N}.$$

Determine the positive integers that occur in the sequence.

249. Let a and b be integers. Prove that $\frac{2a^2-1}{b^2+2}$ is not an integer.

250. Suppose that $n > m \geq 1$ are integers such that the string of digits 143 occurs somewhere in the decimal representation of the fraction $\frac{m}{n}$. Prove that $n > 125$.

251. Prove that the sequence $5, 12, 19, 26, 33, \cdots$ contains no term of the form $2^n - 1$.

252. Let n be a positive integer. Prove that the number of ways to express n as a sum of distinct positive integers (up to order) and the number of ways to express n as a sum of odd positive integers (up to order) are the same.

253. Find the number of positive integers n satisfying $\phi(n)|n$ such that

$$\sum_{m=1}^{\infty} \left(\left[\frac{n}{m}\right] - \left[\frac{n-1}{m}\right] \right) = 1992$$

What is the largest number among them? As usual, $\phi(n)$ is the number of positive integers less than or equal to n and relatively prime to n.

254. Let a, b, c be integers. Prove that there are integers $p_1, q_1, r_1, p_2, q_2, r_2$ such that

$$a = q_1 r_2 - q_2 r_1, b = r_1 p_2 - r_2 p_1, c = p_1 q_2 - p_2 q_1.$$

255. Integers $a_1, a_2, ..., a_n$ satisfy $|a_k| = 1$ and

$$\sum_{k=1}^{n} a_k a_{k+1} a_{k+2} a_{k+3} = 2,$$

where $a_{n+j} = a_j$. Prove that $n \neq 1992$.

256. Suppose that n numbers $x_1, x_2, ..., x_n$ are chosen randomly from the set $\{1, 2, 3, 4, 5\}$. Prove that the probability that $x_1^2 + x_2^2 + \cdots + x_n^2 \equiv 0 \pmod 5$ is at least $\frac{1}{5}$.

257. Solve $19^x + 7^y = z^3$ in positive integers.

258. Prove that the equation $3y^2 = x^4 + x$ doesn't have any positive integer solutions.

259. An *Egyptian number* is a positive integer that can be expressed as a sum of positive integers, not necessarily distinct, such that the sum of their reciprocals is 1. For example, $32 = 2 + 3 + 9 + 18$ is Egyptian because $\frac{1}{2} + \frac{1}{3} + \frac{1}{9} + \frac{1}{18} = 1$. Prove that all integers greater than 23 are *Egyptian*.

260. Find all triples (x, y, z) of integers such that

$$\frac{1}{x^2} + \frac{2}{y^2} + \frac{3}{z^2} = \frac{2}{3}.$$

1.1.14 Amir Hossein - Part 14

261. Let $\phi(n, m), m \neq 1$, be the number of positive integers less than or equal to n that are coprime with m. Clearly, $\phi(m, m) = \phi(m)$, where $\phi(m)$ is Euler's phi function. Find all integers m that satisfy the following inequality:

$$\frac{\phi(n, m)}{n} \geq \frac{\phi(m)}{m}$$

for every positive integer n.

262. Let m be a positive integer and x_0, y_0 integers such that x_0, y_0 are relatively prime, y_0 divides $x_0^2 + m$, and x_0 divides $y_0^2 + m$. Prove that there exist positive integers x and y such that x and y are relatively prime, y divides $x^2 + m$, x divides $y^2 + m$, and $x + y \leq m + 1$.

263. Find four positive integers each not exceeding 70000 and each having more than 100 divisors.

264. Let $P_n = (19 + 92)(19^2 + 92^2) \cdots (19^n + 92^n)$ for each positive integer n. Determine, with proof, the least positive integer m, if it exists, for which P_m is divisible by 33^{33}.

265. Let $(a_n)_{n\in\mathbb{N}}$ be the sequence of integers defined recursively by $a_1 = a_2 = 1, a_{n+2} = 7a_{n+1} - a_n - 2$ for $n \geq 1$. Prove that a_n is a perfect square for every n.

266. Determine all pairs of positive integers (x, y) satisfying the equation $p^x - y^3 = 1$, where p is a given prime number.

267. Given an integer $n \geq 2$, determine all n-digit numbers $M_0 = \overline{a_1 a_2 \cdots a_n}$ $(a_i \neq 0, i = 1, 2, ..., n)$ divisible by the numbers $M_1 = \overline{a_2 a_3 \cdots a_n a_1}$, $M_2 = \overline{a_3 a_4 \cdots a_n a_1 a_2}$, $\cdots$, $M_{n-1} = \overline{a_n a_1 a_2 ... a_{n-1}}$.

268. For given positive integers r, v, n let $S(r, v, n)$ denote the number of n-tuples of non-negative integers $(x_1, \cdots, x_n)$ satisfying the equation $x_1 + \cdots + x_n = r$ and such that $x_i \leq v$ for $i = 1, \cdots, n$. Prove that

$$S(r, v, n) = \sum_{k=0}^{m} (-1)^k \binom{n}{k} \binom{r - (v+1)k + n - 1}{n - 1}$$

Where $m = \left\{ n, \left[\frac{r}{v+1} \right] \right\}$.

269. Find, with proof, all solutions of the equation $\frac{1}{x}+\frac{2}{y}-\frac{3}{z}=1$ in positive integers x, y, z.

270. The positive integers $x_1, \cdots, x_n$, $n \geq 3$, satisfy $x_1 < x_2 < \cdots < x_n < 2x_1$. Set $P = x_1 x_2 \cdots x_n$. Prove that if p is a prime number, k a positive integer, and P is divisible by pk, then $\frac{P}{p^k} \geq n!$.

271. Given a positive integer k, find the least integer n_k for which there exist five sets S_1, S_2, S_3, S_4, S_5 with the following properties:

$$|S_j| = k \text{ for } j = 1, \cdots, 5, \quad |\bigcup_{j=1}^{5} S_j| = n_k;$$

$$|S_i \cap S_{i+1}| = 0 = |S_5 \cap S_1|, \quad \text{for } i = 1, \cdots, 4.$$

272. Find the last eight digits of the binary development of 27^{1986}.

273. Solve $(x^2 - y^2)^2 = 16y + 1$ in integers.

274. Let k be a positive integer. Prove that the equation $\varphi(n) = k!$ has solutions for $n \in \mathbb{N}$.

275. Let F_n be the n-th term of Fibonacci sequence. If $n > 4$ prove that $4|\varphi(F_n)$.

276. Let a, b, c, d be primes such that

$$a^2 - b^2 + c^2 - d^2 = 1749, \quad a > 3b > 6c > 12d$$

Find $a^2 + b^2 + c^2 + d^2$.

277. Find all integers x, y, z such that

$$x^3 + y^3 + z^3 = x + y + z = 8.$$

278. Let $n = p_1^{\alpha_1} p_2^{\alpha_2} \cdots p_k^{\alpha_k}$ be a positive integer where p_i are primes and α_i are positive integers.

Define $f(n) = p_1\alpha_1 + p_2\alpha_2 + \cdots + p_k\alpha_k + 1$. Prove that the number 8 appears in the sequence $n, f(n), f(f(n)), f(f(f(n))), \cdots$ if $n > 6$.

279.
- **(a)** Prove that we can represent any multiple of 6 as sum of cubes of three integers.
- **(b)** Prove that we can represent any integer as sum of cubes of five integers.

280. Solve the equation $x^5 - y^2 = 52$ in positive integers.

1.1.15 Amir Hossein - Part 15

281. Let n be a positive integer and $p > 3$ be a prime. Find at least $3n+3$ integer solutions to the equation

$$xyz = p^n(x+y+z).$$

282. Let n, k be positive integers and $n > 2$. Prove that the equation $x^n - y^n = 2^k$ doesn't have any positive integer solutions.

283. Solve in $\mathbb{Q}$ the equation $y^2 = x^3 - x$.

284. Find all integer solutions to $x^3 + y^4 = z^5$.

285. Find all positive integers m, n such that

$$m^2 + n^2 | m^3 + n, \quad m^2 + n^2 | n^3 + m.$$

286.
- **(a)** Let n, d be positive integers such that $d | 2n^2$. Prove that $n^2 + d$ is not a perfect square.
- **(b)** Let p be a prime and n be a positive integer. Prove that pn^2 has at most one divisor d such that $n^2 + d$ is a perfect square.

287. Let a, b be distinct real numbers such that the numbers

$$a-b,\ a^2-b^2,\ a^3-b^3, \dots$$

Are all integers. Prove that a, b are integers.

288. Find all positive integers such that we can represent them as

$$\frac{(a+b+c)^2}{abc}$$

Where a, b, c are positive integers.

289. Find all (m, n) pairs of integers such that

$$m^4 - 3m^3 + 5m^2 - 9m = n^4 - 3n^3 + 5n^2 - 9n$$

and $m \neq n$.

290. Find all positive integers n such that

$$\lfloor \sqrt[4]{1} \rfloor + \lfloor \sqrt[4]{2} \rfloor + \cdots + \lfloor \sqrt[4]{n} \rfloor = 2n.$$

291. Let $\{a_n\}_{n \geq 1}$ be a sequence of positive integers such that if $i \neq j$, then

$$\gcd(a_i, a_j) = \gcd(i, j)$$

Prove that $a_n = n, \quad \forall n \geq 1$.

292. Solve the system of equations in integers

$$\begin{cases} (4x)_5 + 7y = 14 \\ (2y)_5 - (3x)_7 = 74 \end{cases}$$

Where $(n)_k$ is the closest multiple of k to n.

293. Prove that for every natural number k ($k \geq 2$) there exists an irrational number r such that for every natural number m,

$$[r^m] \equiv -1 \pmod{k}.$$

294. Let $n \geq 2$ be an integer. Prove that if $k^2 + k + n$ is prime for all integers k such that $0 \leq k \leq \sqrt{\frac{n}{3}}$, then $k^2 + k + n$ is prime for all integers k such that $0 \leq k \leq n - 2$.

295.
- **(a)** Let $\gcd(m,k) = 1$. Prove that there exist integers $a_1, a_2, \ldots, a_m$ and $b_1, b_2, \ldots, b_k$ such that each product $a_i b_j$ ($i = 1, 2, \cdots, m$; $j = 1, 2, \cdots, k$) gives a different residue when divided by mk.
- **(b)** Let $\gcd(m,k) > 1$. Prove that for any integers $a_1, a_2, ..., a_m$ and $b_1, b_2, ..., b_k$ there must be two products $a_i b_j$ and $a_s b_t$ ($(i,j) \neq (s,t)$) that give the same residue when divided by mk.

296. Determine the least odd number $a > 5$ satisfying the following conditions: There are positive integers m_1, m_2, n_1, n_2 such that $a = m_1^2 + n_1^2$, $a^2 = m_2^2 + n_2^2$, and $m_1 - n_1 = m_2 - n_2$.

297. Prove that for every given positive integer n, there exists a prime p and an integer m such that

- **(a)** $p \equiv 5 \pmod{6}$
- **(b)**$p \nmid n$
- **(c)**$n \equiv m^3 \pmod{p}$

298. Find all (m,n) pairs of integers such that

$$m^2 - 1 | 3^m + (n! - 1)^m.$$

299. Prove that the equation

$$\frac{1}{10^n} = \frac{1}{n_1!} + \frac{1}{n_2!} + \cdots + \frac{1}{n_k!}$$

Doesn't have any integer solutions such that $1 \leq n_1 < n_2 < \cdots < n_k$.

300. Find all primes p, q such that $\frac{(5^p - 2^p)(5^q - 2^q)}{pq} \in \mathbb{Z}$.

1.1.16 Amir Hossein - Part 16

301. Let n be a positive integer and A be an infinite set of positive integers such that for every prime that doesn't divide n, then this prime doesn't divide infinite members of A. Prove that for any positive integer $m > 1$ such that $\gcd(m, n) = 1$, there exists a finite subset of A and:

$$S \equiv 1 \pmod{m} \quad \text{and} \quad S \equiv 0 \pmod{n}.$$

Where S is sum of the members of that subset.

302. Let n be a positive integer. We know that $A(n) = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}$ is a rational number, so we can write it as an irreducible fraction. Let $P(n)$ be the numerator of this fraction. Find-with proof- all positive integers m such that $3|P(m)$.

303. m, n are odd positive integers and $m^2 - n^2 + 1 | n^2 - 1$. Prove that $m^2 - n^2 + 1$ is a perfect square.

304. Find all positive integers m, n such that

$$mn - 1 | m^2 + n^2.$$

305. Find all integer solutions to the equation

$$x^2 + y^2 + z^2 = 16(xy + yz + zx - 1).$$

306. Find all (m, n) pairs of positive integers such that

$$n | m^2 + 1 \qquad , \qquad m | n^3 + 1.$$

307. Find all integer solutions to the equation $2x^2 + 21y = z^2$ such that x, y, z be in a geometric progression.

308. Prove that for any positive integer $n \geq 3$ there exist positive integers $a_1, a_2, \cdots, a_n$ such that

$$a_1 a_2 \cdots a_n \equiv a_i \pmod{a_i^2} \qquad \forall i \in \{1, 2, \cdots, n\}.$$

309. Find all primes p, q such that

$$p^q - q^p = pq^2 - 19.$$

310. Let m, n be two positive integers and $m \geq 2$. We know that for every positive integer a such that $\gcd(a, n) = 1$ we have $n | a^m - 1$. Prove that $n \leq 4m(2^m - 1)$.

311. Find all functions $f : \mathbb{N} \to \mathbb{N}$ such that:

- **i)** $f^{2000}(m) = f(m)$.

- **ii)** $f(mn) = \frac{f(m)f(n)}{f(\gcd(m,n))}$
- **iii)** $f(m) = 1 \iff m = 1$

312. Prove that the equation $x^3 + y^3 + z^3 = t^4$ has infinitely many solutions in positive integers such that $\gcd(x, y, z, t) = 1$.

313. For a positive integer n, denote $rad(n)$ as product of prime divisors of n. And also $rad(1) = 1$. Define the sequence $\{a_i\}_{i=1}^{\infty}$ in this way: $a_1 \in \mathbb{N}$ and for every $n \in \mathbb{N}$, $a_{n+1} = a_n + rad(a_n)$. Prove that for every $N \in \mathbb{N}$, there exist N consecutive terms of this sequence which are in an arithmetic progression.

314. Prove that $2^{2^n} + 2^{2^{n-1}} + 1$ has at least n distinct prime divisors.

315. Does there exists a subset of positive integers with infinite members such that for every two members a, b of this set

$$a^2 - ab + b^2 | (ab)^2.$$

316. Find all $a, b, c \in \mathbb{N}$ such that

$$a^2 b | a^3 + b^3 + c^3, \qquad b^2 c | a^3 + b^3 + c^3, \qquad c^2 a | a^3 + b^3 + c^3.$$

317. Let $\{a_i\}_{i=1}^{\infty}$ be a sequence of positive integers such that $a_1 < a_2 < a_3 \cdots$ and all of primes are members of this sequence. Prove that for every $n < m$,

$$\frac{1}{a_n} + \frac{1}{a_{n+1}} + \cdots + \frac{1}{a_m} \notin \mathbb{N}.$$

318. Find all subsets of $\mathbb{N}$ like S such that

$$\forall m, n \in S \implies \frac{m+n}{\gcd(m,n)} \in S.$$

319. Let $p \geq 3$ be a prime and $a_1, a_2, \cdots, a_{p-2}$ be a sequence of positive integers such that for every $k \in \{1, 2, \cdots, p-2\}$ neither a_k nor $a_k^k - 1$ is divisible by p. Prove that product of some of members of this sequence is equivalent to 2 modulo p.

320. Let $a > 1$ be a positive integer. Prove that the set $\{a^2 + a - 1, a^3 + a - 1, \cdots\}$ have a subset S with infinite members and for any two members of S like x, y we have $\gcd(x, y) = 1$. Then prove that the set of primes has infinite members.

1.1.17 Amir Hossein - Part 17

321. Solve the equation $4xy - x - y = z^2$ in positive integers.

322. Let m, n, k be positive integers and $1 + m + n\sqrt{3} = (2 + \sqrt{3})^{2k+1}$. Prove that m is a perfect square.

323. Let p be a prime such that $p \equiv 3 \pmod 4$. Prove that we can't partition the numbers $a, a+1, a+2, \cdots, a+p-2$,$(a \in \mathbb{Z})$ in two sets such that product of members of the sets be equal.

324. Let a, b be two positive integers and $b^2 + a - 1 | a^2 + b - 1$. Prove that $b^2 + a - 1$ has at least two prime divisors.

325. Prove that the equation

$$y^3 = x^2 + 5$$

does not have any solutions in $\mathbb{Z}$.

326. $a, b \in \mathbb{Z}$ and for every $n \in \mathbb{N}_0$, the number $2^n a + b$ is a perfect square. Prove that $a = 0$.

327. Solve the equation

$$5x^2 + 3y^3 = p(p-4)$$

Where x, y are positive integers and p is a prime.

328. Find all positive integer solutions to $2z^2 - y^4 = x^2$.

329. Let $a, b, c, d \in \mathbb{N}$ and

$$b^2 + 1 = ac, \qquad c^2 + 1 = bd.$$

Prove that $a = 3b - c$ and $d = 3c - b$.

330. Let $F(x) = (x^2 - 17)(x^2 - 19)(x^2 - 323)$. Prove that for each positive integer m the equation

$$F(x) \equiv 0 \pmod m$$

has solution for $x \in \mathbb{N}$. But $F(x) = 0$ doesn't have any integer solutions (even rational solution).

331. Find all integer solutions to $x^2 + y^2 = 5(xy - 1)$.

332. Let p be an odd prime. Prove that

$$1^{p-1} + 2^{p-1} + \cdots + (p-1)^{p-1} \equiv p + (p-1)! \pmod{p}^2.$$

333. Let $n \geq 5$ be a positive integer and $k+1$ be the smallest prime greater than n. Prove that $n!$ is divisible by $1, 2, \cdots k$.

334. Prove that we can choose 2^n numbers from 2^{n+1} positive integers such that sum of them is divisible by 2^n.

335. Let p, q, n be positive integers such that $\gcd(p, q) = 1$ and

$$\frac{1}{n+1} < \frac{p}{q} < \frac{1}{n}.$$

Prove that $\frac{p}{q} - \frac{1}{n+1}$ is a fraction that numerator of it, in the simplest form, is less than p. Then prove that every fraction like $\frac{p}{q}$ with $0 < p < q$ can be written as
$$\frac{p}{q} = \frac{1}{n_1} + \frac{1}{n_2} + \cdots + \frac{1}{n_k},$$
where n_i are positive and distinct integers.

336. For a natural number n , let $d(n)$ be the greatest odd divisor of n. Let
$$D(n) = d(1) + d(2) + \cdots + d(n) \qquad , T(n) = 1 + 2 + \cdots + n.$$
Prove that there exist infinitely n such that $3D(n) = 2T(n)$.

337. Let $p_1 < p_2 < p_3 < \cdots < p_{15}$ be 15 primes in an arithmetic progression with the common difference d. Prove that d is divisible by $2, 3, 5, 7, 11$ and 13.

338. Find all nonzero integers $a > b > c > d$ such that
$$ab + cd = 34 \qquad , \qquad ac - bd = 19.$$

339. Find all positive integer solutions to $2x^2 + 5y^2 = 11(xy - 11)$.

340. Let m, n be two positive integers and $\gcd(m, n) = 1$. Prove that
$$\Phi(5^m - 1) \neq 5^n - 1.$$

1.1.18 Amir Hossein - Part 18

341. Solve the equation $p = x^2 + y^x$ for Positive integers x, y and prime p.

342. Let $f(x)$ be a polynomial with integer coefficients, prove that there are infinitely many primes p such that the equation $f(x) \equiv 0 \pmod{p}$ has at least one solution in integers.

343.

- **(a)** Find all integers m, n such that
$$m^3 - 4mn^2 = 8n^3 - 2m^2n.$$
- **(b)** In the answers of **(a)**, find those which satisfy $m + n^2 = 3$.

344. Find all $x, y \in \mathbb{N}$ such that
$$x^2 + 615 = 2^y.$$

345. I have chosen a number $\in \{0, 1, 2, \cdots, 15\}$. You can ask 7 questions and I'll answer them with "YES" or "NO". And I can lie just one time. Find my number.

346. Show that at least 99% of the numbers
$$10^1 + 1, 10^2 + 1, \cdots, 10^{2010} + 1$$
are composite.

347. Find all integers $n \geq 2$, such that

$$n^n|(n-1)^{n^{n+1}} + (n+1)^{n^{n-1}}.$$

348. T is a subset of $\{1, 2, \ldots, n\}$ which has this property: for all distinct $i, j \in T$, $2j$ is not divisible by i. Prove that:

$$|T| \leq \frac{4}{9}n + \log_2 n + 2.$$

349. Let m, n be two positive integers and $m > 1$. And for all a with $\gcd(a, n) = 1$, we know that $n|a^m - 1$. Prove that

$$n \leq 4m(2^m - 1).$$

350. Find all $n \in \mathbb{N}$ such that

$$k = \frac{n}{1!} + \frac{n}{2!} + \cdots + \frac{n}{n!} \in \mathbb{Z}.$$

351. Find all prime numbers x, y such that

$$x^y - y^x = xy^2 - 19.$$

352. Find all integer solutions of

$$x^4 + y^2 = z^4.$$

353. n is a positive integer. d is the least positive integer such that for each a that $\gcd(a, n) = 1$ we know that $a^d \equiv 1 \pmod{n}$. Prove that there exist a positive integer b such that $ord_n b = d$.

354. Find all (a, b, c) triples of positive integers such that $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{4}{5}$.

355. Find all integer solutions of $x^3 - y^3 = xy + 61$.

356. Let a, b be two positive integers and $a > b$. We know that $\gcd(a - b, ab + 1) = 1$ and $\gcd(a + b, ab - 1) = 1$. Prove that $(a - b)^2 + (ab + 1)^2$ is not a perfect square.

357. There are 2010 positive integers (not necessary distinct) not greater than 2^{1389}. Let S be the sum of these numbers. We show S in base 2. What is the Maximum number of 1's in this show?

358. Let n be a positive integer such that $m = 2 + 2\sqrt{28n^2 + 1}$ is an integer. Prove that m is a perfect square.

359. Let $z^2 = (x^2 - 1)(y^2 - 1) + n$ such that $x, y \in \mathbb{Z}$. Is there a solution x, y, z if:

- **(a)** $n = 1981$?
- **(b)** $n = 1984$?
- **(c)** $n = 1985$?

360. Find all integer solutions of $x^2 - y^2 = 2xyz$.

1.1.19 Amir Hossein - Part 19

361. For $n \geq 3$ prove that there exist odd $x, y \in \mathbb{Z}$ such that $2^n = 7x^2 + y^2$.

362. Let a, b, c, d be integers such that $a^2 + ab + b^2 = c^2 + cd + d^2$. Prove that $a + b + c + d$ is not a prime number.

363. Let $x, y > 1$ be two positive integers such that $2x^2 - 1 = y^{15}$. Prove that $5|x$.

364. Factorise $5^{1985} - 1$ as a product of three integers, each greater than 5^{100}.

365. Find all triples $(a, b, c) \in \mathbb{Z}^3$ such that:

$$\begin{cases} a|b^2-1, & a|c^2-1 \\ b|a^2-1, & b|c^2-1 \\ c|a^2-1, & c|b^2-1 \end{cases}$$

366. Find the smallest positive prime that divides $n^2 + 5n + 23$ for some integer n.

367. Prove that the equation

$$2x^2 - 73y^2 = 1.$$

Does not have any solutions in $\mathbb{N}$.

368. Let $n > 1$ be a fixed positive integer, and call an n-tuple $(a_1, a_2, \dots, a_n)$ of integers greater than 1 *good* if and only if $a_i \mid \frac{a_1 a_2 \cdots a_n}{a_i} - 1$ for $i = 1, 2, \dots, n$. Prove that there are finitely many good n-tuples.

369. Let $p \geq 5$ be a prime. Show that

$$\sum_{k=0}^{(p-1)/2} \binom{p}{k} 3^k \equiv 2^p - 1 \pmod{p^2}.$$

370. Prove that $n^3 - n - 3$ is not a perfect square for any integer n.

371. Are there positive integers m, n such that there exist at least 2012 positive integers x such that both $m - x^2$ and $n - x^2$ are perfect squares?

372. Prove that if a and b are positive integers and $ab > 1$, then

$$\left\lfloor \frac{(a-b)^2 - 1}{ab} \right\rfloor = \left\lfloor \frac{(a-b)^2 - 1}{ab - 1} \right\rfloor.$$

Here $\lfloor x \rfloor$ denotes the greatest integer not exceeding x.

373. Do there exist positive integers $b, n > 1$ such that when n is expressed in base b, there are more than n distinct permutations of its digits? For example, when $b = 4$ and $n = 18$, $18 = 102_4$, but 102 only has 6 digit arrangements. (Leading zeros are allowed in the permutations.)

374. Find all positive integers n such that $4^n + 6^n + 9^n$ is a square.

375. Let $\omega = e^{2\pi i/5}$ and $p > 5$ be a prime. Show that

$$\frac{1+\omega^p}{(1+\omega)^p} + \frac{(1+\omega)^p}{1+\omega^p}$$

is an integer congruent to 2 $\pmod{p^2}$.

376. Find all integer solutions to $y^2 = 20x^4 - 4x^2 + 1$.

377. Find the set S of primes such that $p \in S$ if and only if there exists an integer x such that $x^{2010} + x^{2009} + \cdots + 1 \equiv p^{2010} \pmod{p^{2011}}$.

378. Prove that there are infinitely many quadruples of integers (a, b, c, d) such that

$$a^2 + b^2 + 3 = 4ab$$
$$c^2 + d^2 + 3 = 4cd$$
$$4c^3 - 3c = a$$

379. Suppose k is some integer. Show that for any integer m, if $k|m$, then $k|m_r$, where m_r is m with its digits reversed. For example, $123_r = 321$.

380. Suppose a, b, and c are positive integers satisfying $a^2 + b^2 + c^2 + 2(ab + ac + bc)$. Show that a, b, and c are perfect squares.

1.2 Andrew

1.2.1 Andrew - Part 1

381. Prove that for any odd positive integer n, $n^{12} - n^8 - n^4 + 1$ is divisible by 2^9.

382. Determine all integers n for which

$$\sqrt{\frac{25}{2} + \sqrt{\frac{625}{4} - n}} + \sqrt{\frac{25}{2} - \sqrt{\frac{625}{4} - n}}$$

is an integer.

383. Let's call a positive integer *interesting* if it is a product of two (distinct or equal) prime numbers. What is the greatest number of consecutive positive integers all of which are interesting?

384. Do there exist positive integers $a > b > 1$ such that for each positive integer k there exists a positive integer n for which $an + b$ is a k-th power of a positive integer?

385. $a_1a_2a_3$ and $a_3a_2a_1$ are two three-digit decimal numbers, with a_1 and a_3 different non-zero digits. Squares of these numbers are five-digit numbers $b_1b_2b_3b_4b_5$ and $b_5b_4b_3b_2b_1$ respectively. Find all such three-digit numbers.

386. Find all positive integers n and k such that $(n+1)^n = 2n^k + 3n + 1$.

387. Determine if the number $\lambda_n = \sqrt{3n^2 + 2n + 2}$ is irrational for all non-negative integers n.

388. Let $a_1, \ldots, a_{10}$ be distinct positive integers, all at least 3 and with sum 678. Does there exist a positive integer n such that the sum of the 20 remainders of n after division by $a_1, a_2, \ldots, a_{10}, 2a_1, 2a_2, \ldots, 2a_{10}$ is 2012?

389. Does there exist natural numbers a, b, c all greater than 10^{10} such that their product is divisible by each of these numbers increased by 2012?

390. Initially, ten consecutive natural numbers are written on the board. In one turn, you may pick any two numbers from the board (call them a and b) and replace them with the numbers $a^2 - 2011b^2$ and ab. After several turns, there were no initial numbers left on the board. Could there, at this point, be again, ten consecutive natural numbers?

391. Let $a_1, \ldots a_{11}$ be distinct positive integers, all at least 2 and with sum 407. Does there exist an integer n such that the sum of the 22 remainders after the division of n by $a_1, a_2, \ldots, a_{11}, 4a_1, 4a_2, \ldots, 4a_{11}$ is 2012?

392. For a positive integer n define $S_n = 1! + 2! + \ldots + n!$. Prove that there exists an integer n such that S_n has a prime divisor greater than 10^{2012}.

393. Consider the sequence $(x_n)_{n\geq 1}$ where $x_1 = 1, x_2 = 2011$ and $x_{n+2} = 4022x_{n+1} - x_n$ for all $n \in \mathbb{N}$. Prove that $\frac{x_{2012}+1}{2012}$ is a perfect square.

394. For any positive integers n and k, let $L(n,k)$ be the least common multiple of the k consecutive integers $n, n+1, \ldots, n+k-1$. Show that for any integer b, there exist integers n and k such that $L(n,k) > bL(n+1,k)$.

395. A sequence $a_1, a_2, \ldots, a_n, \ldots$ of natural numbers is defined by the rule

$$a_{n+1} = a_n + b_n \ (n = 1, 2, \ldots)$$

where b_n is the last digit of a_n. Prove that such a sequence contains infinitely many powers of 2 if and only if a_1 is not divisible by 5.

396. Given a prime number p congruent to 1 modulo 5 such that $2p+1$ is also prime, show that there exists a matrix of 0s and 1s containing exactly $4p$ (respectively, $4p+2$) 1s no sub-matrix of which contains exactly $2p$ (respectively, $2p+1$) 1s.

