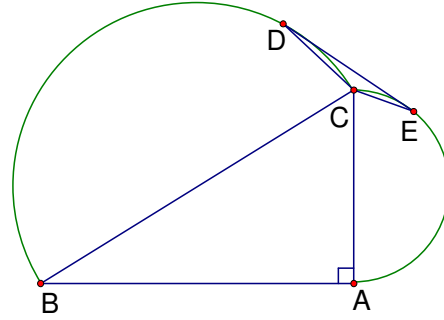


102 北一女二招 計算第四題

如右圖於直角三角形  $ABC$  的斜邊與一股作半圓

， $DE$  為兩圓的公切線， $\angle CBA = 30^\circ$

， $\angle CDE = \alpha$ ，試求  $\cot \alpha = ?$



<解>

令  $\overline{AC} = 1$ ， $\overline{BC} = 2$ ， $\angle CED = \beta$

因為  $\angle CBD = \frac{1}{2}\widehat{CD} = \angle CDE = \alpha$  且  $\angle BDC = 90^\circ$

所以  $\overline{CD} = \overline{BC} \sin \alpha = 2 \sin \alpha$ ，

同理  $\overline{CE} = \overline{AC} \sin \beta = \sin \beta$ ，

又因為  $\angle BCD = 90^\circ - \alpha$ ， $\angle ACE = 90^\circ - \beta \Rightarrow \angle DCE = 360^\circ - (90^\circ - \alpha) - 60^\circ - (90^\circ - \beta) = 120^\circ + \alpha + \beta$

由  $\triangle CDE$  的內角和可知  $(120^\circ + \alpha + \beta) + \alpha + \beta = 180^\circ$ ， $\alpha + \beta = 30^\circ$

故  $\begin{cases} \overline{CD} \sin \alpha = \overline{DE} \text{邊上的高} = \overline{CE} \sin \beta \\ \alpha + \beta = 30^\circ \end{cases}$

$$\begin{cases} 2 \sin^2 \alpha = \sin^2 \beta \\ \sin \beta = \sin(30^\circ - \alpha) = \frac{1}{2} \cos \alpha - \frac{\sqrt{3}}{2} \sin \alpha \end{cases}$$

( $\alpha, \beta$  皆小於  $90^\circ$ )

$$\Rightarrow \sqrt{2} \sin \alpha = \sin \beta = \frac{1}{2} \cos \alpha - \frac{\sqrt{3}}{2} \sin \alpha \Rightarrow \left( \sqrt{2} + \frac{\sqrt{3}}{2} \right) \sin \alpha = \frac{1}{2} \cos \alpha$$

$$\text{故 } \cot \alpha = \frac{\cos \alpha}{\sin \alpha} = 2\sqrt{2} + \sqrt{3}$$

題目由網友 [drexler5422](http://math.pro/db/thread-1590-1-1.html) 提供，網址為 <http://math.pro/db/thread-1590-1-1.html>

解答作者為 [Joy091](#)