397. Find all integers $n \geq 3$ for which the following statement is true: Any arithmetic progression $a_1, \ldots, a_n$ with n terms for which $a_1 + 2a_2 + \ldots + na_n$ is rational contains at least one rational term.

398. Let $k, n > 1$ be integers such that the number $p = 2k - 1$ is prime. Prove that, if the number $\binom{n}{2} - \binom{k}{2}$ is divisible by p, then it is divisible by p^2.

399. Find all positive integers n which have exactly $\sqrt{n}$ positive divisors.

400. Prove that there are infinitely many positive integers n such that $2^{2^n+1}+1$ is divisible by n but $2^n + 1$ is not.

1.2.2 Andrew - Part 2

401. Consider the set $M = \{a^2 - 2ab + 2b^2 \mid a, b \in \mathbb{Z}\}$. Show that $2012 \notin M$. Prove that M is a closed subset under multiplication.

402. Solve in $\mathbb{Z}$ the equation $\frac{3}{\sqrt{x}} + \frac{2}{\sqrt{y}} = \frac{1}{\sqrt{2}}$.

403. How many positive integers satisfy the following three conditions:

- **a)** All digits of the number are from the set $\{1, 2, 3, 4, 5\}$;
- **b)** The absolute value of the difference between any two consecutive digits is 1;
- **c)** The integer has 1994 digits?

404. Find all pairs of positive integers (a, b) such that $2^a + 3^b$ is the square of an integer.

405. Show that for any integer $a \geq 5$ there exist integers b and c, $c \geq b \geq a$, such that a, b, c are the lengths of the sides of a right-angled triangle.

406. Let $p > 2$ be a prime number and

$$1 + \frac{1}{2^3} + \frac{1}{3^3} + \ldots + \frac{1}{(p-1)^3} = \frac{m}{n}$$

where m and n are relatively prime. Show that m is a multiple of p.

407. Prove that any irreducible fraction p/q, where p and q are positive integers and q is odd, is equal to a fraction $\frac{n}{2^k-1}$ for some positive integers n and k.

408. An integer $n \geq 1$ is called balanced if it has an even number of distinct prime divisors. Prove that there exist infinitely many positive integers n such that there are exactly two balanced numbers among $n, n+1, n+2$ and $n+3$.

409. Let $p = 3$ be a prime number. Show that there is a non-constant arithmetic sequence of positive integers $x_1, x_2, \ldots, x_p$ such that the product of the terms of the sequence is a cube.

410. Determine all pairs (p, q) of primes for which both $p^2 + q^3$ and $q^2 + p^3$ are perfect squares.

411. Determine all positive integers d such that whenever d divides a positive integer n, d will also divide any integer obtained by rearranging the digits of n.

412. Let a be any integer. Define the sequence $x_0, x_1, \ldots$ by $x_0 = a$, $x_1 = 3$, and for all $n > 1$

$$x_n = 2x_{n-1} - 4x_{n-2} + 3.$$

Determine the largest integer k_a for which there exists a prime p such that p^{k_a} divides $x_{2011} - 1$.

413. Let $a < b < c$ be three positive integers. Prove that among any $2c$ consecutive positive integers there exist three different numbers x, y, z such that abc divides xyz.

414. Josh is older than Fred. Josh notices that if he switches the two digits of his age (an integer), he gets Fred's age. Moreover, the difference between the squares of their ages is a square of an integer. How old are Josh and Fred?

415. The positive integers a, b, c are pairwise relatively prime, a and c are odd and the numbers satisfy the equation $a^2 + b^2 = c^2$. Prove that $b + c$ is the square of an integer.

416. Let a and k be positive integers such that $a^2 + k$ divides $(a-1)a(a+1)$. Prove that $k \geq a$.

417. Find all triples (x, y, z) of positive integers satisfying the system of equations

$$\begin{cases} x^2 = 2(y+z) \\ x^6 = y^6 + z^6 + 31(y^2 + z^2) \end{cases}.$$

418. Prove that for all positive integers n, there exists a positive integer m which is a multiple of n and the sum of the digits of m is equal to n.

419. Find all functions $f : \mathbb{N}_0 \to \mathbb{N}_0$ for which $f(0) = 0$ and

$$f(x^2 - y^2) = f(x)f(y)$$

for all $x, y \in \mathbb{N}_0$ with $x > y$.

420. Let x be a real number with the following property: for each positive integer q, there exists an integer p, such that

$$\left| x - \frac{p}{q} \right| < \frac{1}{3q}.$$

Prove that x is an integer.

1.2.3 Andrew - Part 3

421. Determine whether or not there exist numbers $x_1, x_2, \ldots, x_{2009}$ from the set $\{-1, 1\}$, such that:

$$x_1x_2 + x_2x_3 + x_3x_4 + \ldots + x_{2008}x_{2009} + x_{2009}x_1 = 999.$$

422. A sequence $a_0, a_1, a_2, \ldots, a_n, \ldots$ of positive integers is constructed as follows:

- if the last digit of a_n is less than or equal to 5 then this digit is deleted and a_{n+1} is the number consisting of the remaining digits. (If a_{n+1} contains no digits the process stops.)
- otherwise $a_{n+1} = 9a_n$.

Can one choose a_0 so that an infinite sequence is obtained?

423.
- **a)** Show that it is possible to pair off the numbers $1, 2, 3, \ldots, 10$ so that the sums of each of the five pairs are five different prime numbers.
- **b)** Is it possible to pair off the numbers $1, 2, 3, \ldots, 20$ so that the sums of each of the ten pairs are ten different prime numbers?

424. Prove that every positive integer n, except a finite number of them, can be represented as a sum of 2004 positive integers: $n = a_1 + a_2 + \cdots + a_{2004}$, where $1 \leq a_1 < a_2 < \cdots < a_{2004}$, and $a_i \mid a_{i+1}$ for all $1 \leq i \leq 2003$.

425. Let $a > 1$ be a positive integer. Show that the set of integers

$$\{a^2 + a - 1, a^3 + a^2 - 1, \ldots, a^{n+1} + a^n - 1, \ldots\}$$

contains an infinite subset of pairwise coprime integers.

426. Given are two integers $a > b > 1$ such that $a + b \mid ab + 1$ and $a - b \mid ab - 1$. Prove that $a < \sqrt{3}b$.

427. Let $p \geq 3$ be a prime number. Show that there exist p positive integers $a_1, a_2, \ldots, a_p$ not exceeding $2p^2$ such that the $\frac{p(p-1)}{2}$ sums $a_i + a_j$ $(i < j)$ are all distinct.

428. Consider the set S of integers k which are products of four distinct primes. Such an integer $k = p_1p_2p_3p_4$ has 16 positive divisors $1 = d_1 < d_2 < \ldots < d_{15} < d_{16} = k$. Find all elements of S less than 2002 such that $d_9 - d_8 = 22$.

429. For each natural number a we denote $\tau(a)$ and $\phi(a)$ the number of natural numbers dividing a and the number of natural numbers less than a that are relatively prime to a. Find all natural numbers n for which n has exactly two different prime divisors and n satisfies $\tau(\phi(n)) = \phi(\tau(n))$.

430. Prove whether or not there exist natural numbers n, k where $1 \leq k \leq n-2$ such that
$$\binom{n}{k}^2 + \binom{n}{k+1}^2 = \binom{n}{k+2}^4.$$

431. A right triangle has integer side lengths, and the sum of its area and the length of one of its legs equals 75. Find the side lengths of the triangle.

432. Prove that for any integer $n > 3$ there exist infinitely many non-constant arithmetic progressions of length $n-1$ whose terms are positive integers whose product is a perfect n-th power.

433. For a positive integer n, let $\alpha(n)$ be the number of 1's in the binary representation of n. Show that for all positive integers r, $2^{2n-\alpha(n)}$ divides $\sum_{k=-n}^{n} \binom{2n}{n+k} k^{2r}$.

434. A positive integer n is known as an *interesting* number if n satisfies
$$\left\{ \frac{n}{10^k} \right\} > \frac{n}{10^{10}}$$
for all $k = 1, 2, \ldots 9$. Find the number of interesting numbers.

435. Let a, b, c be positive integers. Prove that it is impossible to have all of the three numbers $a^2 + b + c, b^2 + c + a, c^2 + a + b$ to be perfect squares.

436. Let $m \geq 2$ be an integer. Find the smallest positive integer $n > m$ such that for any partition with two classes of the set $\{m, m+1, \ldots, n\}$ at least one of these classes contains three numbers a, b, c (not necessarily different) such that $a^b = c$.

437. Denote by $d(n)$ the number of distinct positive divisors of a positive integer n (including 1 and n). Let $a > 1$ and $n > 0$ be integers such that $a^n + 1$ is a prime. Prove that $d(a^n - 1) \geq n$.

438. Let n and k be integers, $1 \leq k \leq n$. Find an integer b and a set A of n integers satisfying the following conditions:

- **(i)** No product of $k-1$ distinct elements of A is divisible by b.
- **(ii)** Every product of k distinct elements of A is divisible by b.
- **(iii)** For all distinct a, a' in A, a does not divide a'.

439. A sequence of integers $a_1, a_2, \ldots$ is such that $a_1 = 1, a_2 = 2$ and for $n \geq 1$,
$$a_{n+2} = \begin{cases} 5a_{n+1} - 3a_n, & \text{if } a_n \cdot a_{n+1} \text{ is even,} \\ a_{n+1} - a_n, & \text{if } a_n \cdot a_{n+1} \text{ is odd,} \end{cases}$$
Prove that $a_n \neq 0$ for all n.

440. Let $a > 1$ be an odd positive integer. Find the least positive integer n such that 2^{2000} is a divisor of $a^n - 1$.

1.2.4 Andrew - Part 4

441. Let p and q be two different prime numbers. Prove that there are two positive integers, a and b, such that the arithmetic mean of the divisors of $n = p^a q^b$ is an integer.

442. Find two positive integers a and b, when their sum and their least common multiple is given. Find the numbers when the sum is 3972 and the least common multiple is 985928.

443. Five points, no three collinear, are given on the plane. Let $l_1, l_2, \ldots, l_{10}$ be the lengths of the ten segments joining any two of the given points. Prove that if $l_1^2, \ldots, l_9^2$ are rational numbers, then l_{10}^2 is also a rational number.

444. For any positive integer n, let a_n denote the closest integer to $\sqrt{n}$, and let $b_n = n + a_n$. Determine the increasing sequence (c_n) of positive integers which do not occur in the sequence (b_n).

445. Let q be a real number with $\frac{1+\sqrt{5}}{2} < q < 2$. If a positive integer n is represented in binary system as $2^k + a_{k-1}2^{k-1} + \cdots + 2a_1 + a_0$, where $a_i \in \{0, 1\}$, define

$$p_n = q^k + a_{k-1}q^{k-1} + \cdots + qa_1 + a_0.$$

Prove that there exist infinitely many positive integers k with the property that there is no $l \in \mathbb{N}$ such that $p_{2k} < p_l < p_{2k+1}$.

446. Let m, n be positive integers with $(m, n) = 1$. Find $\gcd(5^m + 7^m, 5^n + 7^n)$.

447. Let $f(x) = x^3 + 17$. Prove that for every integer $n \geq 2$ there exists a natural number x for which $f(x)$ is divisible by 3^n but not by 3^{n+1}.

448. Let p be a prime number and A an infinite subset of the natural numbers. Let $f_A(n)$ be the number of different solutions of $x_1 + x_2 + \ldots + x_p = n$, with $x_1, x_2, \ldots x_p \in A$. Does there exist a number N for which $f_A(n)$ is constant for all $n < N$?

449. A *pucelana* sequence is an increasing sequence of 16 consecutive odd numbers whose sum is a perfect cube. How many pucelana sequences are there with 3-digit numbers only?

450. Find all four-digit numbers which are equal to the cube of the sum of their digits.

451. Consider the points $O(0, 0)$ and $A(0, 1/2)$ on the coordinate plane. Prove that there is no finite sequence of rational points $P_1, P_2, \ldots, P_n$ in the plane such that

$$OP_1 = P_1P_2 = \ldots = P_{n-1}P_n = P_nA = 1$$

452. Let m, n be positive integers of distinct parities and such that $m < n < 5m$. Show that there exists a partition with two element subsets of the set $\{1, 2, 3, \ldots, 4mn\}$ such that the sum of numbers in each set is a perfect square.

453. Let $(a_n)_{n\geq 1}$ be a sequence of positive integers defined as $a_1, a_2 > 0$ and a_{n+1} is the least prime divisor of $a_{n-1} + a_n$, for all $n \geq 2$.

Prove that a real number x whose decimals are digits of the numbers

$$a_1, a_2, \dots a_n, \dots$$

written in order, is a rational number.

454. Let a, b be positive real numbers. For any positive integer n, denote by x_n the sum of digits of the number $[an+b]$ in it's decimal representation. Show that the sequence $(x_n)_{n\geq 1}$ contains a constant subsequence.

455. Let $P(x)$ and $Q(x)$ be integer polynomials of degree p and q respectively. Assume that $P(x)$ divides $Q(x)$ and all their coefficients are either 1 or 2002. Show that $p+1$ is a divisor of $q+1$.

456. Find all numbers $p \leq q \leq r$ such that all the numbers

$$pq + r, pq + r^2, qr + p, qr + p^2, rp + q, rp + q^2$$

are prime.

457. Prove that for any prime number p the equation $2^p + 3^p = a^n$ has no solution (a, n) in integers greater than 1.

458. Let n be a positive integer and let k be an odd positive integer. Moreover, let a, b and c be integers (not necessarily positive) satisfying the equations

$$a^n + kb = b^n + kc = c^n + ka$$

Prove that $a = b = c$.

459. Find all functions $f : \mathbb{Z}_{>0} \to \mathbb{Z}_{>0}$ that satisfy the following two conditions:

- $f(n)$ is a perfect square for all $n \in \mathbb{Z}_{>0}$,
- $f(m+n) = f(m) + f(n) + 2mn$ for all $m, n \in \mathbb{Z}_{>0}$.

460. If we add 1996 to 1997, we first add the unit digits 6 and 7. Obtaining 13, we write down 3 and "carry" 1 to the next column. Thus we make a carry. Continuing, we see that we are to make three carries in total.
Does there exist a positive integer k such that adding $1996 \cdot k$ to $1997 \cdot k$ no carry arises during the whole calculation?

1.2.5 Andrew - Part 5

461. In a sequence $u_0, u_1, \dots$ of positive integers, u_0 is arbitrary, and for any non-negative integer n,

$$u_{n+1} = \begin{cases} \frac{1}{2}u_n & \text{for even } u_n \\ a + u_n & \text{for odd } u_n \end{cases}$$

where a is a fixed odd positive integer. Prove that the sequence is periodic from a certain step.

462. Find all triples (a, b, c) of non-negative integers satisfying $a \geq b \geq c$ and

$$1 \cdot a^3 + 9 \cdot b^2 + 9 \cdot c + 7 = 1997.$$

463. Let P and Q be polynomials with integer coefficients. Suppose that the integers a and $a + 1997$ are roots of P, and that $Q(1998) = 2000$. Prove that the equation $Q(P(x)) = 1$ has no integer solutions.

464. The worlds in the Worlds' Sphere are numbered $1, 2, 3, \ldots$ and connected so that for any integer $n \geq 1$, Gandalf the Wizard can move in both directions between any worlds with numbers $n, 2n$ and $3n + 1$. Starting his travel from an arbitrary world, can Gandalf reach every other world?

465. Prove that in every sequence of 79 consecutive positive integers written in the decimal system, there is a positive integer whose sum of digits is divisible by 13.

466. A rectangle can be divided into n equal squares. The same rectangle can also be divided into $n + 76$ equal squares. Find n.

467. Suppose that for some $m, n \in \mathbb{N}$ we have $\varphi(5^m - 1) = 5^n - 1$, where φ denotes the Euler function. Show that $(m, n) > 1$.

468. Find all pairs (x, y) of positive integers such that $y^{x^2} = x^{y+2}$.

469. Let n be a non-negative integer. Find all non-negative integers a, b, c, d such that

$$a^2 + b^2 + c^2 + d^2 = 7 \cdot 4^n.$$

470. Determine a right parallelepiped with minimal area, if its volume is strictly greater than 1000, and the lengths of it sides are integer numbers.

471. Determine all positive integers in the form $a < b < c < d$ with the property that each of them divides the sum of the other three.

472. Three students write on the blackboard next to each other three two-digit squares. In the end, they observe that the 6-digit number thus obtained is also a square. Find this number.

473. Find all $n \in \mathbb{Z}$ such that the number $\sqrt{\frac{4n-2}{n+5}}$ is rational.

474. Let n be a positive integer and $f(x) = a_m x^m + \ldots + a_1 x + a_0$, with $m \geq 2$, a polynomial with integer coefficients such that:

- **a)** $a_2, a_3 \ldots a_m$ are divisible by all prime factors of n,
- **b)** a_1 and n are relatively prime.

Prove that for any positive integer k, there exists a positive integer c, such that $f(c)$ is divisible by n^k.

475. For every rational number $m > 0$ we consider the function $f_m : \mathbb{R} \to \mathbb{R}, f_m(x) = \frac{1}{m}x + m$. Denote by G_m the graph of the function f_m. Let p, q, r be positive rational numbers.

- **a)** Show that if p and q are distinct then $G_p \cap G_q$ is non-empty.
- **b)** Show that if $G_p \cap G_q$ is a point with integer coordinates, then p and q are integer numbers.
- **c)** Show that if p, q, r are consecutive natural numbers, then the area of the triangle determined by intersections of G_p, G_q and G_r is equal to 1.

476. Show that there exist no integers a and b such that $a^3 + a^2b + ab^2 + b^3 = 2001$.

477. Find all pairs of integers a and b for which

$$7a + 14b = 5a^2 + 5ab + 5b^2.$$

478. Find all pairs of integers (p, q) for which all roots of the trinomials $x^2 + px + q$ and $x^2 + qx + p$ are integers.

479. Let $A_1, A_2, \ldots, A_{n+1}$ be positive integers such that $(A_i, A_{n+1}) = 1$ for every $i = 1, 2, \ldots, n$. Show that the equation

$$x_1^{A_1} + x_2^{A_2} + \ldots + x_n^{A_n} = x_{n+1}^{A_{n+1}}$$

has an infinite set of solutions $(x_1, x_2, \ldots, x_{n+1})$ in positive integers.

480. Find all numbers $N = \overline{a_1a_2\ldots a_n}$ for which $9 \times \overline{a_1a_2\ldots a_n} = \overline{a_n\ldots a_2a_1}$ such that at most one of the digits $a_1, a_2, \ldots, a_n$ is zero.

1.2.6 Andrew - Part 6

481. Given any integer $m > 1$ prove that there exist infinitely many positive integers n such that the last m digits of 5^n are a sequence $a_m, a_{m-1}, \ldots, a_1 = 5$ $(0 \le a_j < 10)$ in which each digit except the last is of opposite parity to its successor (i.e., if a_i is even, then a_{i-1} is odd, and if a_i is odd, then a_{i-1} is even).

482. Let n and z be integers greater than 1 and $(n, z) = 1$. Prove:

- **(a)** At least one of the numbers $z_i = 1+z+z^2+\cdots+z^i,\ i = 0, 1, \ldots, n-1$, is divisible by n.
- **(b)** If $(z-1, n) = 1$, then at least one of the numbers z_i is divisible by n.

483. Let a be an odd digit and b an even digit. Prove that for every positive integer n there exists a positive integer, divisible by 2^n, whose decimal representation contains no digits other than a and b.

484. Let P be a polynomial with integer coefficients. Suppose that for $n = 1, 2, 3, \ldots, 1998$ the number $P(n)$ is a three-digit positive integer. Prove that the polynomial P has no integer roots.

485. A triple (a, b, c) of positive integers is called *quasi-Pythagorean* if there exists a triangle with lengths of the sides a, b, c and the angle opposite to the side c equal to 120°. Prove that if (a, b, c) is a quasi-Pythagorean triple then c has a prime divisor bigger than 5.

486. Find all functions f of two variables, whose arguments x, y and values $f(x, y)$ are positive integers, satisfying the following conditions (for all positive integers x and y):

$$f(x, x) = x,$$

$$f(x, y) = f(y, x),$$

$$(x + y)f(x, y) = yf(x, x + y).$$

487. Prove that the product of two natural numbers with their sum cannot be the third power of a natural number.

488. Let u_n be the Fibonacci sequence, i.e., $u_0 = 0, u_1 = 1, u_n = u_{n-1} + u_{n-2}$ for $n > 1$. Prove that there exist infinitely many prime numbers p that divide u_{p-1}.

489. Denote by $x_n(p)$ the multiplicity of the prime p in the canonical representation of the number $n!$ as a product of primes. Prove that $\frac{x_n(p)}{n} < \frac{1}{p-1}$ and $\lim_{n\to\infty} \frac{x_n(p)}{n} = \frac{1}{p-1}$.

490. Let m and n denote integers greater than 1, and let $\nu(n)$ be the number of primes less than or equal to n. Show that if the equation $\frac{n}{\nu(n)} = m$ has a solution, then so does the equation $\frac{n}{\nu(n)} = m - 1$.

491. Let x and y be positive integers and assume that $z = \frac{4xy}{x+y}$ is an odd integer. Prove that at least one divisor of z can be expressed in the form $4n - 1$ where n is a positive integer.

492. A sequence $(x_n)_{n\geq 0}$ is defined as follows: $x_0 = a, x_1 = 2$ and $x_n = 2x_{n-1}x_{n-2} - x_{n-1} - x_{n-2} + 1$ for all $n > 1$. Find all integers a such that $2x_{3n} - 1$ is a perfect square for all $n \geq 1$.

493. Let a, b, c and d be prime numbers such that $a > 3b > 6c > 12d$ and $a^2 - b^2 + c^2 - d^2 = 1749$. Determine all possible values of $a^2 + b^2 + c^2 + d^2$.

494. Let m be a positive integer such that $m = 2 \pmod 4$. Show that there exists at most one factorization $m = ab$ where a and b are positive integers satisfying

$$0 < a - b < \sqrt{5 + 4\sqrt{4m + 1}}.$$

495. Does there exist a finite sequence of integers $c_1, c_2, \ldots, c_n$ such that all the numbers $a+c_1, a+c_2, \ldots, a+c_n$ are primes for more than one but not infinitely many different integers a?

496. Find the smallest positive integer k which is representable in the form $k = 19^n - 5^m$ for some positive integers m and n.

497. Determine all positive integers n with the property that the third root of n is obtained by removing its last three decimal digits.

498. Find all positive integers n such that n is equal to 100 times the number of positive divisors of n.

499. Determine all positive real numbers x and y satisfying the equation

$$x + y + \frac{1}{x} + \frac{1}{y} + 4 = 2 \cdot (\sqrt{2x+1} + \sqrt{2y+1}).$$

500. Let n be a positive integer not divisible by 2 or 3. Prove that for all integers k, the number $(k+1)^n - k^n - 1$ is divisible by $k^2 + k + 1$.

1.2.7 Andrew - Part 7

501. Let $a_1, a_2, \ldots, a_n$ be an arithmetic progression of integers such that $i | a_i$ for $i = 1, 2, \ldots, n-1$ and $n \nmid a_n$. Prove that n is a prime power.

502. Let a, b, c, d, p and q be positive integers satisfying $ad - bc = 1$ and $\frac{a}{b} > \frac{p}{q} > \frac{c}{d}$. Prove that:

- **(a)** $q \geq b + d$
- **(b)** If $q = b + d$, then $p = a + c$.

503. Let k and n be positive integers with $n > 2$. Show that the equation:

$$x^n - y^n = 2^k$$

has no positive integer solutions.

504. For any number $n \in \mathbb{N}, n \geq 2$, denote by $P(n)$ the number of pairs (a, b) whose elements are of positive integers such that

$$\frac{n}{a} \in (0,1), \quad \frac{a}{b} \in (1,2) \quad \text{and} \quad \frac{b}{n} \in (2,3).$$

- **a)** Calculate $P(3)$.
- **b)** Find n such that $P(n) = 2002$.

505. Prove that any real number $0 < x < 1$ can be written as a difference of two positive and less than 1 irrational numbers.

506. The polynomial $W(x) = x^2 + ax + b$ with integer coefficients has the following property: for every prime number p there is an integer k such that both $W(k)$ and $W(k+1)$ are divisible by p. Show that there is an integer m such that $W(m) = W(m+1) = 0$.

507. Find all positive integers n for which $n^n + 1$ and $(2n)^{2n} + 1$ are prime numbers.

508. A 12-digit positive integer consisting only of digits $1, 5$ and 9 is divisible by 37. Prove that the sum of its digits is not equal to 76.

509. Does there exist a sequence $a_1, a_2, a_3, \dots$ of positive integers such that the sum of every n consecutive elements is divisible by n^2 for every positive integer n?

510. For a positive integer n let a_n denote the last digit of $n^{(n^n)}$. Prove that the sequence (a_n) is periodic and determine the length of the minimal period.

511. Are there 4 distinct positive integers such that adding the product of any two of them to 2006 yields a perfect square?

512. Let a and b be positive integers, $b < a$, such that $a^3 + b^3 + ab$ is divisible by $ab(a-b)$. Prove that ab is a perfect cube.

513. Let r and k be positive integers such that all prime divisors of r are greater than 50. A positive integer, whose decimal representation (without leading zeroes) has at least k digits, will be called *nice* if every sequence of k consecutive digits of this decimal representation forms a number (possibly with leading zeroes) which is a multiple of r.
Prove that if there exist infinitely many nice numbers, then the number $10^k - 1$ is nice as well.

514. Let a, b, c, d be non-zero integers, such that the only quadruple of integers (x, y, z, t) satisfying the equation

$$ax^2 + by^2 + cz^2 + dt^2 = 0$$

is $x = y = z = t = 0$. Does it follow that the numbers a, b, c, d have the same sign?

515. Let x, y, z be positive integers such that $\frac{x+1}{y} + \frac{y+1}{z} + \frac{z+1}{x}$ is an integer. Let d be the greatest common divisor of x, y and z. Prove that $d \le \sqrt[3]{xy + yz + zx}$.

516. Let a and b be rational numbers such that $s = a + b = a^2 + b^2$. Prove that s can be written as a fraction where the denominator is relatively prime to 6.

517. An integer $n > 0$ is written in decimal system as $\overline{a_m a_{m-1} \dots a_1}$. Find all n such that

$$n = (a_m + 1)(a_{m-1} + 1) \cdots (a_1 + 1).$$

518. Show that the equation $2x^2 - 3x = 3y^2$ has infinitely many solutions in positive integers.

519. Determine all positive integers n for which there exists a partition of the set

$$\{n, n+1, n+2, \ldots, n+8\}$$

into two subsets such that the product of all elements of the first subset is equal to the product of all elements of the second subset.

520. Determine all positive integers n for which $2^{n+1} - n^2$ is a prime number.

1.2.8 Andrew - Part 8

521. Determine all triples (x, y, z) of integers greater than 1 with the property that x divides $yz - 1$, y divides $zx - 1$ and z divides $xy - 1$.

522. Determine all triples (x, y, z) of positive integers such that

$$\frac{13}{x^2} + \frac{1996}{y^2} = \frac{z}{1997}.$$

523. Determine all positive integers n for which there exists an infinite subset A of the set $\mathbb{N}$ of positive integers such that for all pairwise distinct $a_1, a_2, \ldots, a_n \in A$ the numbers $a_1 + \ldots + a_n$ and $a_1 a_2 \ldots a_n$ are coprime.

524. For which k do there exist k pairwise distinct primes $p_1, p_2, \ldots p_k$ such that

$$p_1^2 + p_2^2 + \ldots + p_k^2 = 2010?$$

525. Let p be a prime number. For each k, $1 \le k \le p-1$, there exists a unique integer denoted by k^{-1} such that $1 \le k^{-1} \le p-1$ and $k^{-1} \cdot k = 1 \pmod{p}$. Prove that the sequence

$$1^{-1}, \quad 1^{-1} + 2^{-1}, \quad 1^{-1} + 2^{-1} + 3^{-1}, \quad \ldots, \quad 1^{-1} + 2^{-1} + \ldots + (p-1)^{-1}$$

(addition modulo p) contains at most $\frac{p+1}{2}$ distinct elements.

526. Find all positive integers n such that the decimal representation of n^2 consists of odd digits only.

527. For a positive integer k, let $d(k)$ denote the number of divisors of k and let $s(k)$ denote the digit sum of k. A positive integer n is said to be *amusing* if there exists a positive integer k such that $d(k) = s(k) = n$. What is the smallest amusing odd integer greater than 1?

528. From a sequence of integers (a, b, c, d) each of the sequences

$$(c, d, a, b), \quad (b, a, d, c), \quad (a + nc, b + nd, c, d), \quad (a + nb, b, c + nd, d)$$

for arbitrary integer n can be obtained by one step. Is it possible to obtain $(3, 4, 5, 7)$ from $(1, 2, 3, 4)$ through a sequence of such steps?

529. What is the smallest positive odd integer having the same number of positive divisors as 360?

530. Let n be a positive integer. Prove that at least $2^{n-1}+n$ numbers can be chosen from the set $\{1,2,3,\ldots,2^n\}$ such that for any two different chosen numbers x and y, $x+y$ is not a divisor of $x\cdot y$.

531. Let f be a real-valued function defined on the positive integers satisfying the following condition: For all $n>1$ there exists a prime divisor p of n such that $f(n)=f\left(\frac{n}{p}\right)-f(p)$. Given that $f(2001)=1$, what is the value of $f(2002)$?

532. Does there exist an infinite non-constant arithmetic progression, each term of which is of the form a^b, where a and b are positive integers with $b\geq 2$?

533. Show that the sequence

$$\binom{2002}{2002},\binom{2003}{2002},\binom{2004}{2002},\ldots$$

considred modulo 2002, is periodic.

534. Find all pairs (a,b) of positive rational numbers such that

$$\sqrt{a}+\sqrt{b}=\sqrt{2+\sqrt{3}}.$$

535. Prove that if m,n,r are positive integers, and:

$$1+m+n\sqrt{3}=(2+\sqrt{3})^{2r-1}$$

then m is a perfect square.

536. Let a,b be coprime positive integers. Show that the number of positive integers n for which the equation $ax+by=n$ has no positive integer solutions is equal to $\frac{(a-1)(b-1)}{2}-1$.

537. Prove that if m and s are integers with $ms=2000^{2001}$, then the equation $mx^2-sy^2=3$ has no integer solutions.

538. Prove that for each prime number p and positive integer n, p^n divides

$$\binom{p^n}{p}-p^{n-1}.$$

539. Find all triples of positive integers (a,b,p) with a,b positive integers and p a prime number such that $2^a+p^b=19^a$.

540. The function $\psi:\mathbb{N}\to\mathbb{N}$ is defined by $\psi(n)=\sum_{k=1}^{n}\gcd(k,n)$.

- **(a)** Prove that $\psi(mn)=\psi(m)\psi(n)$ for every two coprime $m,n\in\mathbb{N}$.
- **(b)** Prove that for each $a\in\mathbb{N}$ the equation $\psi(x)=ax$ has a solution.

1.2.9 Andrew - Part 9

541. Find all positive rational numbers $r \neq 1$ such that $r^{\frac{1}{r-1}}$ is rational.

542. Suppose that the sum of all positive divisors of a natural number n, n excluded, plus the number of these divisors is equal to n. Prove that $n = 2m^2$ for some integer m.

543. Let a and b be positive integers. Show that if $a^3 + b^3$ is the square of an integer, then $a + b$ is not a product of two different prime numbers.

544. Every integer is to be coloured blue, green, red, or yellow. Can this be done in such a way that if a, b, c, d are not all 0 and have the same colour, then $3a - 2b = 2c - 3d$?

545. All the positive divisors of a positive integer n are stored into an increasing array. Mary is writing a programme which decides for an arbitrarily chosen divisor $d > 1$ whether it is a prime. Let n have k divisors not greater than d. Mary claims that it suffices to check divisibility of d by the first $\left\lceil \frac{k}{2} \right\rceil$ divisors of n: d is prime if and only if none of them but 1 divides d. Is Mary right?

546. Find all pairs of positive integers (a, b) such that $a - b$ is a prime number and ab is a perfect square.

547. Prove that any real solution of $x^3 + px + q = 0$, where p, q are real numbers, satisfies the inequality $4qx \leq p^2$.

548. Suppose that a sequence $(a_n)_{n=1}^{\infty}$ of integers has the following property: For all n large enough (i.e. $n \geq N$ for some N), a_n equals the number of indices i, $1 \leq i < n$, such that $a_i + i \geq n$. Find the maximum possible number of integers which occur infinitely many times in the sequence.

549. Find all triples of nonnegative integers (m, n, k) satisfying $5^m + 7^n = k^3$.

550. Prove that there are are no positive integers x and y such that $x^5 + y^5 + 1 = (x + 2)^5 + (y - 3)^5$.

551. Find all integers x and y such that $x^3 \pm y^3 = 2001p$, where p is prime.

552. Let $x_k = \frac{k(k+1)}{2}$ for all integers $k \geq 1$. Prove that for any integer $n \geq 10$, between the numbers $A = x_1 + x_2 + \ldots + x_{n-1}$ and $B = A + x_n$ there is at least one square.

553. The discriminant of the equation $x^2 - ax + b = 0$ is the square of a rational number and a and b are integers. Prove that the roots of the equation are integers.

554. Find all the three-digit numbers $\overline{abc}$ such that the 6003-digit number $\overline{abcabc \ldots abc}$ is divisible by 91.

555. Let P_n $(n = 3, 4, 5, 6, 7)$ be the set of positive integers $n^k + n^l + n^m$, where k, l, m are positive integers. Find n such that:

- **i)** In the set P_n there are infinitely many squares.
- **ii)** In the set P_n there are no squares.

556. Find the positive integers n that are not divisible by 3 if the number $2^{n^2-10} + 2133$ is a perfect cube.

557. Find all pairs of integers (m, n) such that the numbers $A = n^2 + 2mn + 3m^2 + 2$, $B = 2n^2 + 3mn + m^2 + 2$, $C = 3n^2 + mn + 2m^2 + 1$ have a common divisor greater than 1.

558. Find all four-digit numbers such that when decomposed into prime factors, each number has the sum of its prime factors equal to the sum of the exponents.

559. Prove that there are no integers x, y, z such that

$$x^4 + y^4 + z^4 - 2x^2y^2 - 2y^2z^2 - 2z^2x^2 = 2000.$$

560. Find all the triples (x, y, z) of positive integers such that $xy + yz + zx - xyz = 2$.

1.3 Goutham

1.3.1 Goutham - Part 1

561. Let $\tau(n)$ be the number of positive divisors of a natural number n, and $\sigma(n)$ be their sum. Find the largest real number α such that

$$\frac{\sigma(n)}{\tau(n)} \geq \alpha\sqrt{n}$$

for all $n \geq 1$.

562. Let $n > 1$ be a positive integer. Find all n-tuples $(a_1, a_2, \ldots, a_n)$ of positive integers which are pairwise distinct, pairwise coprime, and such that for each i in the range $1 \leq i \leq n$,

$$(a_1 + a_2 + \ldots + a_n) | (a_1^i + a_2^i + \ldots + a_n^i).$$

563. Let $S(k)$ denote the digit-sum of a positive integer k(in base 10). Determine the smallest positive integer n such that

$$S(n^2) = S(n) - 7.$$

564. Does the sequence

$$11, 111, 1111, 11111, \ldots$$

contain any fifth power of a positive integer? Justify your answer.

565. Let a, b, c be positive integers for which

$$ac = b^2 + b + 1$$

Prove that the equation

$$ax^2 - (2b+1)xy + cy^2 = 1$$

has an integer solution.

566. Prove that, for any positive integer n, there exists a polynomial $p(x)$ of degree at most n whose coe?cients are all integers such that, $p(k)$ is divisible by 2^n for every even integer k, and $p(k) - 1$ is divisible by 2^n for every odd integer k.

567. Let $(a_n)_{n\geq 1}$ be a sequence of integers that satis?es

$$a_n = a_{n-1} - \min(a_{n-2}, a_{n-3})$$

for all $n \geq 4$. Prove that for every positive integer k, there is an n such that a_n is divisible by 3^k.

568. A positive integer is called *monotonic* if when written in base 10, the digits are weakly increasing. Thus 12226778 is monotonic. Note that a positive integer cannot have first digit which is 0. Prove that for every positive integer n, there is an n-digit monotonic number which is a perfect square.

569. Prove that every positive integer can be represented in the form

$$3^{u_1} \ldots 2^{v_1} + 3^{u_2} \ldots 2^{v_2} + \ldots + 3^{u_k} \ldots 2^{v_k}$$

with integers $u_1, u_2, \ldots, u_k, v_1, \ldots, v_k$ such that $u_1 > u_2 > \ldots > u_k \geq 0$ and $0 \leq v_1 < v_2 < \ldots < v_k$.

570. Find all positive integers k such that there exist two positive integers m and n satisfying

$$m(m+k) = n(n+1).$$

571. An increasing arithmetic progression consists of one hundred positive integers. Is it possible that every two of them are relatively prime?

572. For any integer $n+1, \ldots, 2n$ (n is a natural number) consider its greatest odd divisor. Prove that the sum of all these divisors equals n^2.

573. Let a real number $\lambda > 1$ be given and a sequence (n_k) of positive integers such that $\frac{n_{k+1}}{n_k} > \lambda$ for $k = 1, 2, \ldots$ Prove that there exists a positive integer c such that no positive integer n can be represented in more than c ways in the form $n = n_k + n_j$ or $n = n_r - n_s$.

574. Let there be given two sequences of integers $f_i(1), f_i(2), \cdots (i = 1, 2)$ satisfying:

- $f_i(nm) = f_i(n)f_i(m)$ if $\gcd(n, m) = 1$;
- for every prime P and all $k = 2, 3, 4, \cdots$, $f_i(P^k) = f_i(P)f_i(P^{k-1}) - P^2 f(P^{k-2})$. Moreover, for every prime P:
- $f_1(P) = 2P$,
- $f_2(P) < 2P$.

Prove that $|f_2(n)| < f_1(n)$ for all n.

575. For any positive integer n, we denote by $F(n)$ the number of ways in which n can be expressed as the sum of three different positive integers, without regard to order. Thus, since $10 = 7 + 2 + 1 = 6 + 3 + 1 = 5 + 4 + 1 = 5 + 3 + 2$, we have $F(10) = 4$. Show that $F(n)$ is even if $n \equiv 2$ or $4 \pmod 6$, but odd if n is divisible by 6.

576. Show that it is possible to express 1 as a sum of 6, and as a sum of 9 reciprocals of odd positive integers. Generalize the problem.

577. Are there positive integers a, b with $b \geq 2$ such that $2^a + 1$ is divisible by $2^b - 1$?

578. Let a_0 and a_n be different divisors of a natural number m, and $a_0, a_1, a_2, \cdots, a_n$ be a sequence of natural numbers such that it satisfifies

$$a_{i+1} = |a_i \pm a_{i-1}| \text{ for } 0 < i < n$$

If $\gcd(a_0, \cdots, a_n) = 1$, show that there is a term of the sequence that is smaller than $\sqrt{m}$.

579. Let A be an infinite set of positive integers. Find all natural numbers n such that for each $a \in A$,

$$a^n + a^{n-1} + \cdots + a^1 + 1 \mid a^{n!} + a^{(n-1)!} + \cdots + a^{1!} + 1.$$

580. In a multiplication table, the entry in the i-th row and the j-th column is the product ij From an $m \times n$ subtable with both m and n odd, the interior $(m - 2)(n - 2)$ rectangle is removed, leaving behind a frame of width 1. The squares of the frame are painted alternately black and white. Prove that the sum of the numbers in the black squares is equal to the sum of the numbers in the white squares.

1.3.2 Goutham - Part 2

581. Alex has a piece of cheese. He chooses a positive number a and cuts the piece into several pieces one by one. Every time he choses a piece and cuts it in the same ratio $1 : a$. His goal is to divide the cheese into two piles of equal masses. Can he do it if
(a) a is irrational?
(b) a is rational, $a \neq 1$?

582. 2010 ships deliver bananas, lemons and pineapples from South America to Russia. The total number of bananas on each ship equals the number of lemons on all other ships combined, while the total number of lemons on each ship equals the total number of pineapples on all other ships combined. Prove that the total number of fruits is a multiple of 31.

583. Can it happen that the sum of digits of some positive integer n equals 100 while the sum of digits of number n^3 equals 100^3?

584. Alex has a piece of cheese. He chooses a positive number $a \neq 1$ and cuts the piece into several pieces one by one. Every time he chooses a piece and cuts it in the same ratio $1 : a$. His goal is to divide the cheese into two piles of equal masses. Can he do it?

585. 101 numbers are written on a blackboard: $1^2, 2^2, 3^2, \cdots, 101^2$. Alex choses any two numbers and replaces them by their positive difference. He repeats this operation until one number is left on the blackboard. Determine the smallest possible value of this number.

586. Each of six fruit baskets contains pears, plums and apples. The number of plums in each basket equals the total number of apples in all other baskets combined while the number of apples in each basket equals the total number of pears in all other baskets combined. Prove that the total number of fruits is a multiple of 31.

587. Let $x = \sqrt{a} + \sqrt{b}$, where a and b are natural numbers, x is not an integer, and $x < 1976$. Prove that the fractional part of x exceeds $10^{-19.76}$.

588. Let $(a_n), n = 0, 1, ...$, be a sequence of real numbers such that $a_0 = 0$ and

$$a_{n+1}^3 = \frac{1}{2}a_n^2 - 1, n = 0, 1, \cdots$$

Prove that there exists a positive number $q, q < 1$, such that for all $n = 1, 2, \ldots,$

$$|a_{n+1} - a_n| \leq q|a_n - a_{n-1}|,$$

and give one such q explicitly.

589. For a positive integer n, let $6^{(n)}$ be the natural number whose decimal representation consists of n digits 6. Let us define, for all natural numbers m, k with $1 \leq k \leq m$

$$\left[\begin{array}{c} m \\ k \end{array} \right] = \frac{6^{(m)}6^{(m-1)} \cdots 6^{(m-k+1)}}{6^{(1)}6^{(2)} \cdots 6^{(k)}}.$$

Prove that for all m, k, $\left[\begin{array}{c} m \\ k \end{array} \right]$ is a natural number whose decimal representation consists of exactly $k(m + k - 1) - 1$ digits.

590. Prove that the number $19^{1976} + 76^{1976}$:

- **(a)** is divisible by the (Fermat) prime number $F_4 = 2^{2^4} + 1$;
- **(b)** is divisible by at least four distinct primes other than F_4.

591. Prove that there is a positive integer n such that the decimal representation of 7^n contains a block of at least m consecutive zeros, where m is any given positive integer.

592. A sequence $\{u_n\}$ of integers is defined by

$$u_1 = 2, u_2 = u_3 = 7,$$

$$u_{n+1} = u_n u_{n-1} - u_{n-2}, \text{ for } n \geq 3.$$

Prove that for each $n \geq 1$, u_n differs by 2 from an integral square.

593. Show that the reciprocal of any number of the form $2(m^2+m+1)$, where m is a positive integer, can be represented as a sum of consecutive terms in the sequence $(a_j)_{j=1}^{\infty}$

$$a_j = \frac{1}{j(j+1)(j+2)}.$$

594. Find all pairs of natural numbers (m, n) for which $2^m 3^n + 1$ is the square of some integer.

595. Numbers $1, 2, \cdots, 16$ are written in a 4×4 square matrix so that the sum of the numbers in every row, every column, and every diagonal is the same and furthermore that the numbers 1 and 16 lie in opposite corners. Prove that the sum of any two numbers symmetric with respect to the center of the square equals 17.

596. The decimal number 13^{101} is given. It is instead written as a ternary number. What are the two last digits of this ternary number?

597. The ternary expansion $x = 0.10101010\cdots$ is given. Give the binary expansion of x. Alternatively, transform the binary expansion $y = 0.110110110\cdots$ into a ternary expansion.

598. If p is a prime number greater than 2 and a, b, c integers not divisible by p, prove that the equation

$$ax^2 + by^2 = pz + c$$

has an integer solution.

599. If $n_1, n_2, \cdots, n_k$ are natural numbers and $n_1 + n_2 + \cdots + n_k = n$, show that

$$\max(n_1 n_2 \cdots n_k) = (t+1)^r t^{k-r},$$

where $t = \left[\frac{n}{k}\right]$ and r is the remainder of n upon division by k; i.e., $n = tk+r, 0 \leq r \leq k-1$.

600. Find values of $n \in \mathbb{N}$ for which the fraction $\frac{3^n-2}{2^n-3}$ is reducible.

1.3.3 Goutham - Part 3

601. Does there exist a $2n$-digit number $\overline{a_{2n}a_{2n-1}\cdots a_1}$(for an arbitrary n) for which the following equality holds:

$$\overline{a_{2n}\cdots a_1} = (\overline{a_n\cdots a_1})^2?$$

602. Find all integer solutions of the equation

$$1 + x + x^2 + x^3 + x^4 = y^4.$$

603. Consider the set S of all the different odd positive integers that are not multiples of 5 and that are less than $30m, m$ being a positive integer. What is the smallest integer k such that in any subset of k integers from S there must be two integers one of which divides the other? Prove your result.

604. If p is a prime greater than 3, show that at least one of the numbers

$$\frac{3}{p^2}, \frac{4}{p^2}, \dots, \frac{p-2}{p^2}$$

is expressible in the form $\frac{1}{x} + \frac{1}{y}$, where x and y are positive integers.

605. A is a $2m$-digit positive integer each of whose digits is 1. B is an m-digit positive integer each of whose digits is 4. Prove that $A + B + 1$ is a perfect square.

606. Find all natural numbers $n < 1978$ with the following property: If m is a natural number, $1 < m < n$, and $(m, n) = 1$ (i.e., m and n are relatively prime), then m is a prime number.

607. N is a natural number such that it is the product of three distinct prime numbers. Find all such natural numbers such that the sum of all its composite divisors is equal to $2N + 1$

608. Find the set of all ordered pairs of integers (a, b) such that $\gcd(a, b) = 1$ and $\frac{a}{b} + \frac{14b}{25a}$ is an integer.

609. Find all natural numbers n such that $n + S(n) + S(S(n)) = 2010$ where $S(n) =$ sum of the digits of n.

610. Prove that for every positive integer n coprime to 10 there exists a multiple of n that does not contain the digit 1 in its decimal representation.

611. Determine the sixth number after the decimal point in the number $(\sqrt{1978} + \lfloor\sqrt{1978}\rfloor)^{20}$

612. Given a natural number n, prove that the number $M(n)$ of points with integer coordinates inside the circle $(O(0, 0), \sqrt{n})$ satisfies

$$\pi n - 5\sqrt{n} + 1 < M(n) < \pi n + 4\sqrt{n} + 1.$$

613. Show that for any natural number n there exist two prime numbers p and $q, p \neq q$, such that n divides their difference.

614. Let a, b, c be nonnegative integers such that $a \leq b \leq c, 2b \neq a + c$ and $\frac{a+b+c}{3}$ is an integer. Is it possible to find three nonnegative integers d, e, and f such that $d \leq e \leq f, f \neq c$, and such that $a^2 + b^2 + c^2 = d^2 + e^2 + f^2$?

615. Let a, b, c be natural numbers such that $a + b + c = 2pq(p^{30} + q^{30}), p > q$ being two given positive integers.

- **(a)** Prove that $k = a^3 + b^3 + c^3$ is not a prime number.
- **(b)** Prove that if $a \cdot b \cdot c$ is maximum, then 1984 divides k.

616. Find a sequence of natural numbers a_i such that $a_i = \sum_{r=1}^{i+4} d_r$, where $d_r \neq d_s$ for $r \neq s$ and d_r divides a_i.

617. Let $n > 1$ and $x_i \in \mathbb{R}$ for $i = 1, \cdots, n$. Set

$$S_k = x_1^k + x_2^k + \cdots + x_n^k$$

for $k \geq 1$. If $S_1 = S_2 = \cdots = S_{n+1}$, show that $x_i \in \{0, 1\}$ for every $i = 1, 2, \cdots, n$.

618. Let $(a_n)_{n\geq 1}$ and $(b_n)_{n\geq 1}$ be two sequences of natural numbers such that $a_{n+1} = na_n + 1, b_{n+1} = nb_n - 1$ for every $n \geq 1$. Show that these two sequences can have only a finite number of terms in common.

619. Denote by $[x]$ the greatest integer not exceeding x. For all real $k > 1$, define two sequences:

$$a_n(k) = [nk] \text{ and } b_n(k) = \left[\frac{nk}{k-1}\right]$$

If $A(k) = \{a_n(k) : n \in \mathbb{N}\}$ and $B(k) = \{b_n(k) : n \in \mathbb{N}\}$, prove that $A(k)$ and $B(k)$ form a partition of $\mathbb{N}$ if and only if k is irrational.

620. A "number triangle" $(t_{n,k})(0 \leq k \leq n)$ is defined by $t_{n,0} = t_{n,n} = 1(n \geq 0)$,

$$t_{n+1,m} = (2 - \sqrt{3})^m t_{n,m} + (2 + \sqrt{3})^{n-m+1} t_{n,m-1} \quad (1 \leq m \leq n).$$

Prove that all $t_{n,m}$ are integers.

1.3.4 Goutham - Part 4

621. Prove that for any natural number n, the number $\binom{2n}{n}$ divides the least common multiple of the numbers $1, 2, \cdots, 2n - 1, 2n$.

622. Decide whether it is possible to color the 1984 natural numbers $1, 2, 3, \cdots, 1984$ using 15 colors so that no geometric sequence of length 3 of the same color exists.

623. The set $\{1, 2, \cdots, 49\}$ is divided into three subsets. Prove that at least one of these subsets contains three different numbers a, b, c such that $a + b = c$.

624. Determine the smallest positive integer m such that $529^n + m \cdot 132^n$ is divisible by 262417 for all odd positive integers n.

625. Consider all the sums of the form

$$\sum_{k=1}^{1985} e_k k^5 = \pm 1^5 \pm 2^5 \pm \cdots \pm 1985^5$$

where $e_k = \pm 1$. What is the smallest nonnegative value attained by a sum of this type?

626. Given a polynomial $f(x)$ with integer coefficients whose value is divisible by 3 for three integers $k, k+1$, and $k+2$. Prove that $f(m)$ is divisible by 3 for all integers m.

627. A boy has a set of trains and pieces of railroad track. Each piece is a quarter of circle, and by concatenating these pieces, the boy obtained a closed railway. The railway does not intersect itself. In passing through this railway, the train sometimes goes in the clockwise direction, and sometimes in the opposite direction. Prove that the train passes an even number of times through the pieces in the clockwise direction and an even number of times in the counterclockwise direction. Also, prove that the number of pieces is divisible by 4.

628. Which natural numbers can be expressed as the difference of squares of two integers?

629. Prove that there are infinitely many positive integers that cannot be expressed as the sum of squares of three positive integers.

630. Let $a_0, a_1, a_2, \cdots$ be determined with $a_0 = 0, a_{n+1} = 2a_n + 2^n$. Prove that if n is power of 2, then so is a_n.

631. Let p and q be two prime numbers greater than 3. Prove that if their difference is 2^n, then for any two integers m and n, the number $S = p^{2m+1} + q^{2m+1}$ is divisible by 3.

632. Let a and b be arbitrary integers. Prove that if k is an integer not divisible by 3, then $(a+b)^{2k} + a^{2k} + b^{2k}$ is divisible by $a^2 + ab + b^2$

633. Prove that there exist infinitely many natural numbers a with the following property: The number $z = n^4 + a$ is not prime for any natural number n.

634. Let us define $u_0 = 0, u_1 = 1$ and for $n \geq 0, u_{n+2} = au_{n+1} + bu_n, a$ and b being positive integers. Express u_n as a polynomial in a and b. Prove the result. Given that b is prime, prove that b divides $a(u_b - 1)$.

635. Let a, b, x, y be positive integers such that a and b have no common divisor greater than 1. Prove that the largest number not expressible in the form $ax+by$ is $ab-a-b$. If $N(k)$ is the largest number not expressible in the form $ax+by$ in only k ways, find $N(k)$.

636. The polynomial $P(x) = a_0x^k + a_1x^{k-1} + \cdots + a_k$, where $a_0, \cdots, a_k$ are integers, is said to be divisible by an integer m if $P(x)$ is a multiple of m for every integral value of x. Show that if $P(x)$ is divisible by m, then $a_0 \cdot k!$ is a multiple of m. Also prove that if a, k, m are positive integers such that $ak!$ is a multiple of m, then a polynomial $P(x)$ with leading term ax^kcan be found that is divisible by m.

637. Consider the integer $d = \frac{a^b-1}{c}$, where a, b, and c are positive integers and $c \leq a$. Prove that the set G of integers that are between 1 and d and relatively prime to d (the number of such integers is denoted by $\phi(d)$) can be partitioned into n subsets, each of which consists of b elements. What can be said about the rational number $\frac{\phi(d)}{b}$?

638. Let $\alpha(n)$ be the number of pairs (x, y) of integers such that $x+y = n, 0 \leq y \leq x$, and let $\beta(n)$ be the number of triples (x, y, z) such that$x + y + z = n$ and $0 \leq z \leq y \leq x$. Find a simple relation between $\alpha(n)$ and the integer part of the number $\frac{n+2}{2}$ and the relation among $\beta(n), \beta(n-3)$ and $\alpha(n)$. Then evaluate $\beta(n)$ as a function of the residue of n modulo 6. What can be said about $\beta(n)$ and $1 + \frac{n(n+6)}{12}$? And what about $\frac{(n+3)^2}{6}$? Find the number of triples (x, y, z) with the property $x + y + z \leq n, 0 \leq z \leq y \leq x$ as a function of the residue of n modulo 6. What can be said about the relation between this number and the number $\frac{(n+6)(2n^2+9n+12)}{72}$?

639. Let n be an integer that is not divisible by any square greater than 1. Denote by x_m the last digit of the number x^m in the number system with base n. For which integers x is it possible for x_m to be 0? Prove that the sequence x_m is periodic with period t independent of x. For which x do we have $x_t = 1$. Prove that if m and x are relatively prime, then $0_m, 1_m, ..., (n-1)_m$ are different numbers. Find the minimal period t in terms of n. If n does not meet the given condition, prove that it is possible to have $x_m = 0 \neq x_1$ and that the sequence is periodic starting only from some number $k > 1$.

640. Let a and b be two nonnegative integers. Denote by $H(a, b)$ the set of numbers n of the form $n = pa + qb$, where p and q are positive integers. Determine $H(a) = H(a, a)$. Prove that if $a \neq b$, it is enough to know all the sets $H(a, b)$ for coprime numbers a, b in order to know all the sets $H(a, b)$. Prove that in the case of coprime numbers a and $b, H(a, b)$ contains all numbers greater than or equal to $\omega = (a-1)(b-1)$ and also $\frac{\omega}{2}$ numbers smaller than ω.

1.3.5 Goutham - Part 5

641. Let d and p be two real numbers. Find the first term of an arithmetic progression $a_1, a_2, a_3, \cdots$ with difference d such that $a_1a_2a_3a_4 = p$. Find the number of solutions in terms of d and p.

642. Let p be a prime odd number. Is it possible to find $p-1$ natural numbers $n+1, n+2, ..., n+p-1$ such that the sum of the squares of these numbers is divisible by the sum of these numbers?

643. Prove that the equation $\sqrt{x^3+y^3+z^3} = 1969$ has no integral solutions.

644. We have $\frac{5}{6} = \frac{a_1}{2!} + \frac{a_2}{3!} + \frac{a_3}{4!} + \frac{a_4}{5!} + \frac{a_5}{6!}$ where $a_i \in \mathbb{R}, 0 \le a_i < i \forall i = 1,2,3,4,5$. Then, find all possible values of $a_1 + a_2 + a_3 + a_4 + a_5$.

645. Prove that for all integers n greater than or equal to 3, we can find unique odd integers x, y such that $2^n = x^2 + 7y^2$.

646. Let a, b, x be positive integers such that $x^{a+b} = a^b b$. Prove that $a = x$ and $b = x^x$.

647. Solve in non-negative integers: $(xy-7)^2 = x^2 + y^2$

648. n is a positive integer greater than 10 whose digits are from the set $\{1,3,7,9\}$ Prove that n has a prime factor greater than 10.

649. Prove that there are infinitely many positive integers m for which there exists consecutive odd positive integers $p_m < q_m$ such that $p_m^2 + p_m q_m + q_m^2$ and $p_m^2 + m \cdot p_m q_m + q_m^2$ are both perfect squares. If m_1, m_2 are two positive integers satisfying this condition, then we have $p_{m_1} \neq p_{m_2}$.

650. Find all positive integers n and $m \geq 2$ such that the product of the first n positive integers and the product of the first m even positive integers are the same.

651. If p is a prime that leaves remainder 1 when divided by 4, prove that there exists an integer x such that p divides $x^2 - \frac{p-1}{4}$.

652. Find all natural n such that the natural number $3^n - n$ is exactly divisible by 17.

653. Find all positive integer solutions for $a+b+c = xyz; x+y+z = abc$.

654. Prove that $n+1$ divides ${}^{2n}C_n$ for all integers n.

655. For a given positive real b, show that there exists no biggest real a, such that $a^2 < b$.

656. Let p_n be the n^{th} prime, $n \in \mathbb{N}$. K is a finite set of distinct integers none of which has a prime factor greater than p_n for some n. Prove that the sum of reciprocals of the elements of K is smaller than $\prod_{i=1}^{n} \frac{p_n}{p_n - 1}$.

657. Find all integers n such that $\frac{7n-12}{2^n} + \frac{2n-14}{3^n} + \frac{24n}{6^n} = 1$.

658. Prove that for a positive integer n, $\left\lfloor \frac{(n-1)!}{n(n+1)} \right\rfloor$ is even.

659. n is a positive integer. $s(n)$ is the sum of digits of n. $t(n)$ is sum of all the numbers obtained by removing several digits(at least one) from the right hand end of decimal representation of n. For example, if $n = 123465, t(n) = 1 + 12 + 123 + 1234 + 12346$. Prove that $n = s(n) + 9t(n)$.

660. For all positive integers n, prove that there exists an integer m such that n divides $2^m + m$.

1.3.6 Goutham - Part 6

661. Prove that every positive integer has a multiple whose decimal expansion starts with 2008.
Can we generalize to start with any natural number?

662. Prove that for a given set of six consecutive positive integers, there exists a prime that divides one and only one of these six integers.

663. Find all positive integers x, y satisfying $x^y = y^{50}$.

664. For positive reals x, y, if $x^3 + y^3 + (x + y)^3 + 30xy = 2000$, Prove that $x + y = 10$.

665. Find all positive integers x, y satisfying $x^y = y^{x-y}$.

666. Prove that primitive root surely exists for all natural numbers of the form p^α or $2p^\alpha$ where p is an odd prime and α is a natural number. (We do not consider the trivial case 2.)

667.
- **1)** Given a positive integer n and integers $x_1, x_2, \cdots, x_k, k \in \mathbb{N}$ such that $x_1 \equiv x_2 \equiv \cdots \equiv x_k mod n$, prove that $(x_1, n) = (x_2, n) = \cdots = (x_k, n)$.
- **2)** Prove for $a_1, a_2, \cdots a_n \in \mathbb{Z}$: $(a_1, a_2, \cdots, a_n)|[a_1, a_2 \cdots, a_n]$.
- **3)** Find all positive integers n such that $(n, \phi(n)) = 1$.

668. Find an infinite family of solutions for $a!b!c!d! + 2a!d! = 2a!c!d! + a!b!d! + e!$.

669. Find an infinite family of solutions for $x!y!z! = y!z! + w!$ for $x, y, z, w > 2$.

670. For a positive integer k, call an integer a *pure* $k - th$ *power* if it can be represented as m^k for some integer m. Show that for every positive integer n, there exists n distinct positive integers such that their sum is a pure $2009-$th power and their product is a pure $2010-$th power.

671. Two $2007 - digit$ numbers are given. It is possible to erase 2007 digits from each of them so as to get the same 2000 digit numbers. Prove that we can insert 7 digits for each number anywhere in the two numbers so as to make them equal.

672. For x, y to be natural numbers, if $x^2 + xy + y^2$ is divisible by 10, prove that it is divisible by 100.

673. Prove or disprove: There exists no positive integer solutions for $x^2+y^2 = kz^2$ where k is a constant which cannot be written as sum of two positive perfect squares.

674. Let F_n denote the n^{th} Fibonnaci number. Prove that if $p > 5$ is a prime, then,

$$F_p \equiv \left(\frac{p}{5}\right) \mod p$$

where $\left(\frac{p}{5}\right)$ is the legendre symbol.

675. Find an unbounded sequence $\{a_i\}_{i=1}^{\infty}$ of integers where for all natural i, there exists integers a and b such that $2a_i|(a+b)$ and $ab = 4a_i(a_i+1)$

676. Let the sum of digits of $\overline{999...9991}^2$, where 9 occurs 222 times be A. Prove that $A = 2008$.

677. Let n be a given positive integer. Prove that there exist infinitely many integer triples (x, y, z) such that

$$nx^2 + y^3 = z^4, \ \gcd(x, y) = \gcd(y, z) = \gcd(z, x) = 1.$$

678. Find the minimum natural number n with the following property: between any collection of n distinct natural numbers in the set $\{1, 2, \dots, 999\}$ it is possible to choose four different $a, \ b, \ c, \ d$ such that: $a + 2b + 3c = d$.

679. Show that $5^n - 4n + 15$ is divisible by 16 for all integers $n \geq 1$.

680. Prove, without using mathematical induction, that $5^{2n+2} - 24n - 25$ is divisible by 576.

1.4 Orlando

1.4.1 Orlando - Part 1

681. Determine all $(x, y) \in \mathbb{N}^2$ which satisfy $x^{2y} + (x+1)^{2y} = (x+2)^{2y}$.

682. Determine the number of all numbers which are represented as $x^2 + y^2$ with $x, y \in \{1, 2, 3, \dots, 1000\}$ and which are divisible by 121.

683. Let N be a natural number and $x_1, \dots, x_n$ further natural numbers less than N and such that the least common multiple of any two of these n numbers is greater than N. Prove that the sum of the reciprocals of these n numbers is always less than 2: $\sum_{i=1}^{n} \frac{1}{x_i} < 2$.

684. Determine all $(m, n) \in \mathbb{Z}^+ \times \mathbb{Z}^+$ which satisfy $3^m - 7^n = 2$.

685. Initially, on a board there a positive integer. If board contains the number x, then we may additionally write the numbers $2x+1$ and $\frac{x}{x+2}$. At some point 2008 is written on the board. Prove, that this number was there from the beginning.

686. Let $p > 7$ be a prime which leaves residue 1 when divided by 6. Let $m = 2^p - 1$, then prove $2^{m-1} - 1$ can be divided by $127m$ without residue.

687. For which $n \geq 2, n \in \mathbb{N}$ are there positive integers $A_1, A_2, \ldots, A_n$ which are not the same pairwise and have the property that the product $\prod_{i=1}^{n}(A_i + k)$ is a power for each natural number k.

688. Let $(a_n)_{n\in\mathbb{N}}$ defined by $a_1 = 1$, and $a_{n+1} = a_n^4 - a_n^3 + 2a_n^2 + 1$ for $n \geq 1$. Show that there is an infinite number of primes p such that none of the a_n is divisible by p.

689. Prove: For each positive integer is the number of divisors whose decimal representations ends with a 1 or 9 not less than the number of divisors whose decimal representations ends with 3 or 7.

690. For each $n \in \mathbb{N}$ we have two numbers p_n, q_n with the following property: For exactly n distinct integer numbers x the number

$$x^2 + p_n \cdot x + q_n$$

is the square of a natural number. (Note the definition of natural numbers includes the zero here.)

691. Let $k \in \mathbb{N}$. A polynomial is called *k-valid* if all its coefficients are integers between 0 and k inclusively. (Here we don't consider 0 to be a natural number.)
a.) For $n \in \mathbb{N}$ let a_n be the number of 5-valid polynomials p which satisfy $p(3) = n$. Prove that each natural number occurs in the sequence $(a_n)_n$ at least once but only finitely often.
b.) For $n \in \mathbb{N}$ let a_n be the number of 4-valid polynomials p which satisfy $p(3) = n$. Prove that each natural number occurs infinitely often in the sequence $(a_n)_n$.

692. Find all quadruple $(m, n, p, q) \in \mathbb{Z}^4$ such that

$$p^m q^n = (p + q)^2 + 1.$$

693. Prove there is an integer k for which $k^3 - 36k^2 + 51k - 97$ is a multiple of $3^{2008.}$

694. Let n be a positive integer, and S_n, be the set of all positive integer divisors of n (including 1 and itself). Prove that at most half of the elements in S_n have their last digits equal to 3.

695. Let n, m be positive integers of different parity, and $n > m$. Find all integers x such that $\frac{x^{2^n}-1}{x^{2^m}-1}$ is a perfect square.

696. Let n be a given positive integer. Find the smallest positive integer u_n such that for any positive integer d, in any u_n consecutive odd positive integers, the number of them that can be divided by d is not smaller than the number of odd integers among $1, 3, 5, \ldots, 2n - 1$ that can be divided by d.

697. Find all prime numbers $p, q < 2005$ such that $q|p^2 + 8$ and $p|q^2 + 8$.

698. Prove that the sequence $(a_n)_{n\geq 0,}, a_n = [n \cdot \sqrt{2}]$, contains an infinite number of perfect squares.

699. Let m be a positive odd integer, $m > 2$. Find the smallest positive integer n such that 2^{1989} divides $m^n - 1$.

700. Let $a, b \in \mathbb{Z}$ which are not perfect squares. Prove that if

$$x^2 - ay^2 - bz^2 + abw^2 = 0$$

has a nontrivial solution in integers, then so does

$$x^2 - ay^2 - bz^2 = 0.$$

1.4.2 Orlando - Part 2

701. Let A be a set of positive integers such that no positive integer greater than 1 divides all the elements of A. Prove that any sufficiently large positive integer can be written as a sum of elements of A. (Elements may occur several times in the sum.)

702. Let $a, b, c, d, m, n \in \mathbb{Z}^+$ such that

$$a^2 + b^2 + c^2 + d^2 = 1989,$$

$$a + b + c + d = m^2,$$

and the largest of a, b, c, d is n^2. Determine, with proof, the values of m and n.

703. Define sequence (a_n) by $\sum_{d|n} a_d = 2^n$. Show that $n|a_n$.

704. Let n be a positive integer. Show that

$$\left(\sqrt{2} + 1\right)^n = \sqrt{m} + \sqrt{m - 1}$$

for some positive integer m.

705. Let a, b, c, d be positive integers such that $ab = cd$ and $a + b = c - d$. Prove that there exists a right-angled triangle the measure of whose sides (in some unit) are integers and whose area measure is ab square units.

706. Let $a_1, \ldots, a_n$ be distinct positive integers that do not contain a 9 in their decimal representations. Prove that the following inequality holds

$$\sum_{i=1}^{n} \frac{1}{a_i} \leq 30.$$

707. Let m be a positive integer and define $f(m)$ to be the number of factors of 2 in $m!$ (that is, the greatest positive integer k such that $2^k|m!$). Prove that there are infinitely many positive integers m such that $m - f(m) = 1989$.

708. Prove: If the sum of all positive divisors of $n \in \mathbb{Z}^+$ is a power of two, then the number/amount of the divisors is a power of two.

709. Determine that all $k \in \mathbb{Z}$ such that $\forall n$ the numbers $4n+1$ and $kn+1$ have no common divisor.

710. Let the positive integers a, b, c chosen such that the quotients $\frac{bc}{b+c}$, $\frac{ca}{c+a}$ and $\frac{ab}{a+b}$ are integers. Prove that a, b, c have a common divisor greater than 1.

711. Prove that for no integer n is $n^7 + 7$ a perfect square.

712. For positive integers n, $f_n = \lfloor 2^n \sqrt{2008} \rfloor + \lfloor 2^n \sqrt{2009} \rfloor$. Prove there are infinitely many odd numbers and infinitely many even numbers in the sequence $f_1, f_2, \ldots$.

713. What is the total number of natural numbes divisible by 9 the number of digits of which does not exceed 2008 and at least two of the digits are 9s?

714. Let $m = 2007^{2008}$, how many natural numbers n are there such that $n < m$ and $n(2n+1)(5n+2)$ is divisible by m (which means that $m \mid n(2n+1)(5n+2)$)?

715. There are only a finite number of solutions $p, q, x, y \in \mathbb{N}$, each greater than 1, of the equation

$$x^p - y^q = k$$

with $k = 1$. Also try to solve the equation for general k. which is element of the non-zero integers.

716. Prove that on the coordinate plane it is impossible to draw a closed broken line such that
(i) the coordinates of each vertex are rational;
(ii) the length each of its edges is 1;
(iii) the line has an odd number of vertices.

717. Find all natural numbers n for which every natural number whose decimal representation has $n-1$ digits 1 and one digit 7 is prime.

718. Let n be a composite natural number and p a proper divisor of n. Find the binary representation of the smallest natural number N such that

$$\frac{(1 + 2^p + 2^{n-p})N - 1}{2^n}$$

is an integer.

719. Prove that every integer k greater than 1 has a multiple that is less than k^4 and can be written in the decimal system with at most four different digits.

720. An eccentric mathematician has a ladder with n rungs that he always ascends and descends in the following way: When he ascends, each step he takes covers a rungs of the ladder, and when he descends, each step he takes covers b rungs of the ladder, where a and b are fixed positive integers. By a sequence of ascending and descending steps he can climb from ground level to the top rung of the ladder and come back down to ground level again. Find, with proof, the minimum value of n, expressed in terms of a and b.

1.4.3 Orlando - Part 3

721. For a given positive integer k denote the square of the sum of its digits by $f_1(k)$ and let $f_{n+1}(k) = f_1(f_n(k))$. Determine the value of $f_{1991}(2^{1990})$.

722. Let $f(0) = f(1) = 0$ and

$$f(n+2) = 4^{n+2} \cdot f(n+1) - 16^{n+1} \cdot f(n) + n \cdot 2^{n^2}, \quad n = 0, 1, 2, \ldots$$

Show that the numbers $f(1989), f(1990), f(1991)$ are divisible by 13.

723. Find the highest degree k of 1991 for which 1991^k divides the number

$$1990^{1991^{1992}} + 1992^{1991^{1990}}.$$

724. Find all positive integer solutions x, y, z of the equation $3^x + 4^y = 5^z$.

725. Let a_n be the last nonzero digit in the decimal representation of the number $n!$. Does the sequence $a_1, a_2, \ldots, a_n, \ldots$ become periodic after a finite number of terms?

726. Let a, b, c be integers and p an odd prime number. Prove that if $f(x) = ax^2 + bx + c$ is a perfect square for $2p - 1$ consecutive integer values of x, then p divides $b^2 - 4ac$.

727. Let $f(x) = x^8 + 4x^6 + 2x^4 + 28x^2 + 1$. Let $p > 3$ be a prime and suppose there exists an integer z such that p divides $f(z)$. Prove that there exist integers $z_1, z_2, \ldots, z_8$ such that if

$$g(x) = (x - z_1)(x - z_2) \cdot \ldots \cdot (x - z_8),$$

then all coefficients of $f(x) - g(x)$ are divisible by p.

728. Does there exist a set M with the following properties?
(i) The set M consists of 1992 natural numbers.
(ii) Every element in M and the sum of any number of elements have the form m^k $(m, k \in \mathbb{N}, k \geq 2)$.

729. Prove that for any positive integer m there exist an infinite number of pairs of integers (x, y) such that
(i) x and y are relatively prime;
(ii) y divides $x^2 + m$;
(iii) x divides $y^2 + m$.
(iv) $x + y \leq m + 1-$ (optional condition)

730. M is a subset of $\{1, 2, 3, \ldots, 15\}$ such that the product of any three distinct elements of M is not a square. Determine the maximum number of elements in M.

731. Let p be an odd prime. Determine positive integers x and y for which $x \leq y$ and $\sqrt{2p} - \sqrt{x} - \sqrt{y}$ is non-negative and as small as possible.

732. Let $\mathbb{Z}$ denote the set of all integers. Prove that for any integers A and B, one can find an integer C for which $M_1 = \{x^2 + Ax + B : x \in \mathbb{Z}\}$ and $M_2 = 2x^2 + 2x + C : x \in \mathbb{Z}$ do not intersect.

733. (a) Let n be a positive integer. Prove that there exist distinct positive integers x, y, z such that

$$x^{n-1} + y^n = z^{n+1}.$$

(b) Let a, b, c be positive integers such that a and b are relatively prime and c is relatively prime either to a or to b. Prove that there exist infinitely many triples (x, y, z) of distinct positive integers x, y, z such that

$$x^a + y^b = z^c.$$

734. Show that the set of positive integers that cannot be represented as a sum of distinct perfect squares is finite.

735. Prove that there exist infinitely many positive integers n such that $p = nr$, where p and r are respectively the semiperimeter and the inradius of a triangle with integer side lengths.

736. Show that there exists a bijective function $f : \mathbb{N}_0 \to \mathbb{N}_0$ such that for all $m, n \in \mathbb{N}_0$:

$$f(3mn + m + n) = 4f(m)f(n) + f(m) + f(n).$$

737. Find all positive integers a and b for which

$$\left\lfloor \frac{a^2}{b} \right\rfloor + \left\lfloor \frac{b^2}{a} \right\rfloor = \left\lfloor \frac{a^2 + b^2}{ab} \right\rfloor + ab.$$

738. A finite sequence of integers $a_0, a_1, \ldots, a_n$ is called quadratic if for each i in the set $\{1, 2 \ldots, n\}$ we have the equality $|a_i - a_{i-1}| = i^2$.
a.) Prove that any two integers b and c, there exists a natural number n and a quadratic sequence with $a_0 = b$ and $a_n = c$.
b.) Find the smallest natural number n for which there exists a quadratic sequence with $a_0 = 0$ and $a_n = 1996$.

739. Four integers are marked on a circle. On each step we simultaneously replace each number by the difference between this number and next number on the circle, moving in a clockwise direction; that is, the numbers a, b, c, d are replaced by $a-b, b-c, c-d, d-a$. Is it possible after 1996 such to have numbers a, b, c, d such the numbers $|bc - ad|, |ac - bd|, |ab - cd|$ are primes?

740. 250 numbers are chosen among positive integers not exceeding 501. Prove that for every integer t there are four chosen numbers a_1, a_2, a_3, a_4, such that $a_1 + a_2 + a_3 + a_4 - t$ is divisible by 23.

1.4.4 Orlando - Part 4

741. Is it possible to arrange on a circle all composite positive integers not exceeding 10^6, so that no two neighbouring numbers are coprime?

742. For which numbers n is there a positive integer k with the following property: The sum of digits for k is n and the number k^2 has sum of digits n^2.

743. Let k be a positive integer. Prove that the number $(4 \cdot k^2 - 1)^2$ has a positive divisor of the form $8kn - 1$ if and only if k is even.

744. Find all surjective functions $f : \mathbb{N} \mapsto \mathbb{N}$ such that for every $m, n \in \mathbb{N}$ and every prime p, the number $f(m+n)$ is divisible by p if and only if $f(m) + f(n)$ is divisible by p.

745. For every integer $k \geq 2$, prove that 2^{3k} divides the number

$$\binom{2^{k+1}}{2^k} - \binom{2^k}{2^{k-1}}$$

but 2^{3k+1} does not.

746. Let X be a set of 10,000 integers, none of them is divisible by 47. Prove that there exists a 2007-element subset Y of X such that $a - b + c - d + e$ is not divisible by 47 for any $a, b, c, d, e \in Y$.

747. Let $b, n > 1$ be integers. Suppose that for each $k > 1$ there exists an integer a_k such that $b - a_k^n$ is divisible by k. Prove that $b = A^n$ for some integer A.

748. A mathematical frog jumps along the number line. The frog starts at 1, and jumps according to the following rule: if the frog is at integer n, then it can jump either to $n + 1$ or to $n + 2^{m_n+1}$ where 2^{m_n} is the largest power of 2 that is a factor of n. Show that if $k \geq 2$ is a positive integer and i is a nonnegative integer, then the minimum number of jumps needed to reach $2^i k$ is greater than the minimum number of jumps needed to reach 2^i.

749. Let p be a prime number and let s be an integer with $0 < s < p$. Prove that there exist integers m and n with $0 < m < n < p$ and

$$\left\{\frac{sm}{p}\right\} < \left\{\frac{sn}{p}\right\} < \frac{s}{p}$$

if and only if s is not a divisor of $p - 1$.
Note: For x a real number, let $\lfloor x \rfloor$ denote the greatest integer less than or equal to x, and let $\{x\} = x - \lfloor x \rfloor$ denote the fractional part of x.

750. Let n be a positive integer. Find the largest nonnegative real number $f(n)$ (depending on n) with the following property: whenever $a_1, a_2, ..., a_n$ are real numbers such that $a_1 + a_2 + \cdots + a_n$ is an integer, there exists some i such that $\left|a_i - \frac{1}{2}\right| \geq f(n)$.

751. Let a, b, n be positive integers, $b > 1$ and $b^n - 1 | a$. Show that the representation of the number a in the base b contains at least n digits different from zero.

752. Let S be the set of all pairs (m, n) of relatively prime positive integers m, n with n even and $m < n$. For $s = (m, n) \in S$ write $n = 2^k \cdot n_o$ where k, n_0 are positive integers with n_0 odd and define

$$f(s) = (n_0, m + n - n_0).$$

Prove that f is a function from S to S and that for each $s = (m, n) \in S$, there exists a positive integer $t \leq \frac{m+n+1}{4}$ such that

$$f^t(s) = s,$$

where

$$f^t(s) = \underbrace{(f \circ f \circ \cdots \circ f)}_{t \text{ times}}(s).$$

If $m+n$ is a prime number which does not divide $2^k - 1$ for $k = 1, 2, \ldots, m+n-2$, prove that the smallest value t which satisfies the above conditions is $\left[\frac{m+n+1}{4}\right]$ where $[x]$ denotes the greatest integer $\leq x$.

753. A natural number n is said to have the property P, if, for all a, n^2 divides $a^n - 1$ whenever n divides $a^n - 1$.
a.) Show that every prime number n has property P.
b.) Show that there are infinitely many composite numbers n that possess property P.

754. Prove that for each positive integer n there exist n consecutive positive integers none of which is an integral power of a prime number.

755. Let $a_1, \ldots, a_n$ be an infinite sequence of strictly positive integers, so that $a_k < a_{k+1}$ for any k. Prove that there exists an infinity of terms a_m, which can be written like $a_m = x \cdot a_p + y \cdot a_q$ with x, y strictly positive integers and $p \neq q$.

756. Determine the greatest number, who is the product of some positive integers, and the sum of these numbers is 1976.

757. Let n be a given number greater than 2. We consider the set V_n of all the integers of the form $1 + kn$ with $k = 1, 2, \ldots$ A number m from V_n is called indecomposable in V_n if there are not two numbers p and q from V_n so that $m = pq$. Prove that there exist a number $r \in V_n$ that can be expressed as the product of elements indecomposable in V_n in more than one way. (Expressions which differ only in order of the elements of V_n will be considered the same.)

758. Let a, b be two natural numbers. When we divide $a^2 + b^2$ by $a + b$, we the the remainder r and the quotient q. Determine all pairs (a, b) for which $q^2 + r = 1977$.

759. Let m and n be positive integers such that $1 \leq m < n$. In their decimal representations, the last three digits of 1978^m are equal, respectively, so the last three digits of 1978^n. Find m and n such that $m+n$ has its least value.

760. If p and q are natural numbers so that

$$\frac{p}{q} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots - \frac{1}{1318} + \frac{1}{1319},$$

prove that p is divisible with 1979.

1.4.5 Orlando - Part 5

761. Prove that we can find an infinite set of positive integers of the from $2^n - 3$ (where n is a positive integer) every pair of which are relatively prime.

762. Determine the maximum value of $m^2 + n^2$, where m and n are integers in the range $1, 2, \ldots, 1981$ satisfying $(n^2 - mn - m^2)^2 = 1$.

763. a.) For which $n > 2$ is there a set of n consecutive positive integers such that the largest number in the set is a divisor of the least common multiple of the remaining $n - 1$ numbers?
b.) For which $n > 2$ is there exactly one set having this property?

764. Prove that if n is a positive integer such that the equation

$$x^3 - 3xy^2 + y^3 = n$$

has a solution in integers x, y, then it has at least three such solutions. Show that the equation has no solutions in integers for $n = 2891$.

765. Let a, b and c be positive integers, no two of which have a common divisor greater than 1. Show that $2abc - ab - bc - ca$ is the largest integer which cannot be expressed in the form $xbc + yca + zab$, where x, y, z are non-negative integers.

766. Find one pair of positive integers a, b such that $ab(a+b)$ is not divisible by 7, but $(a+b)^7 - a^7 - b^7$ is divisible by 7^7.

767. Let n and k be relatively prime positive integers with $k < n$. Each number in the set $M = \{1, 2, 3, \ldots, n-1\}$ is colored either blue or white. For each i in M, both i and $n - i$ have the same color. For each $i \neq k$ in M both i and $|i - k|$ have the same color. Prove that all numbers in M must have the same color.

768. Let d be any positive integer not equal to $2, 5$ or 13. Show that one can find distinct a, b in the set $\{2, 5, 13, d\}$ such that $ab - 1$ is not a perfect square.

769. Let $n \geq 2$ be an integer. Prove that if $k^2 + k + n$ is prime for all integers k such that $0 \leq k \leq \sqrt{\frac{n}{3}}$, then $k^2 + k + n$ is prime for all integers k such that $0 \leq k \leq n - 2$.

770. Determine all integers $n > 1$ such that

$$\frac{2^n + 1}{n^2}$$

is an integer.

771. Let $n > 6$ be an integer and $a_1, a_2, \cdots, a_k$ be all the natural numbers less than n and relatively prime to n. If

$$a_2 - a_1 = a_3 - a_2 = \cdots = a_k - a_{k-1} > 0,$$

prove that n must be either a prime number or a power of 2.

772. For each positive integer n, $S(n)$ is defined to be the greatest integer such that, for every positive integer $k \le S(n)$, n^2 can be written as the sum of k positive squares.
a.) Prove that $S(n) \le n^2 - 14$ for each $n \ge 4$.
b.) Find an integer n such that $S(n) = n^2 - 14$.
c.) Prove that there are infintely many integers n such that $S(n) = n^2 - 14$.

773. Let p, q, n be three positive integers with $p + q < n$. Let $(x_0, x_1, \cdots, x_n)$ be an $(n+1)$-tuple of integers satisfying the following conditions :
(a) $x_0 = x_n = 0$, and
(a) For each i with $1 \le i \le n$, either $x_i - x_{i-1} = p$ or $x_i - x_{i-1} = -q$.
Show that there exist indices $i < j$ with $(i, j) \ne (0, n)$, such that $x_i = x_j$.

774. The positive integers a and b are such that the numbers $15a + 16b$ and $16a - 15b$ are both squares of positive integers. What is the least possible value that can be taken on by the smaller of these two squares?

775. Let $n + 1, n \ge 1$ positive integers be formed by taking the product of n given prime numbers (a prime number can appear several times or also not appear at all in a product formed in this way.) Prove that among these $n + 1$ one can find some numbers whose product is a perfect square.

776. Let a, b, c be integers different from zero. It is known that the equation $a \cdot x^2 + b \cdot y^2 + c \cdot z^2 = 0$ has a solution (x, y, z) in integer numbers different from the solutions $x = y = z = 0$. Prove that the equation

$$a \cdot x^2 + b \cdot y^2 + c \cdot z^2 = 1$$

has a solution in rational numbers.

777. A positive integer is called a *double number* if its decimal representation consists of a block of digits, not commencing with 0, followed immediately by an identical block. So, for instance, 360360 is a double number, but 36036 is not. Show that there are infinitely many double numbers which are perfect squares.

778. For each positive integer k and n, let $S_k(n)$ be the base k digit sum of n. Prove that there are at most two primes p less than 20,000 for which $S_{31}(p)$ are composite numbers with at least two distinct prime divisors.

779. Find all positive integers x such that the product of all digits of x is given by $x^2 - 10 \cdot x - 22$.

780. Let p be the product of two consecutive integers greater than 2. Show that there are no integers $x_1, x_2, \ldots, x_p$ satisfying the equation

$$\sum_{i=1}^{p} x_i^2 - \frac{4}{4 \cdot p + 1} \left(\sum_{i=1}^{p} x_i \right)^2 = 1$$

1.4.6 Orlando - Part 6

781. Given integers $a_1, \ldots, a_{10}$, prove that there exist a non-zero sequence $\{x_1, \ldots, x_{10}\}$ such that all x_i belong to $\{-1, 0, 1\}$ and the number $\sum_{i=1}^{10} x_i \cdot a_i$ is divisible by 1001.

782. Let $g(n)$ be defined as follows:

$$g(1) = 0, g(2) = 1$$

and

$$g(n+2) = g(n) + g(n+1) + 1, n \geq 1.$$

Prove that if $n > 5$ is a prime, then n divides $g(n) \cdot (g(n) + 1)$.

783. Let $P(x)$ be a polynomial with integer coefficients. We denote $\deg(P)$ its degree which is ≥ 1. Let $n(P)$ be the number of all the integers k for which we have $(P(k))^2 = 1$. Prove that $n(P) - \deg(P) \leq 2$.

784. Prove that for any n natural, the number

$$\sum_{k=0}^{n} \binom{2n+1}{2k+1} 2^{3k}$$

cannot be divided by 5.

785. Let $a_i, b_i, i = 1, 2, \ldots, k$ be positive integers. For any i, a_i and b_i we have that $\gcd(i, a_i, b_i) = 1$. Let m be the smallest common multiple of $b_1, b_2, \ldots, b_k$. Prove the equality between the greatest common divisor of $(a_i m / b_i), i = 1, 2, \ldots, k$ and the greatest common divisor of $ai, i = 1, 2, \ldots, k$.

786. Express the number 1988 as the sum of some positive integers in such a way that the product of these positive integers is maximal.

787. Let a and b be two positive integers such that $a \cdot b + 1$ divides $a^2 + b^2$. Show that $\frac{a^2+b^2}{a \cdot b+1}$ is a perfect square.

788. Let a be the greatest positive root of the equation $x^3 - 3 \cdot x^2 + 1 = 0$. Show that $\left[a^{1788}\right]$ and $\left[a^{1988}\right]$ are both divisible by 17. Here $[x]$ denotes the integer part of x.

789. Does there exist an integer such that its cube is equal to $3n^2 + 3n + 7$, where n is an integer.

790. In what case does the system of equations

$$x + y + mz = a$$

$$x + my + z = b$$

$$mx + y + z = c$$

have a solution? Find conditions under which the unique solution of the above system is an arithmetic progression.

791. Let $p_1, p_2, \ldots, p_{25}$ are primes which don't exceed 2004. Find the largest integer T such that every positive integer $\le T$ can be expressed as sums of distinct divisors of $(p_1 \cdot p_2 \cdot \ldots \cdot p_{25})^{2004}$.

792. Let u be a fixed positive integer. Prove that the equation $n! = u^\alpha - u^\beta$ has a finite number of solutions (n, α, β).

793. Find all integer solutions to $2x^4 + 1 = y^2$.

794. For all primes $p \ge 3$, define $F(p) = \sum_{k=1}^{\frac{p-1}{2}} k^{120}$ and $f(p) = \frac{1}{2} - \left\{ \frac{F(p)}{p} \right\}$, where $\{x\} = x - [x]$, find the value of $f(p)$.

795. For any prime p, prove that there exists integer x_0 such that $p|(x_0^2 - x_0 + 3)$ $\Leftrightarrow$ there exists integer y_0 such that $p|(y_0^2 - y_0 + 25)$.

796. Let f be a function $f : \mathbb{N} \cup \{0\} \mapsto \mathbb{N}$, and satisfies the following conditions:
(1) $f(0) = 0, f(1) = 1$,
(2) $f(n + 2) = 23 \cdot f(n + 1) + f(n), n = 0, 1, \ldots$.
Prove that for any $m \in \mathbb{N}$, there exist a $d \in \mathbb{N}$ such that $m|f(f(n)) \iff d|n$.

797. Prove that for every integer power of 2, there exists a multiple of it with all digits (in decimal expression) not zero.

798. Let $v_0 = 0, v_1 = 1$ and $v_{n+1} = 8 \cdot v_n - v_{n-1}$, $n = 1, 2, \ldots.$ Prove that in the sequence $\{v_n\}$ there aren't terms of the form $3^\alpha \cdot 5^\beta$ with $\alpha, \beta \in \mathbb{N}$.

799. Let $f(x) = 3x + 2$. Prove that there exists $m \in \mathbb{N}$ such that $f^{100}(m)$ is divisible by 1988.

800. Let n be a positive integer. Prove that the number $2^n + 1$ has no prime divisor of the form $8 \cdot k - 1$, where k is a positive integer.

1.4.7 Orlando - Part 7

801. Prove that there exists an integer n, $n \geq 2002$, and n distinct positive integers $a_1, a_2, \ldots, a_n$ such that the number $N = a_1^2 a_2^2 \cdots a_n^2 - 4(a_1^2 + a_2^2 + \cdots + a_n^2)$ is a perfect square.

802. Let m be a given positive integer which has a prime divisor greater than $\sqrt{2m}+1$. Find the minimal positive integer n such that there exists a finite set S of distinct positive integers satisfying the following two conditions:
I. $m \leq x \leq n$ for all $x \in S$;
II. the product of all elements in S is the square of an integer.

803. Let d be a positive divisor of $5+1998^{1998}$. Prove that $d = 2\cdot x^2 + 2\cdot x \cdot y + 3 \cdot y^2$, where x, y are integers if and only if d is congruent to 3 or 7 $\pmod{20}$.

804. Find all integer polynomials $P(x)$, the highest coefficent is 1 such that: there exist infinitely irrational numbers a such that $p(a)$ is a positive integer.

805. For each positive integer n, let $f(n)$ be the maximal natural number such that: $2^{f(n)}$ divides $\sum_{i=0}^{\left\lfloor \frac{n-1}{2} \right\rfloor} \binom{n}{2\cdot i+1} 3^i$. Find all n such that $f(n) = 1996$.

806. Consider the equation

$$x^2 + y^2 + z^2 + t^2 - N \cdot x \cdot y \cdot z \cdot t - N = 0$$

where N is a given positive integer.
a) Prove that for an infinite number of values of N, this equation has positive integral solutions (each such solution consists of four positive integers x, y, z, t),
b) Let $N = 4 \cdot k \cdot (8 \cdot m + 7)$ where k, m are no-negative integers. Prove that the considered equation has no positive integral solutions.

807. Find all pair of positive integers (x, y) satisfying the equation

$$x^2 + y^2 - 5 \cdot x \cdot y + 5 = 0.$$

808. Let two natural number $n > 1$ and m be given. Find the least positive integer k which has the following property: Among k arbitrary integers $a_1, a_2, \ldots, a_k$ satisfying the condition $a_i - a_j$ ($1 \leq i < j \leq k$) is not divided by n, there exist two numbers a_p, a_s $(p \neq s)$ such that $m + a_p - a_s$ is divided by n.

809. For every natural number n we define $f(n)$ by the following rule: $f(1) = 1$ and for $n > 1$ then $f(n) = 1 + a_1 \cdot p_1 + \ldots + a_k \cdot p_k$, where $n = p_1^{a_1} \cdots p_k^{a_k}$ is the canonical prime factorisation of n ($p_1, \ldots, p_k$ are distinct primes and $a_1, \ldots, a_k$ are positive integers). For every positive integer s, let $f_s(n) = f(f(\ldots f(n)) \ldots)$, where on the right hand side there are exactly s symbols f. Show that for every given natural number a, there is a natural number s_0 such that for all $s > s_0$, the sum $f_s(a) + f_{s-1}(a)$ does not depend on s.

810. Given an integer $n > 1$, denote by P_n the product of all positive integers x less than n and such that n divides $x^2 - 1$. For each $n > 1$, find the remainder of P_n on division by n.

811. Let p be an odd prime and n a positive integer. In the coordinate plane, eight distinct points with integer coordinates lie on a circle with diameter of length p^n. Prove that there exists a triangle with vertices at three of the given points such that the squares of its side lengths are integers divisible by p^{n+1}.

812. For positive integer $a \geq 2$, denote N_a as the number of positive integer k with the following property: the sum of squares of digits of k in base a representation equals k. Prove that:
a.) N_a is odd;
b.) For every positive integer M, there exist a positive integer $a \geq 2$ such that $N_a \geq M$.

813. Find all prime numbers p which satisfy the following condition: For any prime $q < p$, if $p = kq + r, 0 \leq r < q$, there does not exist an integer $q > 1$ such that $a^2 \mid r$.

814. For any $h = 2^r$ (r is a non-negative integer), find all $k \in \mathbb{N}$ which satisfy the following condition: There exists an odd natural number $m > 1$ and $n \in \mathbb{N}$, such that $k \mid m^h - 1, m \mid n^{\frac{m^h-1}{k}} + 1$.

815. Find the smallest prime number p that cannot be represented in the form $|3^a - 2^b|$, where a and b are non-negative integers.

816. Find all sets comprising of 4 natural numbers such that the product of any 3 numbers in the set leaves a remainder of 1 when divided by the remaining number.

817. Find all positive integer n such that the equation $x^3+y^3+z^3 = n\cdot x^2\cdot y^2\cdot z^2$ has positive integer solutions.

818. Let $p \in \mathbb{P}, p > 3$. Calcute:
a)$S = \sum_{k=1}^{\frac{p-1}{2}} \left[\frac{2k^2}{p}\right] - 2\cdot\left[\frac{k^2}{p}\right]$ if $p \equiv 1 \mod 4$
b) $T = \sum_{k=1}^{\frac{p-1}{2}} \left[\frac{k^2}{p}\right]$ if $p \equiv 1 \mod 8$

819. Lesha put numbers from 1 to 22^2 into cells of 22×22 board. Can Oleg always choose two cells, adjacent by the side or by vertex, the sum of numbers in which is divisible by 4?

820. Ten mutually distinct non-zero reals are given such that for any two, either their sum or their product is rational. Prove that squares of all these numbers are rational.

1.4.8 Orlando - Part 8

821. Find the least positive integer, which may not be represented as $\frac{2^a-2^b}{2^c-2^d}$, where a, b, c, d are positive integers.

822. Positive integers $x > 1$ and y satisfy an equation $2x^2 - 1 = y^{15}$. Prove that 5 divides x.

823. Integers $x > 2$, $y > 1$, $z > 0$ satisfy an equation $x^y + 1 = z^2$. Let p be a number of different prime divisors of x, q be a number of different prime divisors of y. Prove that $p \geq q + 2$.

824. Let m be a positive integer and define $f(m) =$ the number of factors of 2 in $m!$ (that is, the greatest positive integer k such that $2^k | m!$). Prove that there are infinite positive integers m such that $m - f(m) = 1989$.

825. Determine all $n \in \mathbb{N}$ for which $n^{10} + n^5 + 1$ is prime.

826. Find all nonnegative integer solutions (x, y, z, w) of the equation

$$2^x \cdot 3^y - 5^z \cdot 7^w = 1.$$

827. Prove that

$$\prod_{k=1}^{n-1} \Gamma\left(\frac{k}{n}\right) = \frac{(2 \cdot \pi)^{\frac{n-1}{2}}}{\sqrt{n}}$$

with $n \in \mathbb{Z}^+$ and Γ denoting the Euler gamma function.

828. Find five integers, the sum of the squares of every four of which is a square.

829. Find three positive integers whose sum, sum of squares, and sum of perfect fourth powers are rational cubes.

830. Find three positive whole numbers whose sum, sum of squares, and sum of cubes are rational cubes.

831. Find three positive whole numbers whose sum, sum of squares, and sum of perfect fourth powers are all rational squares.

832. Find four positive cube numbers, the sum of any three of which is a cube.

833. Find four square numbers such that the sum of every two of them is a square.

834. Find three square numbers whose sum is a square, such that the sum of every two is also a square.

835. Find four positive whole numbers, the sum of every two of which is a perfect fourth power.

836. Find four perfect fourth power numbers whose is a perfect fourth power.

837. Find three perfect fourth power numbers whose sum is a perfect fourth power.

838. Let $A_1 A_2 \ldots A_n$ be a regular polygon. Find all points P in the polygon's plane with the property: the squares of distances from P to the polygon's vertices are consecutive terms of an arithmetic sequence.

839. Prove that $2^{147} - 1$ is divisible by 343.

840. Let a be a non-zero real number. For each integer n, we define $S_n = a^n + a^{-n}$. Prove that if for some integer k, the sums S_k and S_{k+1} are integers, then the sums S_n are integers for all integer n.

1.4.9 Orlando - Part 9

841. Prove the following statement: If r_1 and r_2 are real numbers whose quotient is irrational, then any real number x can be approximated arbitrarily well by the numbers of the form $z_{k_1,k_2} = k_1 r_1 + k_2 r_2$ integers, i.e. for every number x and every positive real number p two integers k_1 and k_2 can be found so that $|x - (k_1 r_1 + k_2 r_2)| < p$ holds.

842. Suppose $\tan\alpha = \dfrac{p}{q}$, where p and q are integers and $q \neq 0$. Prove that the number $\tan\beta$ for which $\tan 2\beta = \tan 3\alpha$ is rational only when $p^2 + q^2$ is the square of an integer.

843. Which fractions $\dfrac{p}{q}$, where p, q are positive integers < 100, is closest to $\sqrt{2}$? Find all digits after the point in decimal representation of that fraction which coincide with digits in decimal representation of $\sqrt{2}$ (without using any table).

844. Let n be a positive integer. Find the maximal number of non-congruent triangles whose sides lengths are integers $\leq n$.

845. Let k, m, n be natural numbers such that $m + k + 1$ is a prime greater than $n + 1$. Let $c_s = s(s + 1)$. Prove that

$$(c_{m+1} - c_k)(c_{m+2} - c_k)\dots(c_{m+n} - c_k)$$

is divisible by the product $c_1 c_2 \dots c_n$.

846. Prove that for any integer a there exist infinitely many positive integers n such that $a^{2^n} + 2^n$ is not a prime.

847. Prove that for every real number M there exists an infinite arithmetic progression such that:

- each term is a positive integer and the common difference is not divisible by 10
- the sum of the digits of each term (in decimal representation) exceeds M.

848. Denote by S the set of all primes such the decimal representation of $\frac{1}{p}$ has the fundamental period divisible by 3. For every $p \in S$ such that $\frac{1}{p}$ has the fundamental period $3r$ one may write

$$\frac{1}{p} = 0, a_1 a_2 \dots a_{3r} a_1 a_2 \dots a_{3r} \dots,$$

where $r = r(p)$; for every $p \in S$ and every integer $k \geq 1$ define $f(k, p)$ by

$$f(k, p) = a_k + a_{k+r(p)} + a_{k+2.r(p)}$$

a) Prove that S is infinite.
b) Find the highest value of $f(k, p)$ for $k \geq 1$ and $p \in S$.

849. Let n, k be positive integers such that n is not divisible by 3 and $k \geq n$. Prove that there exists a positive integer m which is divisible by n and the sum of its digits in decimal representation is k.

850. Prove that there exists two strictly increasing sequences (a_n) and (b_n) such that $a_n(a_n+1)$ divides b_n^2+1 for every natural n.

851. Prove that every positive rational number can be represented in the form $\dfrac{a^3+b^3}{c^3+d^3}$ where a,b,c,d are positive integers.

852. Find all the pairs of positive integers (x, p) such that p is a prime, $x \leq 2p$ and x^{p-1} is a divisor of $(p-1)^x+1$.

853. Let $a_0, a_1, a_2, \ldots$ be an increasing sequence of nonnegative integers such that every nonnegative integer can be expressed uniquely in the form $a_i + 2a_j + 4a_k$, where i, j and k are not necessarily distinct. Determine a_{1998}.

854. Prove that for each positive integer n, there exists a positive integer with the following properties: It has exactly n digits. None of the digits is 0. It is divisible by the sum of its digits.

855. For any positive integer n, let $\tau(n)$ denote the number of its positive divisors (including 1 and itself). Determine all positive integers m for which there exists a positive integer n such that $\frac{\tau(n^2)}{\tau(n)} = m$.

856. Determine all positive integers n for which there exists an integer m such that $2^n - 1$ is a divisor of m^2+9.

857. A sequence of integers $a_1, a_2, a_3, \ldots$ is defined as follows: $a_1 = 1$ and for $n \geq 1$, a_{n+1} is the smallest integer greater than a_n such that $a_i + a_j \neq 3a_k$ for any i, j and k in $\{1, 2, 3, \ldots, n+1\}$, not necessarily distinct. Determine a_{1998}.

858. Determine the smallest integer $n \geq 4$ for which one can choose four different numbers a, b, c and d from any n distinct integers such that $a+b-c-d$ is divisible by 20.

859. Determine all pairs (a, b) of real numbers such that $a\lfloor bn \rfloor = b\lfloor an \rfloor$ for all positive integers n. (Note that $\lfloor x \rfloor$ denotes the greatest integer less than or equal to x.)

860. Determine all pairs (x, y) of positive integers such that $x^2y + x + y$ is divisible by $xy^2 + y + 7$.

1.4.10 Orlando - Part 10

861. Let p be a prime number and let A be a set of positive integers that satisfies the following conditions:
(1) The set of prime divisors of the elements in A consists of $p-1$ elements;
(2) for any nonempty subset of A, the product of its elements is not a perfect p-th power.
What is the largest possible number of elements in A?

862. Let m be a fixed integer greater than 1. The sequence x_0, x_1, x_2, … is defined as follows:
$x_i = 2^i$ if $0 \le i \le m-1$ and $x_i = \sum_{j=1}^{m} x_{i-j}$, if $i \ge m$.

863. Is it possible to find 100 positive integers not exceeding 25,000, such that all pairwise sums of them are different?

864. Let $a > b > c > d$ be positive integers and suppose that

$$ac + bd = (b + d + a - c)(b + d - a + c).$$

Prove that $ab + cd$ is not prime.

865. Let $p \ge 5$ be a prime number. Prove that there exists an integer a with $1 \le a \le p-2$ such that neither $a^{p-1} - 1$ nor $(a+1)^{p-1} - 1$ is divisible by p^2.

866. Let $a_1 = 11^{11}$, $a_2 = 12^{12}$, $a_3 = 13^{13}$, and $a_n = |a_{n-1} - a_{n-2}| + |a_{n-2} - a_{n-3}|, n \ge 4$. Determine $a_{14^{14}}$.

867. Consider the system $x + y = z + u, 2xy = zu$. Find the greatest value of the real constant m such that $m \le x/y$ for any positive integer solution (x, y, z, u) of the system, with $x \ge y$.

868. Prove that there is no positive integer n such that, for $k = 1, 2, \ldots, 9$, the leftmost digit (in decimal notation) of $(n + k)!$ equals k.

869. Find all pairs of positive integers $m, n \ge 3$ for which there exist infinitely many positive integers a such that

$$\frac{a^m + a - 1}{a^n + a^2 - 1}$$

is itself an integer.

870. Let $m, n \ge 2$ be positive integers, and let $a_1, a_2, \ldots, a_n$ be integers, none of which is a multiple of m^{n-1}. Show that there exist integers $e_1, e_2, \ldots, e_n$, not all zero, with $|\, e_i \,| < m$ for all i, such that $e_1a_1 + e_2a_2 + \ldots + e_na_n$ is a multiple of m^n.

871. Is there a positive integer m such that the equation

$$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{abc} = \frac{m}{a + b + c}$$

has infinitely many solutions in positive integers a, b, c?

872. Let $p_1, p_2, \ldots, p_n$ be distinct primes greater than 3. Show that $2^{p_1p_2\cdots p_n} + 1$ has at least 4^n divisors.

873. Let $n \ge 2$ be a positive integer, with divisors $1 = d_1 < d_2 < \ldots < d_k = n$. Prove that $d_1d_2 + d_2d_3 + \ldots + d_{k-1}d_k$ is always less than n^2, and determine when it is a divisor of n^2.

874. What is the smallest positive integer t such that there exist integers $x_1, x_2, \ldots, x_t$ with
$$x_1^3 + x_2^3 + \ \ldots \ + x_t^3 = 2002^{2002}?$$

875. Find all pairs of positive integers $(x;\ y)$ satisfying the equation $2^x = 3^y + 5$.

876. A given natural number N is being decomposed in a sum of some consecutive integers.
a.) Find all such decompositions for $N = 500$.
b.) How many such decompositions does the number $N = 2^\alpha 3^\beta 5^\gamma$ (where α, β and γ are natural numbers) have? Which of these decompositions contain natural summands only?
c.) Determine the number of such decompositions (= decompositions in a sum of consecutive integers; these integers are not necessarily natural) for an arbitrary natural N.

877. Determine the digits x, y and z if it is known that the equation

$$\sqrt{\underbrace{\overline{xx\ldots x}}_{2n \text{ digits}} - \underbrace{\overline{yy\ldots y}}_{n \text{ digits}}} = \underbrace{\overline{zz\ldots z}}_{n \text{ digits}}$$

holds, at least, for two different values of the natural number n. Find all values for n for which this equation remains true.

878. How many real solutions does the equation

$$x = 1964 \sin x - 189$$

have ?

879. Does there exist a natural number z which can be represented in the form $z = x! + y!$, where x and y are natural numbers satisfying the inequality $x \le y$, in (at least) two different ways ?

880. Let $a, b, c, d > 1$ natural numbers which satisfy $a^{b^{c^d}} = d^{c^{b^a}}$. Prove that $a = d$ and $b = c$.

1.5 Valentin

1.5.1 Valentin - Part 1

881. Solve the equation

$$x^3 + 2y^3 - 4x - 5y + z^2 = 2012,$$

in the set of integers.

882. We consider the proposition $p(n)$: $n^2 + 1$ divides $n!$, for positive integers n. Prove that there are infinite values of n for which $p(n)$ is true, and infinite values of n for which $p(n)$ is false.

883. Prove that if $n \geq 4$, $n \in \mathbb{Z}$ and $\left\lfloor \frac{2^n}{n} \right\rfloor$ is a power of 2, then n is also a power of 2.

884. The world-renowned Marxist theorist *Joric* is obsessed with both mathematics and social egalitarianism. Therefore, for any decimal representation of a positive integer n, he tries to partition its digits into two groups, such that the difference between the sums of the digits in each group be as small as possible. Joric calls this difference the *defect* of the number n. Determine the average value of the defect (over all positive integers), that is, if we denote by $\delta(n)$ the defect of n, compute

$$\lim_{n\to\infty} \frac{\sum_{k=1}^{n} \delta(k)}{n}.$$

885. i) Find all infinite arithmetic progressions formed with positive integers such that there exists a number $N \in \mathbb{N}$, such that for any prime p, $p > N$, the p-th term of the progression is also prime.
ii) Find all polynomials $f(X) \in \mathbb{Z}[X]$, such that there exist $N \in \mathbb{N}$, such that for any prime p, $p > N$, $|f(p)|$ is also prime.

886. Consider the set $E = \{1, 2, \ldots, 2n\}$. Prove that an element $c \in E$ can belong to a subset $A \subset E$, having n elements, and such that any two distinct elements in A do not divide one each other, if and only if

$$c > n\left(\frac{2}{3}\right)^{k+1},$$

where k is the exponent of 2 in the factoring of c.

887. Prove that the function $f : \mathbb{N} \longrightarrow \mathbb{Z}$ defined by $f(n) = n^{2007} - n!$, is injective.

888. Let a_i, $i = 1, 2, \ldots, n$, $n \geq 3$, be positive integers, having the greatest common divisor 1, such that

$$a_j \text{ divide } \sum_{i=1}^{n} a_i$$

for all $j = 1, 2, \ldots, n$. Prove that

$$\prod_{i=1}^{n} a_i \text{ divides } \Big(\sum_{i=1}^{n} a_i\Big)^{n-2}.$$

889. Let n be a positive integer of the form $4k+1$, $k \in \mathbb{N}$ and $A = \{a^2 + nb^2 \mid a, b \in \mathbb{Z}\}$. Prove that there exist integers x, y such that $x^n + y^n \in A$ and $x + y \notin A$.

890. Find all non-negative integers m, n, p, q such that

$$p^m q^n = (p+q)^2 + 1.$$

891. For all integers $n \geq 1$ we define $x_{n+1} = x_1^2 + x_2^2 + \cdots + x_n^2$, where x_1 is a positive integer. Find the least x_1 such that 2006 divides x_{2006}.

892. Find all triplets of positive rational numbers (m, n, p) such that the numbers $m + \frac{1}{np}$, $n + \frac{1}{pm}$, $p + \frac{1}{mn}$ are integers.

893. For each integer $n \geq 3$, find the smallest natural number $f(n)$ having the following property:

$\star$ For every subset $A \subset \{1, 2, \ldots, n\}$ with $f(n)$ elements, there exist elements $x, y, z \in A$ that are pairwise coprime.

894. For all positive integers m, n define $f(m, n) = m^{3^{4n}+6} - m^{3^{4n}+4} - m^5 + m^3$. Find all numbers n with the property that $f(m, n)$ is divisible by 1992 for every m.

895. Let p be a prime and $m \geq 2$ be an integer. Prove that the equation

$$\frac{x^p + y^p}{2} = \left(\frac{x+y}{2}\right)^m$$

has a positive integer solution $(x, y) \neq (1, 1)$ if and only if $m = p$.

896. Let a and b be natural numbers with $a > b$ and having the same parity. Prove that the solutions of the equation

$$x^2 - (a^2 - a + 1)(x - b^2 - 1) - (b^2 + 1)^2 = 0$$

are natural numbers, none of which is a perfect square.

897. Let $p > 5$ be a prime. Consider the set $X = \{p - n^2 \mid n \in \mathbb{N}\}$. Prove that there exist two distinct elements $x, y \in X$ such that $x \neq 1$ and $x \mid y$.

898. Consider the finite sequence $\left\lfloor \frac{k^2}{1998} \right\rfloor$, for $k = 1, 2, \ldots, 1997$. How many distinct terms are there in this sequence?

899. Prove that the following equation has no solution in integer numbers:

$$x^2 + 4 = y^5.$$

900. Let p be an odd prime congruent to 2 modulo 3. Prove that at most $p - 1$ members of the set $\{m^2 - n^3 - 1 \mid 0 < m,\ n < p\}$ are divisible by p.

1.5.2 Valentin - Part 2

901. Let a, b, n be positive integers such that $2^n - 1 = ab$. Let $k \in \mathbb{N}$ such that $ab + a - b - 1 \equiv 0 \pmod{2^k}$ and $ab + a - b - 1 \not\equiv 0 \pmod{2^{k+1}}$. Prove that k is even.

902. Show that for any n we can find a set X of n distinct integers greater than 1, such that the average of the elements of any subset of X is a square, cube or higher power.

903. For which pairs of positive integers (m, n) there exists a set A such that for all positive integers x, y, if $|x-y| = m$, then at least one of the numbers x, y belongs to the set A, and if $|x-y| = n$, then at least one of the numbers x, y does not belong to the set A?

904. Find all positive integers n and p if p is prime and

$$n^8 - p^5 = n^2 + p^2.$$

905. For positive integers $a_1, a_2, \dots, a_{2006}$ such that $\frac{a_1}{a_2}, \frac{a_2}{a_3}, \dots, \frac{a_{2005}}{a_{2006}}$ are pairwise distinct, find the minimum possible amount of distinct positive integers in the set$\{a_1, a_2, ..., a_{2006}\}$.

906. Solve in positive integers the equation

$$n = \varphi(n) + 420,$$

where $\varphi(n)$ is the number of positive integers less than n having no common prime factors with n.

907. Find all positive integer solutions to $2^x - 5 = 11^y$.

908. For any positive integer k, let f_k be the number of elements in the set $\{k+1, k+2, \dots, 2k\}$ whose base 2 representation contains exactly three 1s.
(a) Prove that for any positive integer m, there exists at least one positive integer k such that $f(k) = m$.
(b) Determine all positive integers m for which there exists [i]exactly one[/i] k with $f(k) = m$.

909. Show that there exists a set A of positive integers with the following property: for any infinite set S of primes, there exist *two* positive integers m in A and n not in A, each of which is a product of k distinct elements of S for some $k \geq 2$.

910. Find all positive integers $n \geq 1$ such that $n^2 + 3^n$ is the square of an integer.

911. Find all positive integers which have exactly 16 positive divisors $1 = d_1 < d_2 < \ldots < d_{16} = n$ such that the divisor d_k, where $k = d_5$, equals $(d_2 + d_4)d_6$.

912. Find all 3-digit positive integers $\overline{abc}$ such that

$$\overline{abc} = abc(a + b + c),$$

where $\overline{abc}$ is the decimal representation of the number.

913. Does there exist a positive integer n such that n has exactly 2000 prime divisors and n divides $2^n + 1$?

914. Prove that for all positive integers $n \geq 1$ the number $\prod_{k=1}^{n} k^{2k-n-1}$ is also an integer number.

915. Find all primes p, q such that $\alpha^{3pq} - \alpha \equiv 0 \pmod{3pq}$ for all integers α.

916. Find the greatest positive integer n for which there exist n nonnegative integers $x_1, x_2, \ldots, x_n$, not all zero, such that for any $\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_n$ from the set $\{-1, 0, 1\}$, not all zero, $\varepsilon_1 x_1 + \varepsilon_2 x_2 + \cdots + \varepsilon_n x_n$ is not divisible by n^3.

917. Let A be the set $A = \{1, 2, \ldots, n\}$. Determine the maximum number of elements of a subset $B \subset A$ such that for all elements x, y from B, $x + y$ cannot be divisible by $x - y$.

918. Find all positive integers A which can be represented in the form:

$$A = \left(m - \frac{1}{n}\right)\left(n - \frac{1}{p}\right)\left(p - \frac{1}{m}\right)$$

where $m \geq n \geq p \geq 1$ are integer numbers.

919. For every positive integer n we denote by $d(n)$ the sum of its digits in the decimal representation. Prove that for each positive integer k there exists a positive integer m such that the equation $x + d(x) = m$ has exactly k solutions in the set of positive integers.

920. Prove that for any integer n, $n \geq 3$, there exist n positive integers $a_1, a_2, \ldots, a_n$ in arithmetic progression, and n positive integers, and n positive integers in geometric progression $b_1, b_2, \ldots, b_n$ such that

$$b_1 < a_1 < b_2 < a_2 < \cdots < b_n < a_n.$$

Give an example of two such progressions having at least five terms.

1.5.3 Valentin - Part 3

921. Prove that for any positive integer n, the number

$$S_n = \binom{2n+1}{0} \cdot 2^{2n} + \binom{2n+1}{2} \cdot 2^{2n-2} \cdot 3 + \cdots + \binom{2n+1}{2n} \cdot 3^n$$

is the sum of two consecutive perfect squares.

922. a) Prove that there exists a sequence of digits $\{c_n\}_{n\geq 1}$ such that or each $n \geq 1$ no matter how we interlace k_n digits, $1 \leq k_n \leq 9$, between c_n and c_{n+1}, the infinite sequence thus obtained does not represent the fractional part of a rational number.
b) Prove that for $1 \leq k_n \leq 10$ there is no such sequence $\{c_n\}_{n\geq 1}$.

923. Let $n \geq 0$ be an integer and let $p \equiv 7 \pmod 8$ be a prime number. Prove that

$$\sum_{k=1}^{p-1} \left\{ \frac{k^{2^n}}{p} - \frac{1}{2} \right\} = \frac{p-1}{2}.$$

924. For m a positive integer, let $s(m)$ be the sum of the digits of m. For $n \geq 2$, let $f(n)$ be the minimal k for which there exists a set S of n positive integers such that $s\left(\sum_{x \in X} x\right) = k$ for any nonempty subset $X \subset S$. Prove that there are constants $0 < C_1 < C_2$ with

$$C_1 \log_{10} n \leq f(n) \leq C_2 \log_{10} n.$$

925. Determine all composite positive integers n for which it is possible to arrange all divisors of n that are greater than 1 in a circle so that no two adjacent divisors are relatively prime.

926. Find the largest positive integer $n > 10$ such that the residue of n when divided by each perfect square between 2 and $\dfrac{n}{2}$ is an odd number.

927. Let m, n be co-prime integers, such that m is even and n is odd. Prove that the following expression does not depend on the values of m and n:

$$\frac{1}{2n} + \sum_{k=1}^{n-1} (-1)^{\left[\frac{mk}{n}\right]} \left\{\frac{mk}{n}\right\}.$$

928. Let $n \geq 1$ be an integer and let X be a set of n^2+1 positive integers such that in any subset of X with $n+1$ elements there exist two elements $x \neq y$ such that $x \mid y$. Prove that there exists a subset $\{x_1, x_2, \ldots, x_{n+1}\} \in X$ such that $x_i \mid x_{i+1}$ for all $i = 1, 2, \ldots, n$.

929. Solve the equation $3^x = 2^x y + 1$ in positive integers.

930. Prove that for all positive integers n there exists a single positive integer divisible with 5^n which in decimal base is written using n digits from the set $\{1, 2, 3, 4, 5\}$.

931. I'm looking for an elementary proof (no group/fields knowledge involved - because I know how to prove using polynomials over finite fields) for the following theorem: if $p \equiv 1 \pmod 8$ is a prime number then

$$2^{\frac{p-1}{2}} \equiv 1 \pmod p.$$

932. Let $\{a_n\}_{n \geq 0}$ be a sequence of positive integers, given by $a_0 = 2$ and for all positive integers n,

$$a_n = 2a_{n-1}^2 - 1.$$

Prove that if p is an odd prime such that $p \mid a_n$ then $p^2 \equiv 1 \pmod{2^{n+3}}$.

933. Let $1 = d_1 < d_2 < .... < d_k = n$ be all different divisors of positive integer n written in ascending order. Determine all n such that:

- $d_4 = 4d_3 - 5d_2$,
- $d_5 = 2d_2d_3d_4 - 1$,

- n is minimum.

934. Let $1 = d_1 < d_2 < .... < d_k = n$ be all different divisors of positive integer n written in ascending order. Determine all n such that

$$d_1^2 + d_2^2 + d_3^2 + d_4^2 = n.$$

935. Let $1 = d_1 < d_2 < .... < d_k = n$ be all different divisors of positive integer n written in ascending order. Determine all n such that

$$d_{d_4-1} = (d_1 + d_2 + d_4)d_8; \quad k = 12.$$

936. Let $1 = d_1 < d_2 < .... < d_k = n$ be all different divisors of positive integer n written in ascending order. Determine all n such that:

$$d_6^2 + d_7^2 - 1 = n.$$

937. Let $1 = d_1 < d_2 < .... < d_k = n$ be all different divisors of positive integer n written in ascending order. Determine all n such that:

$$d_7^2 + d_{10}^2 = \left(\frac{n}{d_{22}}\right)^2.$$

938. Let $\mathbb{N}$ be the set of positive integers. Let $n \in \mathbb{N}$ and let $d(n)$ be the number of divisors of n. Let $\varphi(n)$ be the Euler-totient function (the number of co-prime positive integers with n, smaller than n).

Find all non-negative integers c such that there exists $n \in \mathbb{N}$ such that

$$d(n) + \varphi(n) = n + c,$$

and for such c find all values of n satisfying the above relationship.

939. Find all integers n, such that the following number is a perfect square

$$N = n^4 + 6n^3 + 11n^2 + 3n + 31.$$

940. Let S be the set of positive integers in which the greatest and smallest elements are relatively prime. For natural n, let S_n denote the set of natural numbers which can be represented as sum of at most n elements (not necessarily different) from S. Let a be greatest element from S. Prove that there are positive integer k and integers b such that $|S_n| = a \cdot n + b$ for all $n > k$.

1.5.4 Valentin - Part 4

941. Let A be a set of positive integers such that
a) if $a \in A$, the all the positive divisors of a are also in A;
b) if $a, b \in A$, with $1 < a < b$, then $1 + ab \in A$.
Prove that if A has at least 3 elements, then A is the set of all positive integers.

942. Let p be a prime number and $f \in \mathbb{Z}[X]$ given by

$$f(x) = a_{p-1}x^{p-2} + a_{p-2}x^{p-3} + \cdots + a_2x + a_1,$$

where $a_i = \left(\frac{i}{p}\right)$ is the Legendre symbol of i with respect to p(i.e. $a_i = 1$ if $i^{\frac{p-1}{2}} \equiv 1 \pmod p$ and $a_i = -1$ otherwise, for all $i = 1, 2, \ldots, p-1$).
a) Prove that $f(x)$ is divisible with $(x-1)$, but not with $(x-1)^2$ iff $p \equiv 3 \pmod 4$;
b) Prove that if $p \equiv 5 \pmod 8$ then $f(x)$ is divisible with $(x-1)^2$ but not with $(x-1)^3$.

943. Let $m \geq 2$ be an integer. A positive integer n has the property that for any positive integer a which is co-prime with n, we have $a^m - 1 \equiv 0 \pmod n$.

Prove that $n \leq 4m(2^m - 1)$.

944. A finite set of positive integers is called [i]isolated [/i]if the sum of the numbers in any given proper subset is co-prime with the sum of the elements of the set.
a) Prove that the set $A = \{4, 9, 16, 25, 36, 49\}$ is isolated;
b) Determine the composite numbers n for which there exist the positive integers a, b such that the set

$$A = \{(a+b)^2, (a+2b)^2, \ldots, (a+nb)^2\}$$

is isolated.

945. Let p be a prime number and let $0 \leq a_1 < a_2 < \cdots < a_m < p$ and $0 \leq b_1 < b_2 < \cdots < b_n < p$ be arbitrary integers. Let k be the number of distinct residues modulo p that $a_i + b_j$ give when i runs from 1 to m, and j from 1 to n. Prove that
a) if $m + n > p$ then $k = p$;
b) if $m + n \leq p$ then $k \geq m + n - 1$.

946. Let a, b, c, d be positive integers such that the number of pairs $(x, y) \in (0, 1)^2$ such that both $ax + by$ and $cx + dy$ are integers is equal with 2004. If $\gcd(a, c) = 6$ find $\gcd(b, d)$.

947. For any positive integer n the sum $1 + \dfrac{1}{2} + \cdots + \dfrac{1}{n}$ is written in the form $\dfrac{P(n)}{Q(n)}$, where $P(n)$ and $Q(n)$ are relatively prime.
a) Prove that $P(67)$ is not divisible by 3;
b) Find all possible n, for which $P(n)$ is divisible by 3.

948. Solve in prime numbers the equation $x^y - y^x = xy^2 - 19$.

949. Find all positive integers n for which there exist distinct positive integers $a_1, a_2, \ldots, a_n$ such that

$$\frac{1}{a_1} + \frac{2}{a_2} + \cdots + \frac{n}{a_n} = \frac{a_1 + a_2 + \cdots + a_n}{n}.$$

950. Let a, b be two positive integers, such that $ab \neq 1$. Find all the integer values that $f(a, b)$ can take, where

$$f(a,b) = \frac{a^2 + ab + b^2}{ab - 1}.$$

951. For which positive integer k there exist positive integers x, y, z such that $(x + y + z)^2 = kxyz$?

952. Find all integer numbers x and y such that:

$$(y^3 + xy - 1)(x^2 + x - y) = (x^3 - xy + 1)(y^2 + x - y).$$

953. Find all the functions from $\mathbb{N}$ to itself such that

$$f(m)^d + f(n) | (m^d + n)^e$$

holds for all $m, n \in \mathbb{N}$, where d, e is given positive constant integers.

954. If the positive integers x and y are such that $3x + 4y$ and $4x + 3y$ are both perfect squares, prove that both x and y are both divisible with 7.

955. Define the sequence of integers $\langle a_n \rangle$ as;

$$a_0 = 1, \quad a_1 = -1, \quad \text{and} \quad a_n = 6a_{n-1} + 5a_{n-2} \quad \forall n \geq 2.$$

Prove that $a_{2012} - 2010$ is divisible by 2011.

956. Let p be prime number. Given an integer a such that $\gcd(a, p!) = 1$, prove that $a^{(p-1)!} - 1$ is divisible by $p!$.

957. Given $F(x) \in \mathbb{Z}[x]$. For all positive integer n , $F(n)$ is divisible by one of $a_1, a_2, \ldots, a_k \in \mathbb{Z}$. Prove that there exists $a_i \in \{a_1, \ldots, a_k\}$ so that for any positive integer n $F(n)$ is divisible by a_i.

958. Let be given 131 distinct natural numbers, each having prime divisors not exceeding 42. Prove that one can choose four of them whose product is a perfect square.

959. How many pairs of integers (x, y) are there such that $3^x = y^2 - 2007$?

960. Find a set of four consecutive positive integers such that the smallest is a multiple of 5, the second is a multiple of 7, the third is a multiple of 9, and the largest is a multiple of 11.

1.6 Darij

1.6.1 Darij - Part 1

961. Let a be a positive integer. A permutation π of the set $\{1, 2, ..., n\}$ will be called a *v-permutation* if for every divisor d of n and for arbitrary i and j from

the set $\{1, 2, ..., n\}$ satisfying $d \mid i - j$, we have $d \mid \pi(i) - \pi(j)$.
For every n, we denote by $v(n)$ the number of all v-permutations of the set $\{1, 2, ..., n\}$.
Find the prime factorization of $v(n)$, given that of n.

962. Let a, b, c be three integers. Prove that there exist six integers x, y, z, x', y', z' such that

$$a = yz' - zy'; b = zx' - xz'; c = xy' - yx'.$$

963. If a_1, a_2, ..., a_n are n nonnegative integers, then show that

$$\frac{(na_1)! \cdot (na_2)! \cdot ... \cdot (na_n)!}{(a_1!)^{n-1} \cdot (a_2!)^{n-1} \cdot ... \cdot (a_n!)^{n-1} \cdot (a_1 + a_2 + ... + a_n)!}$$

is an integer.

964. Let n be a positive integer which has more than k different prime factors. Prove that there exists a convex n-gon in the plane such that
1. all angles of the n-gon are equal;
2. the lengths of the sides of the n-gon are the numbers 1^k, 2^k, ..., n^k in some order.

965. If x and y are two distinct positive integers, and k is a positive integer, then $k \mid \varphi\left(x^k + y^k\right)$.

966. Show that there exist four integers a, b, c, d whose absolute values are all > 1000000 and which satisfy $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d} = \frac{1}{abcd}$.

967. Let a_1, a_2, ..., a_{10} be positive integers such that $a_1 < a_2 < ... < a_{10}$. For every k, denote by b_k the greatest divisor of a_k such that $b_k < a_k$. Assume that $b_1 > b_2 > ... > b_{10}$. Show that $a_{10} > 500$.

968. If an integer $a > 1$ is given such that $(a-1)^3 + a^3 + (a+1)^3$ is the cube of an integer, then show that $4 \mid a$.

969. The sum and the product of two purely periodic decimal fractions a and b are purely periodic decimal fractions of period length T. Show that the lengths of the periods of the fractions a and b are not greater than T.
Note. A *purely periodic decimal fraction* is a periodic decimal fraction without a non-periodic starting part.

970. Let a, b, c and n be positive integers such that a^n is divisible by b, such that b^n is divisible by c, and such that c^n is divisible by a. Prove that $(a+b+c)^{n^2+n+1}$ is divisible by abc.

971. For any positive integer n, let $w(n)$ denote the number of different prime divisors of the number n. (For instance, $w(12) = 2$.) Show that there exist infinitely many positive integers n such that $w(n) < w(n+1) < w(n+2)$.

972. Prove that the equation $4x^3 - 3x + 1 = 2y^2$ has at least 31 solutions in positive integers x and y with $x \leq 2005$.

973. Prove that there are no three nonzero integers a, b, c satisfying

$$\frac{b+c}{a} + \frac{c+a}{b} + \frac{a+b}{c} = 0.$$

974. Given six consecutive positive integers, prove that there exists a prime such that one and only one of these six integers is divisible by this prime.

975. Let a, b, c be three integers such that $a^3 + b^3 + c^3 = (b+c)(c+a)(a+b)$. Prove that at least one of the integers a, b, c is 0.

976. Let b and c be any two positive integers. Define an integer sequence a_n, for $n \geq 1$, by $a_1 = 1$, $a_2 = 1$, $a_3 = b$ and $a_{n+3} = ba_{n+2}a_{n+1} + ca_n$.

Find all positive integers r for which there exists a positive integer n such that the number a_n is divisible by r.

977. **(a)** Does there exist a positive integer n such that the decimal representation of $n!$ ends with the string 2004, followed by a number of digits from the set $\{0;\ 4\}$?
(b) Does there exist a positive integer n such that the decimal representation of $n!$ starts with the string 2004 ?

978. Find the smallest positive integer n with the following property:

For any integer m with $0 < m < 2004$, there exists an integer k such that

$$\frac{m}{2004} < \frac{k}{n} < \frac{m+1}{2005}.$$

979. A positive integer is called *nice* if the sum of its digits in the number system with base 3 is divisible by 3.

Calculate the sum of the first 2005 nice positive integers.

980. Find all positive integers x, y, z and t such that $2^x 3^y + 5^z = 7^t$.

1.6.2 Darij - Part 2

981. Let Q(n) denote the sum of the digits of a positive integer n. Prove that $Q\left(Q\left(Q\left(2005^{2005}\right)\right)\right) = 7$.

982. Let s be a positive real.
Consider a two-dimensional Cartesian coordinate system. A *lattice point* is defined as a point whose coordinates in this system are both integers. At each lattice point of our coordinate system, there is a lamp.
Initially, only the lamp in the origin of the Cartesian coordinate system is turned on; all other lamps are turned off. Each minute, we additionally turn on every lamp L for which there exists another lamp M such that

- the lamp M is already turned on, and

- the distance between the lamps L and M equals s.

Prove that each lamp will be turned on after some time ...

- ... if $s = 13$.
- ... if $s = 2005$.
- ... if s is an integer of the form $s = p_1p_2...p_k$ if p_1, p_2, ..., p_k are different primes which are all $\equiv 1 \mod 4$.
- ... if s is an integer whose prime factors are all $\equiv 1 \mod 4$.

983. Let p be a prime such that $p \equiv 1 \pmod 4$. Evaluate with reasons, $\sum_{k=1}^{\frac{p-1}{2}} \left\{ \frac{k^2}{p} \right\}$.

984. Let a, b, c, d and n be positive integers such that $7 \cdot 4^n = a^2 + b^2 + c^2 + d^2$. Prove that the numbers a, b, c, d are all $\geq 2^{n-1}$.

985. Let $d_n = [1, \ldots, n]$ be the least common multiple of $1, \ldots, n$. Prove by a (literary) one-line elementary argument that $d_n > 2^n$ for $n \geq 9$.

986. Find all sets X consisting of at least two positive integers such that for every two elements $m, n \in X$, where $n > m$, there exists an element $k \in X$ such that $n = mk^2$.

987. A positive integer is written on each of the six faces of a cube. For each vertex of the cube we compute the product of the numbers on the three adjacent faces. The sum of these products is 1001. What is the sum of the six numbers on the faces?

988. Let $f(x)$ be a non-constant polynomial with integer coefficients, and let u be an arbitrary positive integer. Prove that there is an integer n such that $f(n)$ has at least u distinct prime factors and $f(n) \neq 0$.

989. Let m and n be two positive integers. Let a_1, a_2, $\ldots$, a_m be m different numbers from the set $\{1, 2, \ldots, n\}$ such that for any two indices i and j with $1 \leq i \leq j \leq m$ and $a_i + a_j \leq n$, there exists an index k such that $a_i + a_j = a_k$. Show that

$$\frac{a_1 + a_2 + ... + a_m}{m} \geq \frac{n+1}{2}.$$

990.

- Find four integers a, b, c, d such that the numbers a^2+b^2, $a^2+b^2+c^2$ and $a^2 + b^2 + c^2 + d^2$ are perfect squares.
- Does there exist an infinite sequence whose members are perfect squares, and which has the property that for any natural n, the sum of the first n members of the sequence is a perfect square?

991. Find all infinite sequences q_1, q_2, q_3, ... of natural numbers such that for any n ¿ 3, we have
$q_n = \frac{q_{n-3}+q_{n-1}}{q_{n-2}}$.

992. The number $\frac{2}{1}+\frac{2^2}{2}+\frac{2^3}{3}+...+\frac{2^n}{n}$ is represented as a non-reducible fraction $\frac{p_n}{q_n}$ ("non-reducible" means that the numbers p_n and q_n are coprime).
a) Prove that p_n is an even number.
b) Prove that if $n > 3$, then p_n is divisible by 8.
c) Prove that for every natural k, one can find a natural number n such that the numbers p_n, p_{n+1}, p_{n+2}, ... are all divisible by 2^k.

993. A sequence c_n is constructed after the following rule: We set $c_1 = 2$ and $c_{n+1} = \left[\frac{3c_n}{2}\right]$ for every $n \geq 1$. [Hereby, $[x]$ denotes the integer part of the number x.] Prove that
a) This sequence c_n contains infinitely many even numbers and infintely many odd numbers.
b) The sequence $e_n = (-1)^{c_n}$ is non-periodical.
c) [Calculus fundamentals required.] There exists a real number G such that $c_n = \left[\left(\frac{3}{2}\right)^n G\right] + 1$ for every natural n.

994. Do there exist **a)** 6; **b)** 1000 different natural numbers, such that if a and b are any two of them, then the sum $a + b$ is divisible by the difference $a - b$?

995. Let $a_1 < a_2 < a_3 < \cdots < a_n < \cdots$ be an infinite sequence of natural numbers such that every natural number either belongs to this sequence or can be written as the sum of two elements of this sequence (these two elements can also be equal).

Prove that for every natural n, we have $a_n \leq n^2$.

996. Let the sequence $(a, a+d, a+2d, \ldots, a+nd, \ldots)$ be an infinite arithmetic progression, where $d \neq 0$. Prove that this sequence contains an infinite subsequence which is a geometric progression if and only if the ratio $\frac{a}{d}$ is rational.

997. Prove that in every arithmetic sequence $a, a+d, a+2d, \ldots, a+nd, \ldots$, where a and d are natural numbers, we can find infinitely many numbers whose canonic representations consist of the same prime factors.

[An example of two numbers whose canonic representations consist of the same prime factors: $8232 = 2^3 \cdot 3 \cdot 7^3$ and $2016 = 2^5 \cdot 3^2 \cdot 7$.]

998. Prove that there exist infinitely many pairs $(x;\ y)$ of different positive rational numbers, such that the numbers $\sqrt{x^2+y^3}$ and $\sqrt{x^3+y^2}$ are both rational.

999. Let k be a positive integer. A natural number m is called k-*typical* if each divisor of m leaves the remainder 1 when being divided by k.

Prove:
a) If the number of all divisors of a positive integer n (including the divisors 1 and n) is k-typical, then n is the k-th power of an integer.
b) If $k > 2$, then the converse of the assertion **a)** is not true.

1000. Given a set S of 11 natural numbers. Prove that this set S has a non-empty subset T such that if you take all the numbers from T and throw all of their digits in a bin, then each digit occurs in this bin an even number of times.

1.7 Vesselin

1.7.1 Vesselin - Part 1

1001. Prove that there exists an infinite sequence of pairwise co-prime positive > 1, none of which divides a number of the form $2^m + 2^n + 1$, where $m, n \in \mathbb{N}$.

1002. Prove that between any pair of consecutive perfect squares it is possible to choose some integers whose product is twice a square. In other words, for every $n \in \mathbb{N}$ there exists a subset $S \subseteq \{n^2+1, n^2+2, \ldots (n+1)^2\}$ with $\prod_{x \in S} x = 2m^2$ for some $m \in \mathbb{N}$.

1003. Show that the primes p which divide some number of the form $2^n + 1$ have natural density $17/24$ in the set of primes.

1004. Let $f(x)$ be any nonconstant integer polynomial. Show that there is a constant $C > 0$ such that, for any $N > 1$, the number of n with $1 \leq n \leq N$ and $|f(n)|$ prime is less than $CN/\log N$.

1005.

- Let $f(x)$ be any nonconstant integer polynomial, and L an arbitrary positive integer. Show that there are infinitely many primes $p \equiv 1 \mod L$ such that $f(x)$ has a zero modulo p.
- Find all cases of f when, in fact, there exists an L such that $f(x)$ has a zero modulo *every* prime $p \equiv 1 \mod L$.
- Find all arithmetic progressions $am + b$, $(a, b) = 1$, with this property: there is an irreducible nonlinear polynomial f that has a zero modulo every prime of the form $am + b$.

1006. Let A be the set of primes p such that $p \mid 2^n - 3$ for some n; and let $\mathbb{P}$ denote the set of all primes. Show that the sets A and $\mathbb{P} \setminus A$ are both infinite.

1007. For any $C > 0$, show that there exists some $n > 0$ such that $n^2 + 1$ has a prime factor bigger than Cn.

1008. Prove that there exist infinitely many n for which $2^n - 1$ and $3^n - 1$ have a common divisor bigger than $\exp(\exp(\sqrt{\log n}))$.

1009. There have been two threads proving separately the existence of infinitely many n with $n, n+1, n+2$ all squarefree (Proof: the density of squarefree numbers is $6/\pi^2 > 1/2$), and infinitely many n with n^2+1 squarefree (Proof: for arbitrarily large N, the proportion of numbers in n in $[1, N]$ with $n^2 + 1$ not squarefree is less than $2\sum_{p \leq N} \lceil N/p^2 \rceil / N \sim 2\sum_{p \leq N} 1/p^2 < 1$, where the sums are over the primes $1 \mod 4$). Unify both results as follows: there are infinitely many n for which $n, n+1, n+2$ and $n^2 + 1$ are simultaneously squarefree.

1010. Let $p \equiv 3 \pmod 4$ be a prime. Show that the set $\{1, 2, \ldots, \frac{p-1}{2}\}$ contains more quadratic residues than quadratic nonresidues.

1011. Construct an infinite arithmetic progression of positive integers that does not contain any number of the form $2^n + p$ with $n \in \mathbb{N}_0$ and p prime.

1012. Let p be an odd prime, and denote by $r(p)$ the least quadratic residue modulo p in $\{1, 2, \ldots, p-1\}$. Prove that $r(p) < \sqrt{p} + \frac{1}{2}$.

1013. Let $r > 1$ be an integer, and let $p := 4^r - 2^r + 1$. If p is prime, show that $p \equiv 1 \pmod{6r}$.

1014. For every $M > 0$, prove that there exist integers $a > b > M$ such that the set of prime divisors of $a^2 + 1$ coincides with that of $b^2 + 1$.

1015. Do there exist infinitely many pairs $(a, b), a \neq b$ of distinct positive integers such that the numbers in each of the pairs $\{a, b\}$ and $\{a+1, b+1\}$ have the same set of prime divisors?

1016. Let $a, b \in \mathbb{N}$ be positive integers. Prove that there exists a positive integer n for which
$$n \nmid a^{2^n} + b^{3^n}.$$

1017. Let $a_1, \ldots, a_n \in \mathbb{N}$, none of which is a perfect square. Prove or disprove that the number $\sqrt{a_1} + \cdots + \sqrt{a_n}$ is always irrational.

1018. Does there exist an integer polynomial $f \in \mathbb{Z}[x, y]$ in two variables which is a bijection : $\mathbb{Z}^2 \to \mathbb{Z}$?

1019. Find all primes $p \geq 3$ with the following property: for any prime $q < p$, the number
$$p - \left\lfloor \frac{p}{q} \right\rfloor q$$
is squarefree (i.e. is not divisible by the square of a prime).

1020. Let n be a positive integer. Determine all positive integers $m \in \mathbb{N}$ for which there exists an integer polynomial
$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0 \in \mathbb{Z}[x]$$
such that $a_n \neq 0$, $\gcd(a_0, a_1, \ldots, a_n, m) = 1$, and $f(k)$ is divisible by m for all integers $k \in \mathbb{N}$.

1.8 Gabriel

1.8.1 Gabriel - Part 1

1021. This is really a strange theorem: let f be a polynomial with integer coefficients having at least one real root. Then there are infinitely many prime numbers of the form $4k + 3$ dividing at least one term of the sequence $f(1), f(2), f(3), \ldots$.

1022. Let p be a prime k a positive integer and r an integer. Prove that for infinitely many n we have $\binom{2n}{n} = r \pmod{()} p^k)$.

1023. Prove as elementarily as possible the existence of a transcendent number x such that $|x - \frac{p}{q}| > \frac{1}{10q^2}$ for all integers p, q.

1024. Express in a (very) closed form the sum of the m-th powers of the primitive roots mod p.

1025. This is very challenging, but also very beautiful: for any sufficiently large prime p, there exists $g < p$ a primitive root mod p which is relatively prime to $p - 1$.

1026. Find all polynomials with integer coefficients such that $P(n)P(n+1)$ is of the form $P(k)$ for some integer and this for all n.

1027. Prove that the equation $3a^4 + 4b^4 = 19c^4$ has no nontrivial rational solutions.

1028. Let N be a positive integer. Prove that all sufficiently large integers can be written as the sum of a family of pairwise distinct integers greater than N and whose sum of inverses is 1.

1029. Show the existence of an absolute positive constant c such that for all positive irational x there are infinitely many rational numbers $\frac{p}{q}$ with $gcd(p, q) = 1$, $q = 1 \pmod 4$ and $|x - \frac{p}{q}| < \frac{c}{q^2}$.

1030. Find the positive integers a with the following property: any positive integer has at least as many divisors of the form $ak + 1$ as of the form $ak - 1$.

1031. Find all couples (a, b) of positive integers with the following property: there are infinitely many n such that $n^2 | a^n + b^n$.

1032. Let $x_0 = 0, x_1 = 1$ and a, b integers greater than 1. Suppose that $\frac{x_n + x_{n-2}}{x_{n-1}}$ is equal to a if n is even and with b otherwise. Then $x_{n+m}x_{n+m-1}...x_{n+1}$ is a multiple of $x_m x_{m-1}$ for all m, n.

1033. Solve the equation $7^x = 3^y + 4$ in positive integers.

1034. Let a, b, c some positive integers and x, y, z some integer numbers such that we have
a) $ax^2 + by^2 + cz^2 = abc + 2xyz - 1$;
b) $ab + bc + ca \geq x^2 + y^2 + z^2$.
Prove that a, b, c are all sums of three squares of integer numbers.

1035. For infinitely many n we have $\phi(n - \phi(n)) > \phi(n)$.

1036. Let a a base. Prove that any number $n > 0$ has a multiple which uses all digits in base a.

1037. Here is a very very nice problem: the largest prime factor of $2^{2^n}+1$ is greater than $2^{n+2}\cdot(n+1)$. Probably this will give an insight for my problem: if $a>1$ then for infinitely many n the largest prime factor of a^n-1 is greater than $n\log_a n$.

1038. We choose random a unitary polynomial of degree n and coefficients in the set $1,2,...,n!$. Prove that the probability for this polynomial to be special is between 0.71 and 0.75, where a polynomial g is called special if for every $k>1$ in the sequence $f(1),f(2),f(3),...$ there are infinitely many numbers relatively prime with k.

1039. For a given k natural number find all couples (a,b) of positive integers such that $a^n+b^n|(kn)!$ for all sufficiently large n.

1040. Find all triples of integers a,b,c such that $a\cdot 2^n+b|c^n+1$ for all natural number n.

1.8.2 Gabriel - Part 2

1041. Consider x_n the coefficient of x^n in $(x^2+x+1)^n$. Prove that for prime p we have: p^2 divides x_p-1.

1042. Find all polynomials f,g with integer coefficients such that any odd prime can be represented either in the form $f(n)$ (with positive integer n) or in the form $g(n)$ with positive integer n.

1043. For any $M>0$ and any positive integer a show that there exist infinitely many positive integers b such that $|a^m-b^n|>M$ for any positive integers m,n.

1044. Prove that for any integers $a_1,...,a_n$ the number $\frac{\text{lcm}(a_1,...,a_n)}{a_1a_2...a_n}\cdot\prod_{i<j}(a_i-a_j)$ is an integer divisible by $1!2!\cdots(n-2)!$. Moreover, there exists n-uplets for which the two numbers are equal.

1045. Find x,y,z such that $xy^2=z^3+1$ and x does not have prime factors of the form $6k+1$.

1046. Prove that for infinitely many n there is a real polynomial f of degree n such that $p(0),p(1),\ldots,p(n+1)$ are different perfect powers of 2.

1047. Show that there exists a constant $C>0$ such that for all n the sum of digits of 2^n is greater than $C\ln n$.

1048. Prove that there exists infinitely many n such that $n,n+1$ and $n+2$ are squarefree.

1049. Find all polynomials with integer coefficients f such that for all $n>2005$ the number $f(n)$ is a divisor of $n^{n-1}-1$.

1050. Show that if $n+1$ is a multiple of 24, then the sum of the divisors of n is also divisible by 24.

1051. Is there a trinomial (like $f(n) = an^2 + bn + c$) with integer coefficients f such that for all natural numbers n, all prime factors of $f(n)$ are of the form $4k + 3$?

1052. Let p be a prime number of the form $4k+1$, where k is an integer. Show that

$$\sum_{i=1}^{k} \left\lfloor \sqrt{ip} \right\rfloor = \left(p^2 - 1\right)/12.$$

1053. Let $a, b > 1$ be integers such that for all $n > 0$ we have $a^n - 1 | b^n - 1$. Then prove that b is a natural power of a.

1054. Find all primes p and q such that $p^2 + 1 | 2003^q + 1$ and $q^2 + 1 | 2003^p + 1$.

1055. Are there arbitrarily long sequences of positive integers no two of which have the same number of divisors?

1056. Let $a_1, \ldots, a_{2004}$ be integers with the property that for all $n \geq 1$ the number $(a_1)^n + (a_2)^n + \cdots + (a_{2004})^n$ is the square of an integer. Prove that at least 21 numbers are 0.

1057. Find the smallest number which is the sum of 2002 numbers (not necessarily different) with the same sum of digits and also the sum of 2003 numbers with the same sum of digits.

1058. Prove that an infinite number of natural numbers cannot be written as $a^3 + b^5 + c^7 + d^9 + e^{11}$, with a, b, c, d, e positive integers.

1059. Is there an infinite sequence of prime numbers $p_1, p_2, \ldots, p_n, p_{n+1}, \ldots$ such that $|p_{n+1} - 2p_n| = 1$ for each $n \in \mathbb{N}$?

1060. Find all polynomials f with integer coefficients such that for any natural number n we have $f(n) | 2^n - 1$.

1.9 April

1.9.1 April - Part 1

1061. Let a and b be distinct integers greater than 1. Prove that there exists a positive integer n such that $(a^n - 1)(b^n - 1)$ is not a perfect square.

1062. Let k be a positive integer. Show that if there exists a consequence $a_0, a_1, \ldots$ of integers satisfying the condition

$$a_n = \frac{a_{n-1} + n^k}{n} \text{ for all } n \geq 1,$$

then $k - 2$ is divisible by 3.

1063. Let $P(x)$ be a non-constant polynomial with integer coefficients. Prove that there is no function T from the set of integers into the set of integers such that the number of integers x with $T^n(x) = x$ is equal to $P(n)$ for every $n \geq 1$, where T^n denotes the n-fold application of T.

1064. Find all positive integers n such that there exists a sequence of positive integers $a_1, a_2, \ldots, a_n$ satisfying:

$$a_{k+1} = \frac{a_k^2 + 1}{a_{k-1} + 1} - 1$$

for every k with $2 \leq k \leq n - 1$.

1065. Let f be a non-constant function from the set of positive integers into the set of positive integer, such that $a - b$ divides $f(a) - f(b)$ for all distinct positive integers a, b. Prove that there exist infinitely many primes p such that p divides $f(c)$ for some positive integer c.

1066. A positive integer N is called *balanced*, if $N = 1$ or if N can be written as a product of an even number of not necessarily distinct primes. Given positive integers a and b, consider the polynomial P defined by $P(x) = (x + a)(x + b)$.
(a) Prove that there exist distinct positive integers a and b such that all the number $P(1), P(2), \ldots, P(50)$ are balanced.
(b) Prove that if $P(n)$ is balanced for all positive integers n, then $a = b$.

1067. **(a)** Do there exist 2009 distinct positive integers such that their sum is divisible by each of the given numbers?
(b) Do there exist 2009 distinct positive integers such that their sum is divisible by the sum of any two of the given numbers?

1068. For a given prime $p > 2$ and positive integer k let

$$S_k = 1^k + 2^k + \ldots + (p-1)^k$$

Find those values of k for which $p \mid S_k$.

1069. For every $n \in \mathbb{N}$ let $d(n)$ denote the number of (positive) divisors of n. Find all functions $f : \mathbb{N} \to \mathbb{N}$ with the following properties:

- $d(f(x)) = x$ for all $x \in \mathbb{N}$.
- $f(xy)$ divides $(x-1)y^{xy-1}f(x)$ for all $x, y \in \mathbb{N}$.[/list]

1070. Let n be a positive integer. Show that the numbers

$$\binom{2^n - 1}{0}, \binom{2^n - 1}{1}, \binom{2^n - 1}{2}, \ldots, \binom{2^n - 1}{2^{n-1} - 1}$$

are congruent modulo 2^n to 1, 3, 5, ..., $2^n - 1$ in some order.

1071. Let $a_0, a_1, a_2, \ldots$ be a sequence of positive integers such that the greatest common divisor of any two consecutive terms is greater than the preceding term; in symbols, $\gcd(a_i, a_{i+1}) > a_{i-1}$. Prove that $a_n \geq 2^n$ for all $n \geq 0$.

1072. Let $a_1, a_2, \ldots, a_n$ be distinct positive integers, $n \geq 3$. Prove that there exist distinct indices i and j such that $a_i + a_j$ does not divide any of the numbers $3a_1, 3a_2, \ldots, 3a_n$.

1073. Let n be a positive integer and let p be a prime number. Prove that if a, b, c are integers (not necessarily positive) satisfying the equations

$$a^n + pb = b^n + pc = c^n + pa$$

then $a = b = c$.

1074. Given strictly increasing sequence $a_1 < a_2 < \ldots$ of positive integers such that each its term a_k is divisible either by 1005 or 1006, but neither term is divisible by 97. Find the least possible value of maximal difference of consecutive terms $a_{i+1} - a_i$.

1075. Given are positive integers $n > 1$ and a so that $a > n^2$, and among the integers $a+1, a+2, \ldots, a+n$ one can find a multiple of each of the numbers $n^2+1, n^2+2, \ldots, n^2+n$. Prove that $a > n^4 - n^3$.

1076. Let x, y be two integers with $2 \leq x, y \leq 100$. Prove that $x^{2^n} + y^{2^n}$ is not a prime for some positive integer n.

1077. Prove that there exist infinitely many pairs (a, b) of natural numbers not equal to 1 such that $b^b + a$ is divisible by $a^a + b$.

1078. Find all pairs (x, y) of integers such that $x^3 - y^3 = 2xy + 8$.

1079. Let a, b and m be positive integers. Prove that there exists a positive integer n such that $(a^n - 1)b$ is divisible by m if and only if $\gcd(ab, m) = \gcd(b, m)$.

1080. Which positive integers are missing in the sequence $\{a_n\}$, with $a_n = n + [\sqrt{n}] + [\sqrt[3]{n}]$ for all $n \geq 1$? ($[x]$ denotes the largest integer less than or equal to x, i.e. g with $g \leq x < g + 1$.)

1.9.2 April - Part 2

1081. **(a)** Does there exist a polynomial $P(x)$ with coefficients in integers, such that $P(d) = \frac{2008}{d}$ holds for all positive divisors of 2008?
(b) For which positive integers n does a polynomial $P(x)$ with coefficients in integers exists, such that $P(d) = \frac{n}{d}$ holds for all positive divisors of n?

1082. Are there integers x, y, not both divisible by 5, such that $x^2 + 19y^2 = 198 \cdot 10^{1989}$?

1083. The Fibonacci sequence is defined by $F_1 = F_2 = 1$ and $F_{n+1} = F_n + F_{n-1}$ for $n > 1$. Let $f(x) = 1985x^2 + 1956x + 1960$. Prove that there exist infinitely many natural numbers n for which $f(F_n)$ is divisible by 1989. Does there exist n for which $f(F_n) + 2$ is divisible by 1989?

1084. Let n be a positive integer, let A be a subset of $\{1, 2, \cdots, n\}$, satisfying for any two numbers $x, y \in A$, the least common multiple of x, y not more than n. Show that $|A| \leq 1.9\sqrt{n} + 5$.

1085. A national number x is called *good* if it satisfies: $x = \frac{p}{q} > 1$ with p, q being positive integers, $\gcd(p, q) = 1$ and there exists constant numbers α, N such that for any integer $n \geq N$,

$$|\{x^n\} - \alpha| \leq \frac{1}{2(p+q)}$$

Find all the good numbers.

1086. A positive integer m is called *good* if there is a positive integer n such that m is the quotient of n by the number of positive integer divisors of n (including 1 and n itself). Prove that $1, 2, \ldots, 17$ are good numbers and that 18 is not a good number.

1087. Let p and q be two coprime positive integers, and n be a non-negative integer. Determine the number of integers that can be written in the form $ip + jq$, where i and j are non-negative integers with $i + j \leq n$.

1088. Assume that α and β are two roots of the equation: $x^2 - x - 1 = 0$. Let $a_n = \frac{\alpha^n - \beta^n}{\alpha - \beta}$, $n = 1, 2, \cdots$.
(1) Prove that for any positive integer n, we have $a_{n+2} = a_{n+1} + a_n$.
(2) Find all positive integers a and b, $a < b$, satisfying $b \mid a_n - 2na^n$ for any positive integer n.

1089. Show that the equation $y^{37} = x^3 + 11 \pmod p$ is solvable for every prime p, where $p \leq 100$.

1090. Find all finite sets of positive integers with at least two elements such that for any two numbers a, b ($a > b$) belonging to the set, the number $\frac{b^2}{a-b}$ belongs to the set, too.

1091. How many pairs (m, n) of positive integers with $m < n$ fulfill the equation $\frac{3}{2008} = \frac{1}{m} + \frac{1}{n}$?

1092. Consider a set A of positive integers such that the least element of A equals 1001 and the product of all elements of A is a perfect square. What is the least possible value of the greatest element of A?

1093. For a positive integer n, let $S(n)$ denote the sum of its digits. Find the largest possible value of the expression $\frac{S(n)}{S(16n)}$.

1094. Consider a subset A of 84 elements of the set $\{1, 2, \ldots, 169\}$ such that no two elements in the set add up to 169. Show that A contains a perfect square.

1095. Is there an integer N such that $\left(\sqrt{1997} - \sqrt{1996}\right)^{1998} = \sqrt{N} - \sqrt{N-1}$?

1096. Let the sum of the first n primes be denoted by S_n. Prove that for any positive integer n, there exists a perfect square between S_n and S_{n+1}.

1097. Let m and n be two distinct positive integers. Prove that there exists a real number x such that $\frac{1}{3} \leq \{xn\} \leq \frac{2}{3}$ and $\frac{1}{3} \leq \{xm\} \leq \frac{2}{3}$. Here, for any real number y, $\{y\}$ denotes the fractional part of y. For example $\{3.1415\} = 0.1415$.

1098. If natural numbers x, y, p, n, k with $n > 1$ odd and p an odd prime satisfy $x^n + y^n = p^k$, prove that n is a power of p.

1099. Let $S(n)$ be the sum of decimal digits of a natural number n. Find the least value of $S(m)$ if m is an integral multiple of 2003.

1100. Consider the polynomial $P(x) = x^3 + 153x^2 - 111x + 38$.
(a) Prove that there are at least nine integers a in the interval $[1, 3^{2000}]$ for which $P(a)$ is divisible by 3^{2000}.
(b) Find the number of integers a in $[1, 3^{2000}]$ with the property from (a).

1.9.3 April - Part 3

1101. Let T be a finite set of positive integers, satisfying the following conditions:

- For any two elements of T, their greatest common divisor and their least common multiple are also elements of T.
- For any element x of T, there exists an element x' of T such that x and x' are relatively prime, and their least common multiple is the largest number in T.

For each such set T, denote by $s(T)$ its number of elements. It is known that $s(T) < 1990$; find the largest value $s(T)$ may take.

1102. The function $f : \mathbb{N} \to \mathbb{Z}$ is defined by $f(0) = 2$, $f(1) = 503$ and $f(n+2) = 503f(n+1) - 1996f(n)$ for all $n \in \mathbb{N}$. Let $s_1, s_2, \ldots, s_k$ be arbitrary integers not smaller than k, and let $p(s_i)$ be an arbitrary prime divisor of $f\left(2^{s_i}\right)$, $(i = 1, 2, \ldots, k)$. Prove that, for any positive integer t $(t \leq k)$, we have $2^t \Big| \sum_{i=1}^{k} p(s_i)$ if and only if $2^t | k$.

1103. Find the greatest real number α for which there exists a sequence of infinitive integers (a_n), $(n = 1, 2, 3, \ldots)$ satisfying the following conditions:
1) $a_n > 1997n$ for every $n \in \mathbb{N}^*$;
2) For every $n \geq 2$, $U_n \geq a_n^{\alpha}$, where $U_n = \gcd\{a_i + a_k | i + k = n\}$.

1104. For any nonnegative integer n, let $f(n)$ be the greatest integer such that $2^{f(n)} | n+1$. A pair (n, p) of nonnegative integers is called nice if $2^{f(n)} > p$. Find all triples (n, p, q) of nonnegative integers such that the pairs (n, p), (p, q) and $(n + p + q, n)$ are all nice.

1105. Find all integers a, b, n greater than 1 which satisfy

$$\left(a^3 + b^3\right)^n = 4(ab)^{1995}.$$

1106. For which natural numbers n do there exist rational numbers a and b which are not integers such that both $a + b$ and $a^n + b^n$ are integers?

1107. Determine all functions f defined on the natural numbers that take values among the natural numbers for which

$$(f(n))^p \equiv n \pmod{f(p)}$$

for all $n \in \mathbb{N}$ and all prime numbers p.

1108. Find all triples of positive integers (a, b, c) such that
(i) a, b, c are prime numbers, and $a < b < c$.
(ii) $a + 1$, $b + 1$, $c + 1$ are three consecutive terms of a geometric progression.

1109. The inradius of triangle ABC is 1 and the side lengths of ABC are all integers. Prove that triangle ABC is right-angled.

1110. Let n be a positive integer and $[\ n] = a$. Find the largest integer n such that the following two conditions are satisfied:
(1) n is not a perfect square;
(2) a^3 divides n^2.

1111. Find all integer solutions of equation: $x^4 + 5y^4 = z^4$.

1112. Determine all positive integers x, y such that $y \,|\, x^2 + 1$ and $x^2 \,|\, y^3 + 1$.

1113. Find all pairs (a, b) of positive integers such that $2a - 1$ and $2b + 1$ are coprime and $a + b$ divides $4ab + 1$.

1114. For an integer n, denote $A = \sqrt{n^2 + 24}$ and $B = \sqrt{n^2 - 9}$. Find all values of n for which $A - B$ is an integer.

1115. If n is is an integer such that $4n + 3$ is divisible by 11, find the from of n and the remainder of n^4 upon division by 11.

1116. Let given two primes p, q satisfying: $p - 1$ divisible by q and $q^3 - 1$ divisible by p. Prove that: $p + q$ is a square number.

1117. Find all positive integers N for which, there exist positive pairwise coprime integers a, b, c such that $S(ab) = S(ac) = S(bc) = N$. Where $S(k)$ denotes sum of digits in decimal representation of k.

1118. Show that, for any fixed integer $n \geq 1$, the sequence

$$2,\ 2^2,\ 2^{2^2},\ 2^{2^{2^2}}, \ldots \pmod{n}$$

is eventually constant.

1119. Find all positive integers a, b, c such that $a^{b^c} = b^{c^a}$.

1120. Find all positive integers n, k such that $(n-1)! = n^k - 1$.

1.10 Arne

1.10.1 Arne - Part 1

1121. Does there exist a multiple of 103, say n, such that $2^{2n+1} \equiv 2 \pmod{n}$?

1122. Let a be a positive integer. Set $n = 2^{2a+1} - 1$ and let $\zeta = \exp\left(\frac{2\pi i}{n}\right)$.
Let S be the smallest subset of $\mathbb{C}$ such that
-if $s \in S$, then $s^2 \in S$, and
-for all $k \in \{1, 2, \cdots, 2^a\}$, $\zeta^k \in S$.
Show that S and $\bar{S}$ (the set of conjugates of elements of S) are disjoint.

1123. Let p be an odd prime. Let A_p and B_p be the sets of quadratic residues and non-residues respectively (modulo p). Let ζ be a primitive p-th root of unity and define $a = \sum_{k \in A_p} \zeta^k$ and $b = \sum_{k \in B_p} \zeta^k$. Show that $1 - 4ab = p \cdot \left(\frac{-1}{p}\right)$.

1124. Let $p(X) \in \mathbb{Z}_p[X]$ be an odd polynomial, where p is prime and $p \equiv 3 \pmod{4}$.

How many solutions (X, Y) does the equation $Y^2 = p(X)$ have in $\mathbb{Z}_p^2$?

1125. Let p be an odd prime. Calculate $\sum_{k=1}^{p-2} \left(\frac{k(k+1)}{p}\right)$.

1126. Show that there is a unique sequence $u_0, u_1, u_2, \cdots$ of positive integers such that

$$u_n^2 = \sum_{r=0}^{n} \binom{n+r}{r} u_{n-r}$$

for all nonnegative integers n.

1127. For each positive integer $n \geq 2$, let $\rho(n)$ be the product of the distinct prime divisors of n. Let $a_1 \geq 2$ be an integer and define $a_{n+1} = a_n + \rho(a_n)$. Show that the sequence (a_n) contains arbitrarily long arithmetic progressions.

1128. For any positive integer n, define a_n to be the product of the digits of n.
(a) Prove that $n \geq a(n)$ for all positive integers n.
(b) Find all n for which $n^2 - 17n + 56 = a(n)$.

1129. Find all positive integers m and n such that $1 + 5 \cdot 2^m = n^2$.

1130. Let n be a positive integer. Prove that some Fibonacci number is divisible by n^2+1.

1131. Let p be prime, let n be a positive integer, show that

$$\gcd\left(\binom{p-1}{n-1}, \binom{p+1}{n}, \binom{p}{n+1}\right) = \gcd\left(\binom{p}{n-1}, \binom{p-1}{n}, \binom{p+1}{n+1}\right).$$

1132. Let x and y be integers such that $\frac{x^2+y^2+6}{xy}$ is an integer. Show that $\frac{x^2+y^2+6}{xy}$ is a perfect cube.

1133. Let $s<1$ be a positive real number. Show that there exist only finitely many positive integers n such that $\varphi(n) < n^s$.

Hence, or otherwise, study the convergence of the series $\sum_{n=1}^{\infty} \frac{1}{\varphi(n)^a}$.

1134. Let p be a prime number.

Find the number of irreducible polynomials over $\mathbb{Z}_p$, of the form X^2+aX+b.

1135. Let n be a positive integer, let p be prime and let q be a divisor of $(n+1)^p - n^p$. Show that p divides $q-1$.

1136. Let $x \in \mathbb{R}$. Prove that x is irrational if and only if there exist sequences $\{p_n\}$ and $\{q_n\}$ of positive integers such that

$$\lim_{n\to+\infty} (q_n x - p_n) = 0$$

and such that

$$x \neq \frac{p_n}{q_n},\ \forall n \in \mathbb{N}.$$

1137. Prove that the sum of the squares of two consecutive positive integers cannot be equal to a sum of the fourth powers of two consecutive positive integers.

1138. The first four digits of a certain positive integer n are 1137. Prove that the digits of n can be shuffled in such a way that the new number is divisible by 7.

1139. Find all rational numbers a, b, c and d such that

$$8a^2 - 3b^2 + 5c^2 + 16d^2 - 10ab + 42cd + 18a + 22b - 2c - 54d = 42,$$

$$15a^2 - 3b^2 + 21c^2 - 5d^2 + 4ab + 32cd - 28a + 14b - 54c - 52d = -22.$$

1140. Find all natural numbers with the property that, when the first digit is moved to the end, the resulting number is $\frac{7}{2}$ times the original one.

1.10.2 Arne - Part 2

1141. Define the sequence $\{u_n\}_{n\geq 1}$ by

$$u_1 = 1, \quad u_n = (n-1)u_{n-1} + 1.$$

Find all positive integers n such that n divides u_n.

1142. For a certain real number x, the differences between x^{1919}, x^{1960} and x^{2001} are all integers. Prove that x is an integer.

1143. A number x_n of the form 10101...1 has n ones. Find all n such that x_n is prime.

1144. Let S be the set of all rational numbers whose denominators are powers of 3. Let a, b and c be given non-zero real numbers. Determine all real-valued functions f that are defined for $x \in S$, satisfy

$$f(x) = af(3x) + bf(3x-1) + cf(3x-2) \text{ if } 0 \leq x \leq 1,$$

and are zero elsewhere.

1145. The sequence $L_1,\ L_2,\ L_3,\ \ldots$ is defined by

$$L_1 = 1, \quad L_2 = 3, \quad L_n = L_{n-1} + L_{n-2} \text{ for } n > 2.$$

Prove that $L_p - 1$ is divisible by p if p is prime.

1146. Prove that

$$\gcd\left(\binom{n}{1}, \binom{n}{2}, \ldots, \binom{n}{n-1}\right)$$

is a prime if n is a power of a prime, and 1 otherwise.

1147. Let f_n be the nth Fibonacci number ($f_0 = 0,\ f_1 = f_2 = 1,\ f_3 = 2$, etc).
Find all positive integers n such that

$$nf_nf_{n+1} = (f_{n+2} - 1)^2.$$

1148. A calculator can perform two different operations:
a) transform a given number x into $2x - 1$,
b) multiply a given number by 2.
Is it possible to transform 2005 into the fifth power of an integer using this calculator?

1149. Consider the sequence $1, 2, 4, 5, 7, 9, 10, 12, 14, 16, 17, \ldots$ obtained by concatenating alternating blocks $\{1\}, \{2, 4\}, \{5, 7, 9\}, \{10, 12, 14, 16\}$, etc of odd and even numbers. Each block contains one more element than the previous one and the first element in each block is one more than the last element of the previous block. Show that the nth term of the sequence equals

$$2n - \left[\frac{1 + \sqrt{8n-7}}{2}\right].$$

1150. Suppose that some positive integer can be written as the sum of 4 odd distinct squares. Prove that it can be written as the sum of 4 even distinct perfect squares!

1151. Given a natural number, one may repeatedly
a) multiply it by any other natural number, and
b) delete zeros in its decimal representations.
Prove that, starting with any natural number, one can perform a sequence of these operations that transforms this number into a one-digit number.

1152. The lengths of the altitudes of a triangle are positive integers, and the inradius is a prime number.

Find the lengths of the sides of the triangle.

1153. Show that if the difference of two consecutive cubes is a square, then it is the square of the sum of two successive squares.

1154. The sequence $1, 3, 4, 9, 10, 12, 13, \ldots$ is the increasing sequence of all positive integers which are powers of 3 or can be written as the sum of distinct powers of 3. Determine the 2005^{th} term of the sequence.

1155. Find all possible integer values of the expression

$$\frac{x}{y^2z^2} + \frac{y}{z^2x^2} + \frac{z}{x^2y^2}$$

where x, y and z are integers.

1156. Prove that there exist infinitely many integer solutions to the equation

$$(a-b)(b-c)(c-a) = a^2 + b^2 + c^2.$$

1157. Suppose coprime positive integers a, b, c satisfy $a^2 - ab + b^2 = c^2$. Prove that if d is a divisor of c, then $d \equiv 1 \pmod 6$.

1158. Solve in positive integers

$$2^x = 5^y + 3.$$

1159. Solve $2^x = 3^y + 5$ in positive integers?

1160. Find all primes p, q such that pq divides $2^p + 2^q$.

1.11 Kunihiko

1.11.1 Kunihiko - Part 1

1161. Let $p \geq 5$ be a prime. Is $\cos \frac{2\pi}{p} \in \mathbb{Q}$?

1162. Find all possible positive integers n such that $n! + 1$ is a perfect square.

1163. Find all possible positive integers n such that $3^n + 4^n$ divides 5^n.

1164. Let $m \geq 2,\ n$ be positive integers.
Find all possible pairs $(m,\ n)$ such that $(n+1)!$ is divisible by m^n.

1165. Let n be positive integer. Denote by $N(n),\ S(n)$ be the number and sum of $n!$ respectively. Is there n finitely or are there n infinitely such that $S(n)$ is divisible by $N(n)$?

1166. Let p be prime. Consider a triplet of $(m,\ n,\ p)$ of positive integers such that $n + p^p = m^2$ Find the smallest n satisfying the condition.

1167. Answer the following questions:
(1) Let $k \geq 0$ be integer. Denote by a_k the number of the pairs of non-negative integers $(x,\ y)$ such that $\frac{x}{3} + \frac{y}{2} \leq k$. Express a_k in terms of k.
(2) Let $n \geq 0$ be integer. Denote by b_n the number of the triplets of non-negative integers $(x,\ y,\ z)$ such that $\frac{x}{3} + \frac{y}{2} + z \leq n$. Express b_n in terms of n.

1168. Let $n \geq 2$ be integer. Call a number x as *n-powered number* such that $x = p^n$ for a positive integer p. Answer the following questions:
(1) Prove that the product of two consecutive positive integers is not *n-powered number*.
(2) Prove that the product of n consecutive positive integers is not *n-powered number*.

1169. Let p be prime. Find all possible integers n such that for all integers x, if $x^n - 1$ is divisible by p, then $x^n - 1$ is divisible by p^2 as well.

1170. Let $p,\ q$ be distinct primes. Prove that $p^{2n} + q^{2n}$ is not a multiple of $p + q$ for each positive integer n.

1171. Find the possible maximum integral value by which, for any integers $a,\ b,\ c$, $(a+b+c)^5 - a^5 - b^5 - c^5$ is divided.

1172. Let S be a set such that $S = \{a^3 + b^3 + c^3 - 3abc | a,\ b,\ c \in \mathbb{Z}\}$.
(1) Find the remainder when the element in S is divided by 9.
(2) Prove that S is closed under multiplication.

1173. Find the remainder when $123^{456^{789}}$ is divided by 11.

1174. Let $a_k \in \mathbb{N}^+ (1 \leq k \leq n)$. Find the maximum value of n such that $\sum_{k=1}^{n} a_k = \frac{n(n+1)}{2},\ \prod_{k=1}^{n} a_k = n!$ implies $\{a_1,\ a_2,\ \cdots,\ a_n\} = \{1,\ 2,\ \cdots,\ a_n\}$.

1175. How many pairs of integers $(a,\ b)$ are there for $a^2b^2 = 4a^5 + b^3$?

1176. Find all positive integers n such that n divides $2^{n-1} + 1$.

1177. Find all prime numbers such that $(p^2 - 1)!$ is divisible by p^4, but is not divisible by p^5.

1178. Let $A,\ B,\ C$ be the sets of positive integers respectively. Suppose that for any elements $a \in A,\ b \in B,\ a+b \in C$. Prove the following contexts.
(1) If all elements of C are odd, then $A \cap B = \phi$.
(2) If all elements of C are even and $A \cap B = \phi$, then for any elements of $m,\ n \in A \cup B$, $m+n$ is even.

1179. Find all of quintuple of positive integers (a, n, p, q, r) such that $a^n - 1 = (a^p - 1)(a^q - 1)(a^r - 1)$.

1180. Prove that $2^{17} - 1$ is prime.

1.11.2 Kunihiko - Part 2

1181. For two odd numbers $a,\ b$, let $m = 11a + b,\ n = 3a + b$. Prove the following contexts (1), (2).
(1) If $d = \gcd(a,\ b)$, then we have $\gcd(m,\ n) = 2d,\ 4d,\ 8d$.
(2) Both of m and n can't be perfect number.

1182. For any two positive integers $m,\ n\ (m \geq 2)$, suppose that n^m is expressed by the sum of n consecutive odd numbers. Express the minimum term in terms of n and m.

1183. Suppose that $n \geq 2$ be integer and $a_1,\ a_2,\ a_3,\ a_4$ satisfy the following condition:

- n and $a_i\ (i = 1,\ 2,\ 3,\ 4)$ are relatively prime.
- $(ka_1)_n + (ka_2)_n + (ka_3)_n + (ka_4)_n = 2n$ for $k = 1,\ 2,\ \cdots,\ n-1$.

Note that $(a)_n$ expresses the divisor when a is divided by n.

Prove that $(a_1)_n,\ (a_2)_n,\ (a_3)_n,\ (a_4)_n$ can be divided into two pair with sum n.

1184. Let $a,\ b$ are relatively prime and a is odd. Define, for positive integer n, $a_n,\ b_n$ such that $(a + b\sqrt{2})^n = a_n + b_n\sqrt{2}$. Prove that a_n is odd and $a_n,\ b_n$ are relatively prime for every n.

1185. Let $k \geq 2$ be integer, $n_1,\ n_2,\ n_3$ be positive integers and $a_1,\ a_2,\ a_3$ be integers from $1,\ 2,\ \cdots,\ k-1$. Let $b_i = a_i \sum_{j=0}^{n_i} k^j\ (i = 1,\ 2,\ 3)$. Find all possible pairs of integers $(n_1,\ n_2,\ n_3)$ such that $b_1 b_2 = b_3$.

1186. Let N be postive integer. Some integers are written in a black board and those satisfy the following conditions.
1. Any numbers written are integers which are from 1 to N.
2. More than one integer which is from 1 to N is written.
3. The sum of numbers written is even.

If we mark X to some numbers written and mark Y to all remaining numbers, then prove that we can set the sum of numbers marked X are equal to that of numbers marked Y.

1187. Find all positive integers n such that $8^n + n$ is divisible by $2^n + n$.

1188. **(1)** Prove that injective continuous mapping $f : \mathbb{R} \longrightarrow \mathbb{R}$ is strictly monotone increasing or strictly monotone decreasing.
(2) Let $S^1 = \{z \in \mathbb{C} | |z| = 1\}$. For a continuous mapping $g : S^1 \longrightarrow S^1$ let denote n as ord(g) when there exists some integer $n \geq 2$ such that $g^n = id$ and $g^l \neq id$ $(\forall l \in \{1, \cdots, n-1\})$ where id expresses identy mapping and call the point x on S^1 with $g(x) = x$ fixed point of g.
(a) If $ord(g) = 2$, prove that g has no fixed point or exactly has 2 fixed point.
(b) If $ord(g) > 3$, prove that g has no fixed point.

1189. For $n \geq 2$, let a_n be an increasing integer sequence such that $\sum_{k=1}^{n} \frac{1}{a_k} = 1$. Prove that a_n has more than one even numbers.

1190. Consider a sequence $c_1, c_2, c_3, \cdots$ such that $c_{n+1} = 8c_n - 7$ $(n = 1, 2, 3, \cdots)$. Find two positive integers of c_1 such that only one prime appeares in the sequence of $c_1, c_2, c_3, \cdots$.

1191. (1) Let $f(x)$ be a polynomial with real coefficients. Prove that the necessary and sufficient condition such that $f(x)$ is integer for any integers x is that $f(0)$ is integer and $f(k) - f(k-1)$ is integer for any integers k.
(2) Find the necessary and sufficient condition such that $ax^2 + bx + c$ is integer for any integers x in terms of a, b, c.

1192. Let $p \geq 3$ be a prime number. Put p points on a circle and write 1 on a point, from the point making proceed one point in clockwise, write 2. Moreover from this point making proceed two points, write 3. Repeating this procedure, from the point written in $p-1$ finally making proceed $p-1$ points, write p Note that we may find the point written in more than two numbers and the point written no number. How many points are there written in some number?

1193. For a positive integer n, denote $\frac{10^n - 1}{9} = \overbrace{111 \cdots 111}^{\text{n's 1}}$ by $\boxed{n}$. For example $\boxed{1} = 1$, $\boxed{2} = 11$, $\boxed{3} = 111$.
(1) Let m be a non negative integer. Prove that $\boxed{3^m}$ is divisible by 3^m and indivisible by 3^{m+1}.
(2) Prove that n is divisible by 27 is the necessary and sufficient condition for which $\boxed{n}$ is divisible by 27.

1194. Let x, y be coprime positive integers such that $xy \neq 1$. If n is positive even, then prove that $x^n + y^n$ is not divisible by $x + y$.

1195. Suppose that n is expressed in the decimal representation : $n = \overline{a_m a_{m-1} \ldots a_1}$,where $a_m \neq 0$.

Find all values of n such that $n = (a_m + 1)(a_{m-1} + 1) \cdot \cdots \cdot (a_1 + 1)$.

1196. Let p be any primes and m be any positive integers. If you select some positive integers nicely, then prove that there exists the part of the series of m consecutive zeros in the decimal representation of p^n.

1197. Find all pairs $(a,\ b)$ of positive integers with $a \geq 2,\ b \geq 2$ such that $a^{a^a} = b^b$.

1198. For postive integers $m,\ n$, Let $f(m,\ n)$ such that $f(m,\ n) = \frac{1}{2}\{(m+n-1)^2 + (m-n+1)\}$.
(1) Find one pair of $(m,\ n)$ such that $f(m,\ n) = 100$.
(2) Let $a,\ b,\ c,\ d$ be integers such that $a^2+b = c^2+d$. If $-a < b \leqq a,\ -c < d \leqq c$ holds, then show that we have $a = c,\ b = d$.
(3) Show that there is one pair of $(m,\ n)$ such that $f(m,\ n) = k$ for any positive integers k.

1199. Can it be existed postive integers n such that there are integers b and non zero integers a_i $(i = 1,\ 2,\ \cdots n)$ for rational numbers r which satisfies $r = b + \sum_{i=1}^{n} \frac{1}{a_i}$?

1200. Let $a,\ b,\ p,\ q$ be positive integers such that $\frac{p^2+q^2}{a} = \frac{pq}{b}$ and $gcd(a,\ b) = 1$.
(1) Show that pq is divisible by b.
(2) Show that $\sqrt{a+2b}$ is positive integer.

1.11.3 Kunihiko - Part 3

1201. Define a sequence $\{a_n\} \in \mathbb{N}$ such that $a_1 = 2,\ a_{n+1} = 2a_n^2 - a_n + 2$ $(n = 1,\ 2,\ \cdots)$. Find the great common divisor of a_{n+2} and a_n.

1202. Let $f(x) = x^3 + x + 1$. Prove that there exists an integer such that 3^n $(n \in \mathbb{N})$ divides only one among $f(1),\ f(2),\ \cdots,\ f(3^n)$.

1203. Let p be prime numbers with three digit such that $p = 100a + 10b + c$ for integers $a,\ b,\ c$. Can the quadratic equation $ax^2 + bx + c = 0$ has integral solution?

1204. Let $p > 3$ be prime and $a,\ b,\ c$ are positive integers. If all of $a+b+c,\ a^2+b^2+c^2,\ a^3+b^3+c^3$ are the multiple of p, prove that all of $a, b,\ c$ are the multiple of p.

1205. For real positive numbers x, the set $A(x)$ is defined by

$$A(x) = \{[nx] \mid n \in \mathbb{N}\},$$

where $[r]$ denotes the greatest integer not exceeding real numbers r.Find all irrational numbers $\alpha > 1$ satisfying the following condition. Condition: If positive real number β satisfies $A(\alpha) \supset A(\beta)$, then $\frac{\beta}{\alpha}$ is integer.

1206. Find the pair of integers (a, b) such that a, b are distinct primes for which the following equation will become integers.

$$\frac{7(a-b) - 24ab(a^{2005} - b^{2005}) + (ab+576)(a^{2006} - b^{2006}) - 24(a^{2007} - b^{2007})}{576 - 24(a+b) + ab}.$$

1207. Find the pairs of integers (m, n) $(m \geq 3,\ n \geq 4)$ for which $\frac{1}{x+y} \leq \frac{1}{mx} + \frac{1}{ny}$ holds for arbiterary positive real numbers x, y.

1208. Find the pair of integer (a, b) such that $0 \leq a,\ 0 \leq b \leq 5$, for which the following system of equation with respect to x, y has integral solution.

$$ax + 3by = 1$$

$$bx + ay = 0$$

1209. Let n be positive integers and let α be real numbers such that $0 < \alpha < 1$. Denote by $N(n)$ the number among $\sqrt{1},\ \sqrt{2},\ \sqrt{3},\ \cdots,\sqrt{n}$ whose decimal part is less than or equal to α.

Prove that $\lim_{n\to\infty} \frac{N(n)}{n} = \alpha$.

1210. For natural numbers m, n such that $GCD(m, n) = 1$, find $GCD(5^m + 7^m,\ 5^n + 7^n)$. Note that $GCD(m, n)$ denotes the Greatest Common devisor of m and n.

1211. Let n be positive integers.For real numbers x, y, z, consider the equation

$$x^n + y^n + z^n = xyz \ \cdots\cdots\cdots\cdots\cdots (*)$$

(1) For $n = 1$, find the pair of positive integers (x, y, z) satifying $(*)$ and $x \leq y \leq z$.
(2) For $n = 3$, prove that there don't exist the pair of positive real numbers satisfying $(*)$.

1212. Determine all integers k for which there exist infinitely the pairs of integers (a, b, c) satisfying the following equation

$$(a^2 - k)(b^2 - k) = c^2 - k.$$

1213. Consider the pair of (x, y, z) satisfying the following condition
$(A):$ x, y, z are positive integers such that $x^2 + y^2 + z^2 = xyz$ and $x \leq y \leq z$.
(1) Find all the pairs (x, y, z) satisfying the condition (A) and $y \leq 3$.
(2) If the pair of (a, b, c) satisfy the condition (A), then prove that there exist z such that the pair of (b, c, z) satisfy the condition (A).
(3) Prove that there are infinitely many pairs of (x, y, z) satisfying the condition (A).

1214. Find all the distinct triplet of positive integers such that the sum of the three numbers are minimized among the pairs in which the sum of any two numbers is square number.Note that the rearranged pairs such as $1, 2, 3$ and $3, 2, 1$ are regarded as the same pair.

1215. Let m, n be non zero integers and a, b be odd numbers.For the point $X(2^m a,\ 2^n b)$ which lie in the first quadrant, denote the rule which make correspond to the integer $m-n$ by (R). If the two points $X(2^m a,\ 2^n b),\ X'(2^{m'} a',\ 2^{n'} b')$ which lie in the first quadrant lie in a line passing through the origin, then prove that the integer $m - n$ equal to $m' - n'$ by coresponding by the rule (R) respectively.

1216. Let $f(x)$ be the polynominal with the coefficient of integer. Prove that if $f(n)$ is prime number for all positive integers n, then $f(x)$ is constant number.

1217. Let m, n be positive integers. Find (m, n) such that $2^m - 3^n = 1$.

1218. Let $n \geq 2$ be an integer and r be a positive integer such that r is not the multiple of n, and let g be the greatest common measure of n and r. Prove that

$$\sum_{i=1}^{n-1} \left\{ \frac{ri}{n} \right\} = \frac{1}{2}(n-g),$$

where $\{x\}$ is the fractional part, that is to say, which means the value by subtracting x from the maximum integer value which is equal or less than x.

1219. Prove the following number is an integer.

$$\sqrt{\frac{2006^n - 2005n - 1}{2006^{n-2} + 2 \cdot 2006^{n-3} + \cdots + (n-2) \cdot 2006 + (n-1)}} \quad (n = 2, 3, \cdots)$$

1220. Let k, l, m, n be non negative integers. Find (k, l, m, n) such that the following equality holds for all x except 0

$$\frac{(x+1)^k}{x^l} - 1 = \frac{(x+1)^m}{x^n}.$$

Chapter 2

Solutions

2.1 Amir Hossein

2.1.1 Amir Hossein - Part 1

1. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2602940

2. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2602931

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2.1.2 Amir Hossein - Part 2

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2.1.5 Amir Hossein - Part 5

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2.1.6 Amir Hossein - Part 6

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2.1.7 Amir Hossein - Part 7

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2.1.8 Amir Hossein - Part 8

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146. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2089878

147. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2087878

148. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2085503

149. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2085462

150. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2085449

151. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2085441

152. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2076523

153. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2070989

154. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2070745

155. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2070729

156. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2070375

157. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2069871

158. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2069759

159. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2069257

160. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2728069

2.1.9 Amir Hossein - Part 9

161. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2066186

162. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2066184

163. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2061559

164. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2061555

165. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2059367

166. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2053600

167. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2052762

168. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2051316

169. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2041758

170. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2041752

171. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2041747

172. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2041738

173. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2040437

174. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2040435

175. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2039366

176. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2039349

177. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2039324

178. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2039309

179. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2037704

180. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2037703

2.1.10 Amir Hossein - Part 10

181. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2034763

182. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2033616

183. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2033594

184. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2031754

185. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2031548

186. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2029537

187. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2029515

188. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2027563

189. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2027347

190. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2026345

191. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2026236

192. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2025366

193. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2024457

194. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2024453

195. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2024448

196. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2023277

197. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2023274

198. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2022556

199. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2022459

200. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2022451

2.1.11 Amir Hossein - Part 11

201. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2022413

202. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2022400

203. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2022396

204. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2019754

205. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2019748

206. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2019731

207. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2019675

208. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2019634

209. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2018336

210. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2018333

211. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2018330

212. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2018322

213. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2018315

214. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2017854

215. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2017849

216. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2017733

217. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2017728

218. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2016281

219. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2016266

220. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2014948

2.1.12 Amir Hossein - Part 12

221. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2014944

222. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2014913

223. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2014850

224. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2014840

225. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2013703

226. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2013701

227. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2013684

228. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2013668

229. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2013598

230. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2012330

231. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2012324

232. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2012322

233. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2012308

234. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2012306

235. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2010145

236. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2010139

237. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2010136

238. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2009127

239. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2009076

240. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2009003

2.1.13 Amir Hossein - Part 13

241. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2008954

242. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2008879

243. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2007867

244. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2007828

245. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2006526

246. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2006525

247. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2006523

248. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2005916

249. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2005749

250. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2005719

251. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2005114

252. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2005112

253. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2005096

254. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2005037

255. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2004841

256. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2004818

257. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2004484

258. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2004483

259. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2004314

260. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2004310

2.1.14 Amir Hossein - Part 14

261. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2004308

262. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2004289

263. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2003226

264. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2002366

265. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2001034

266. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2001027

267. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2001006

268. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2000981

269. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2000912

270. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2000897

271. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2000225

272. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2000148

273. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1999752

274. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1999743

275. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1999740

276. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1998819

277. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1998622

278. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1998615

279. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1998582

280. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1998579

2.1.15 Amir Hossein - Part 15

281. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1995405

282. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1994674

283. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1994272

284. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1994075

285. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1993139

286. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1992185

287. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1992148

288. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1992146

289. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1990913

290. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1990911

291. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1990863

292. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1989585

293. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1989424

294. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1989414

295. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1989380

296. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1987092

297. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1986978

298. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1986784

299. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1986782

300. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1983921

2.1.16 Amir Hossein - Part 16

301. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1980119

302. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1978670

303. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1977433

304. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1977297

305. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1974672

306. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1974190

307. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1968098

308. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1963013

309. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1963010

310. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1963008

311. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1962975

312. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1962971

313. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1962968

314. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1962941

315. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1962939

316. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1962933

317. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1962924

318. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1962914

319. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1962913

320. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1962900

2.1.17 Amir Hossein - Part 17

321. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1962893

322. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1962889

323. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1962883

324. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1962879

325. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1962871

326. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1962870

327. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1962565

328. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1956993

329. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1947046

330. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1946919

331. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1946896

332. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1946616

333. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1946610

334. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1945021

335. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1945017

336. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1939609

337. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1937275

338. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1926230

339. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1926226

340. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1926224

2.1.18 Amir Hossein - Part 18

341. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1896892

342. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1890016

343. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1888075

344. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1888071

345. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1883586

346. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1883583

347. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1882422

348. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1882413

349. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1879852

350. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1879847

351. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1878721

352. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1878122

353. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1878120

354. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1874754

355. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1873342

356. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1862069

357. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1859074

358. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1849251

359. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1848582

360. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1848577

2.1.19 Amir Hossein - Part 19

361. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1848570

362. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1848528

363. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1827752

364. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1826878

365. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1809750

366. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1800703

367. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1772072

368. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2729548

369. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2729547

370. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2729546

371. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2728420

372. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2728415

373. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2728413

374. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2728409

375. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2183726

376. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2130981

377. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2731494

378. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2731490

379. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1597397

380. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1482873

2.2 Andrew

2.2.1 Andrew - Part 1

381. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2712168

382. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2712166

383. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2712165

384. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2712164

385. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2712157

386. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2705480

387. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2705217

388. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2699686

389. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2699674

390. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2699669

391. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2699649

392. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2699459

393. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2681670

394. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2677392

395. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2668279

396. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2655216

397. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2620811

398. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2620798

399. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2620775

400. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2617973

2.2.2 Andrew - Part 2

401. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2603114

402. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2603110

403. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2549476

404. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2549461

405. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2549443

406. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2549432

407. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2549429

408. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2498571

409. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2498568

410. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2498566

411. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2498564

412. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2498559

413. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2463300

414. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2463298

415. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2463295

416. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2463291

417. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2463288

418. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2455611

419. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2455509

420. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2455501

2.2.3 Andrew - Part 3

421. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2455485

422. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2455464

423. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2455441

424. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2447949

425. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2439474

426. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2387453

427. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2315676

428. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2315652

429. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2288520

430. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2288476

431. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2284535

432. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2284508

433. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2275406

434. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2275398

435. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2274367

436. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2248506

437. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2210875

438. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2210871

439. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2210863

440. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2180178

2.2.4 Andrew - Part 4

441. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2173660

442. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2173653

443. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2173581

444. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2173579

445. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2173576

446. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2173571

447. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2173565

448. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2173562

449. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2173543

450. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2172861

451. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2172813

452. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2167312

453. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2167294

454. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2167269

455. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2167262

456. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2164532

457. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2163610

458. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2160726

459. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2160721

460. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2160121

2.2.5 Andrew - Part 5

461. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2160108

462. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2160095

463. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2160094

464. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2160090

465. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2160088

466. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2160068

467. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2153615

468. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2153595

469. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2151183

470. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2151124

471. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2151090

472. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2151064

473. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2151054

474. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2148453

475. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2146761

476. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2146719

477. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2143924

478. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2143913

479. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2143893

480. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2143890

2.2.6 Andrew - Part 6

481. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2143864

482. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2143850

483. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2143626

484. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2143623

485. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2143618

486. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2143616

487. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2136115

488. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2136111

489. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2135026

490. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2134967

491. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2130626

492. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2130591

493. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2126731

494. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2126729

495. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2126726

496. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2126723

497. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2126645

498. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2122041

499. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2122025

500. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2122016

2.2.7 Andrew - Part 7

501. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2121995

502. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2118102

503. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2112628

504. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2112595

505. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2112560

506. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2110497

507. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2110492

508. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2107867

509. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2107866

510. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2107862

511. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2107856

512. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2103090

513. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2103089

514. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2103085

515. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2103082

516. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2103079

517. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2099928

518. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2099849

519. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2099680

520. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2099677

2.2.8 Andrew - Part 8

521. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2094765

522. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2093608

523. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2091466

524. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2091464

525. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2091460

526. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2091453

527. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2091450

528. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2089173

529. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2089168

530. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2089162

531. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2089158

532. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2084497

533. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2084491

534. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2084409

535. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2083431

536. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2080872

537. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2080789

538. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2079766

539. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2079709

540. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2079690

2.2.9 Andrew - Part 9

541. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2077705

542. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2077417

543. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2077416

544. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2077407

545. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2077402

546. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2077386

547. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2075476

548. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2070780

549. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2070777

550. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2068595

551. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2068587

552. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2068583

553. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2068577

554. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2068569

555. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2068551

556. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2068539

557. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2068487

558. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2068483

559. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2068365

560. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2068350

2.3 Goutham

2.3.1 Goutham - Part 1

561. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2558862

562. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2558812

563. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2558810

564. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2558809

565. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2558807

566. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2558806

567. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2558802

568. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2558799

569. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2318392

570. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2318369

571. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2317614

572. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2317589

573. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2295880

574. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2295819

575. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2292476

576. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2210092

577. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2210090

578. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2202142

579. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2202137

580. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2173998

2.3.2 Goutham - Part 2

581. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2170864

582. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2170853

583. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2169402

584. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2169391

585. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2169254

586. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2169238

587. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2159996

588. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2136815

589. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2136813

590. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2136807

591. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2136803

592. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2136799

593. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2136792

594. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2136770

595. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2109983

596. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2109978

597. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2109975

598. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2109973

599. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2109972

600. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2109965

2.3.3 Goutham - Part 3

601. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2109961

602. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2072853

603. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2072850

604. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2068101

605. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2068096

606. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2065894

607. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2063428

608. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2062303

609. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2062302

610. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2058004

611. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2056670

612. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2056651

613. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2050074

614. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2049160

615. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2049156

616. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2048109

617. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2048107

618. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2048093

619. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2048074

620. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2048055

2.3.4 Goutham - Part 4

621. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2047664

622. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2046274

623. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2046271

624. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2046262

625. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2044164

626. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2038470

627. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2038430

628. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2038285

629. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2038280

630. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2038272

631. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2038205

632. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2033987

633. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2033981

634. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2033979

635. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2033967

636. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2033966

637. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2033965

638. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2033964

639. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2033957

640. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2033956

2.3.5 Goutham - Part 5

641. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2033953

642. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2033939

643. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2033928

644. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2000692

645. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1997886

646. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1889631

647. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1884417

648. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1884416

649. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1883912

650. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1882479

651. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1881746

652. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1881743

653. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1881292

654. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1880787

655. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1880784

656. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1880782

657. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1880779

658. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1880775

659. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1880770

660. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1880762

2.3.6 Goutham - Part 6

661. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1879623

662. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1879493

663. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1879491

664. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1879490

665. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1879489

666. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1877322

667. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1876473

668. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1875408

669. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1874664

670. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1868939

671. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1857023

672. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1857020

673. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1855383

674. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1853144

675. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1849157

676. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1697137

677. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2639969

678. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=484472

679. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2732915

680. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2732885

2.4 Orlando

2.4.1 Orlando - Part 1

681. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2361875

682. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2361870

683. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2361848

684. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2361782

685. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2361724

686. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2361714

687. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1564940

688. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1564930

689. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1392508

690. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1392054

691. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1371872

692. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1365045

693. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1364660

694. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1359447

695. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1357721

696. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1357699

697. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1353597

698. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1257937

699. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1257930

700. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1257920

2.4.2 Orlando - Part 2

701. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1257919

702. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1257846

703. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1257818

704. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1257814

705. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1257799

706. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1257789

707. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1257772

708. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1251798

709. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1251784

710. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1249366

711. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1247502

712. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1236876

713. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1236769

714. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1236757

715. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1230272

716. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1225290

717. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1225284

718. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1225256

719. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1225251

720. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1225228

2.4.3 Orlando - Part 3

721. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1225217

722. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1225213

723. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1225024

724. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1225021

725. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1225019

726. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1225016

727. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1222819

728. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1222532

729. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1222505

730. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1219674

731. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1219477

732. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1219467

733. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1219050

734. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1218963

735. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1218962

736. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1218613

737. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1218612

738. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1218610

739. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1218606

740. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1195251

2.4.4 Orlando - Part 4

741. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1195234

742. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1192915

743. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1187273

744. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1187222

745. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1187214

746. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1187204

747. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1187198

748. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=490682

749. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=490569

750. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=462343

751. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=462313

752. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=454802

753. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=454615

754. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=372271

755. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=367455

756. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=367432

757. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=367407

758. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=367399

759. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=367368

760. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=367332

2.4.5 Orlando - Part 5

761. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=366676

762. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=366642

763. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=366641

764. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=366630

765. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=366618

766. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=366605

767. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=366589

768. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=366557

769. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=366550

770. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=366466

771. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=366443

772. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=366420

773. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=365174

774. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=365167

775. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=365153

776. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=365142

777. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=365111

778. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=361343

779. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=361332

780. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=361331

2.4.6 Orlando - Part 6

781. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=361317

782. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=361277

783. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=358045

784. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=358034

785. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=358033

786. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=352716

787. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=352683

788. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=352664

789. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=347350

790. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=347335

791. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=347239

792. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=346573

793. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=269097

794. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=269092

795. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=269087

796. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=269083

797. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=269075

798. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=269060

799. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=269053

800. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=268397

2.4.7 Orlando - Part 7

801. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=268383

802. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=268377

803. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=268338

804. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=268337

805. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=268326

806. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=268042

807. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=267989

808. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=267973

809. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=267961

810. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=258123

811. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=258097

812. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=239372

813. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=239205

814. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=239202

815. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=234864

816. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=234853

817. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=234095

818. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=230042

819. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=220242

820. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=220237

2.4.8 Orlando - Part 8

821. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=220231

822. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=220224

823. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=220217

824. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=201585

825. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=166047

826. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=155821

827. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=151421

828. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=151407

829. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=151403

830. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=151400

831. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=151397

832. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=151394

833. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=151389

834. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=151384

835. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=151380

836. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=151377

837. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=151375

838. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=139811

839. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=137476

840. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=137473

2.4.9 Orlando - Part 9

841. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=137427

842. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=137416

843. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=137380

844. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=137341

845. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=137234

846. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=135064

847. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=131825

848. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=131821

849. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=131819

850. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=131815

851. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=131812

852. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=131811

853. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=124444

854. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=124442

855. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=124439

856. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=124436

857. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=124434

858. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=124432

859. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=124430

860. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=124428

2.4.10 Orlando - Part 10

861. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=119994

862. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=119990

863. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=119218

864. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=119217

865. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=119215

866. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=119213

867. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=119211

868. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=119209

869. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=118695

870. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=118694

871. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=118691

872. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=118690

873. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=118687

874. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=118686

875. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=113396

876. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=113388

877. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=113326

878. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=113324

879. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=113322

880. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=19835

2.5 Valentin

2.5.1 Valentin - Part 1

881. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2659580

882. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1115397

883. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1114548

884. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=859305

885. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=848078

886. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=848075

887. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=848062

888. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=848060

889. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=518101

890. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=515081

891. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=508187

892. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=498992

893. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=495869

894. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=495856

895. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=495854

896. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=495437

897. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=495423

898. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=495389

899. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=495384

900. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=495342

2.5.2 Valentin - Part 2

901. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=495321

902. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=495308

903. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=492878

904. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=457341

905. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=419642

906. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=415141

907. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=409056

908. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=365587

909. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=365586

910. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=358042

911. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=358021

912. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=357995

913. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=354115

914. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=338285

915. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=335786

916. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=335760

917. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=334916

918. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=334911

919. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=334405

920. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=334311

2.5.3 Valentin - Part 3

921. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=334308

922. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=214545

923. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=214544

924. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=213014

925. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=213007

926. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=199648

927. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=198931

928. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=198301

929. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=198299

930. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=198166

931. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=150745

932. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=150340

933. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=141617

934. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=141616

935. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=141615

936. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=141613

937. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=141611

938. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=124059

939. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=124049

940. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=18291

2.5.4 Valentin - Part 4

941. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=19650

942. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=19645

943. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=19483

944. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=19278

945. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=19012

946. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=19011

947. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=19008

948. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=18142

949. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=17643

950. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=17641

951. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=8423

952. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2685814

953. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2725814

954. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=101575

955. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2144391

956. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2730996

957. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2726898

958. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2731857

959. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2732867

960. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2734018

2.6 Darij

2.6.1 Darij - Part 1

961. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1455222

962. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=963215

963. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=898341

964. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=718166

965. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=564088

966. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=506222

967. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=506200

968. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=506193

969. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=504804

970. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=481709

971. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=471019

972. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=428906

973. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=404096

974. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=372389

975. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=278829

976. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=248876

977. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=245459

978. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=245455

979. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=234837

980. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2724129

2.6.2 Darij - Part 2

981. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=228589

982. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=227631

983. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=137099

984. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=136652

985. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=136648

986. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=133894

987. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=133871

988. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=133870

989. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=124924

990. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=117538

991. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=117509

992. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=114649

993. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=114648

994. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=114647

995. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=114646

996. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=114645

997. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=114644

998. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=113307

999. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=113306

1000. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=110134

2.7 Vesselin

2.7.1 Vesselin - Part 1

1001. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=842750

1002. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=840396

1003. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=816763

1004. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=725692

1005. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=721076

1006. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=720569

1007. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=719580

1008. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=719074

1009. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=713535

1010. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=675993

1011. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=480179

1012. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=480178

1013. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=276132

1014. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=233135

1015. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=233097

1016. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=169807

1017. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=21087

1018. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=20251

1019. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=19725

1020. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=19724

2.8 Gabriel

2.8.1 Gabriel - Part 1

1021. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1194191

1022. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=894744

1023. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=882202

1024. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=881479

1025. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=879669

1026. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=847885

1027. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=841218

1028. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=831880

1029. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=646611

1030. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=642197

1031. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=633345

1032. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=308903

1033. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=306415

1034. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=296955

1035. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=296105

1036. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=291925

1037. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=291840

1038. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=281962

1039. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=274020

1040. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=273993

2.8.2 Gabriel - Part 2

1041. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=256002

1042. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=238103

1043. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=218646

1044. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=216982

1045. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=177189

1046. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=189091

1047. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=172298

1048. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=171134

1049. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=141088

1050. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=120836

1051. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=98347

1052. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=19845

1053. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=14248

1054. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=8480

1055. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=6066

1056. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=6064

1057. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=5692

1058. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=5239

1059. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=5207

1060. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=5039

2.9 April

2.9.1 April - Part 1

1061. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1932947

1062. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1932946

1063. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1932945

1064. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1932944

1065. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1932942

1066. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1932941

1067. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1602385

1068. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1602380

1069. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1555934

1070. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1555932

1071. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1555931

1072. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1555929

1073. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1555927

1074. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1500068

1075. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1498263

1076. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1495364

1077. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1404078

1078. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1404067

1079. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1404062

1080. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1397387

2.9.2 April - Part 2

1081. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1397371

1082. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1392900

1083. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1392897

1084. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1359755

1085. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1359751

1086. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1358808

1087. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1358641

1088. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1358200

1089. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1353555

1090. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1327937

1091. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1327936

1092. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1327931

1093. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1327926

1094. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1327924

1095. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1306096

1096. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1298779

1097. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1298771

1098. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1295024

1099. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1293971

1100. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1293828

2.9.3 April - Part 3

1101. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1205683

1102. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1204127

1103. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1204113

1104. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1203294

1105. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1203291

1106. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1107306

1107. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1106215

1108. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=901859

1109. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=901828

1110. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=901827

1111. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=899103

1112. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=853339

1113. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=833940

1114. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=816382

1115. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=816377

1116. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=683363

1117. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2694849

1118. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=113561

1119. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2684751

1120. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2545368

2.10 Arne

2.10.1 Arne - Part 1

1121. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1136226

1122. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1136223

1123. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=811555

1124. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=798940

1125. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=798178

1126. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=640472

1127. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=606603

1128. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=504706

1129. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=504698

1130. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=473677

1131. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=471577

1132. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=471520

1133. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=455401

1134. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=376840

1135. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=364980

1136. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=359014

1137. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=347459

1138. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=347454

1139. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=346622

1140. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=343339

2.10.2 Arne - Part 2

1141. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=339396

1142. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=338296

1143. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=337979

1144. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=337940

1145. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=337937

1146. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=337897

1147. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=335380

1148. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=330442

1149. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=329491

1150. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=327845

1151. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=325481

1152. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=318943

1153. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=311208

1154. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=292298

1155. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=272511

1156. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=269998

1157. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=247018

1158. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=171028

1159. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=170948

1160. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=158224

2.11 Kunihiko

2.11.1 Kunihiko - Part 1

1161. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2730352

1162. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2658118

1163. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2658115

1164. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2653579

1165. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2636518

1166. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2630114

1167. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2613179

1168. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2610950

1169. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2597744

1170. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2508387

1171. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2449188

1172. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2446361

1173. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2436341

1174. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2432514

1175. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2386851

1176. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2345559

1177. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2326068

1178. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2201029

1179. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2173260

1180. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=2110423

2.11.2 Kunihiko - Part 2

1181. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1928285

1182. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1756979

1183. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1671105

1184. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1421941

1185. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1414602

1186. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1414593

1187. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1414577

1188. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1383949

1189. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1340905

1190. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1305039

1191. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1182592

1192. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1167797

1193. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1133515

1194. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1058936

1195. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1056318

1196. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1056310

1197. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1042963

1198. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1037475

1199. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1032740

1200. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=1004434

2.11.3 Kunihiko - Part 3

1201. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=974140

1202. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=967994

1203. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=843620

1204. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=825585

1205. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=769825

1206. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=580681

1207. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=558972

1208. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=503070

1209. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=462919

1210. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=460378

1211. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=451706

1212. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=444438

1213. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=439888

1214. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=429744

1215. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=425644

1216. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=406099

1217. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=369280

1218. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=350861

1219. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=335338

1220. http://www.artofproblemsolving.com/Forum/viewtopic.php?p=332194